Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.1.3-g-tan-^p-a+b-sin-^m

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3.174 $\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)} \, dx$ . . . 856

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3.178 $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$ 875
3.179 $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$ 880
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3.194 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ 962
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3.196 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ 970
3.197 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ 974
3.198 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ 979
3.199 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ 986
3.200 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ 992
3.201 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ 999
3.202 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$ 1006

3.203 $\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx$ 1015
3.204 $\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx$ 1022
3.205 $\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx$ 1028
3.206 $\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$ 1032
3.207 $\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$ 1036
3.208 $\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$ 1040

4 Listing of Grading functions **1043**

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [208]. This is test number [72].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.04 (206)	% 0.96 (2)
Mathematica	% 97.6 (203)	% 2.4 (5)
Maple	% 85.58 (178)	% 14.42 (30)
Maxima	% 68.27 (142)	% 31.73 (66)
Fricas	% 85.58 (178)	% 14.42 (30)
Sympy	% 1.92 (4)	% 98.08 (204)
Giac	% 67.31 (140)	% 32.69 (68)

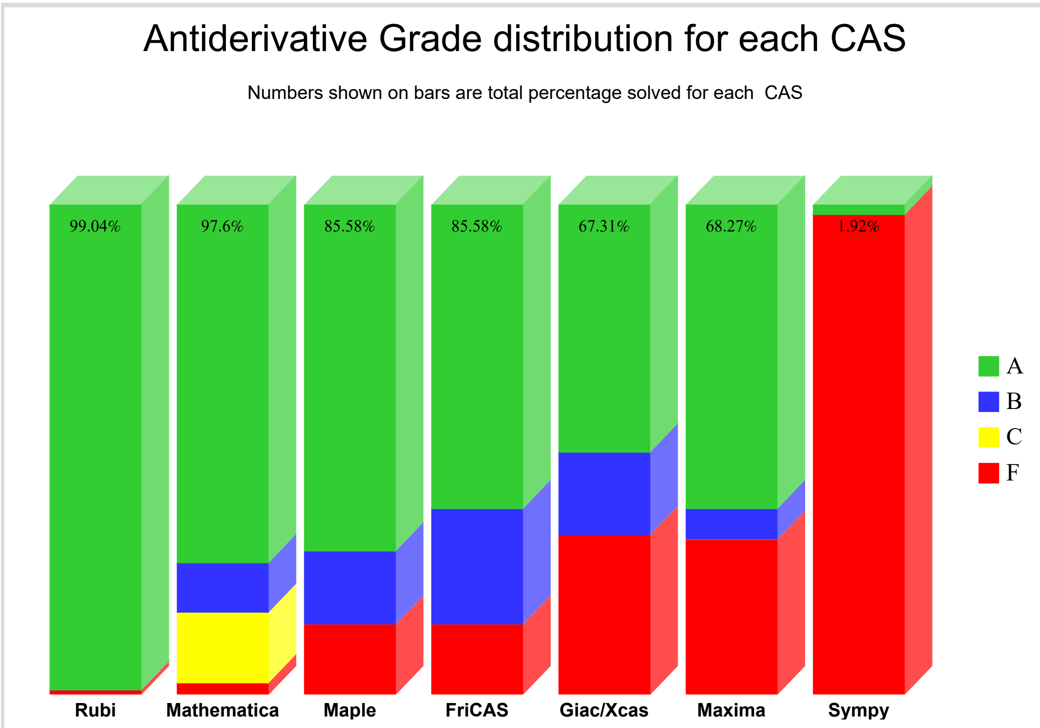
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

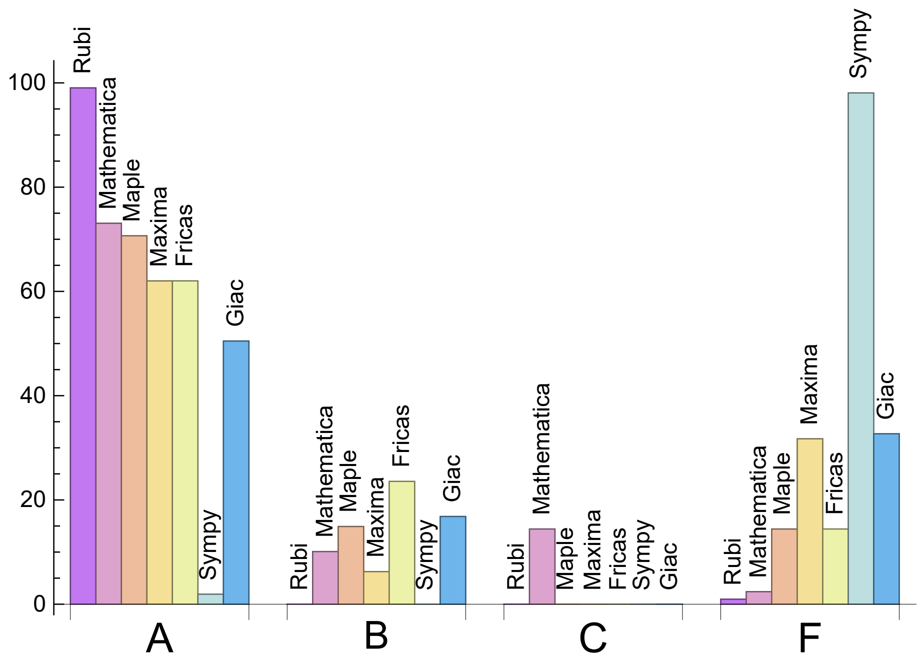
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.04	0.	0.	0.96
Mathematica	73.08	10.1	14.42	2.4
Maple	70.67	14.9	0.	14.42
Maxima	62.02	6.25	0.	31.73
Fricas	62.02	23.56	0.	14.42
Sympy	1.92	0.	0.	98.08
Giac	50.48	16.83	0.	32.69

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.23	133.35	1.01	113.	1.
Mathematica	2.93	548.75	4.84	113.	1.
Maple	0.15	183.64	1.39	136.	1.3
Maxima	1.63	169.07	1.56	128.	1.35
Fricas	1.83	643.02	4.51	411.	3.84
Sympy	1.29	112.5	1.85	99.5	1.83
Giac	3.02	586.31	7.02	198.5	2.

1.4 list of integrals that has no closed form antiderivative

{208}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {110, 114, 115, 117, 118, 123, 124, 126, 127, 128, 129, 136, 138, 158, 168, 179, 190, 201, 203, 204, 205, 206, 207}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

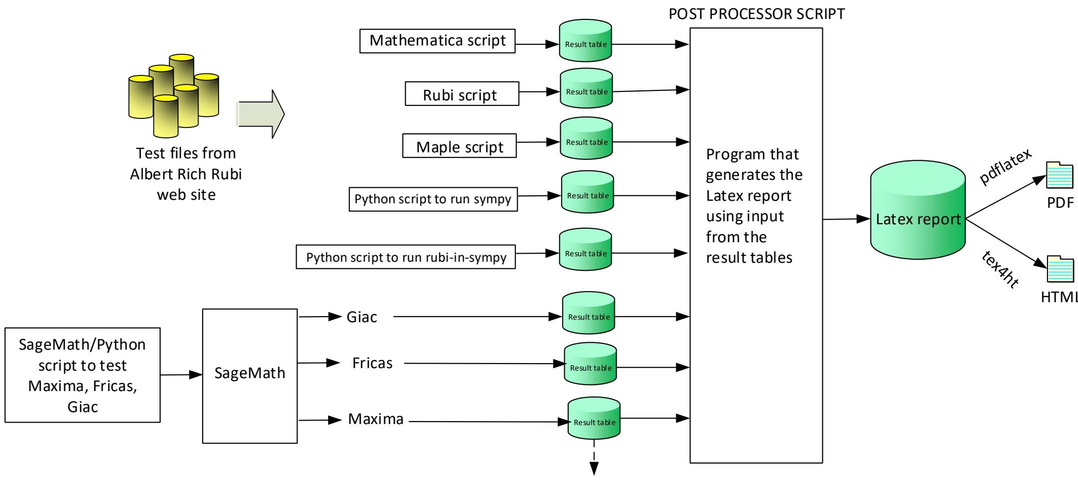
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 208 }

B grade: { }

C grade: { }

F grade: { 206, 207}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 94, 96, 97, 98, 99, 100, 101, 102, 120, 130, 131, 132, 133, 134, 137, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208 }

B grade: { 21, 55, 56, 57, 58, 59, 60, 88, 89, 90, 93, 105, 106, 110, 126, 127, 128, 129, 158, 179, 206 }

C grade: { 11, 12, 13, 91, 92, 95, 103, 104, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 136, 138, 147, 148, 149, 203, 204, 205 }

F grade: { 121, 122, 125, 135, 139}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 19, 20, 21, 22, 23, 27, 28, 31, 32, 33, 36, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 181, 182, 183, 184, 185, 186, 187, 188, 192, 193, 194, 195, 196, 197, 208 }

B grade: { 14, 15, 17, 18, 24, 25, 26, 29, 30, 34, 35, 38, 39, 53, 54, 58, 59, 60, 160, 164, 178, 179, 180, 189, 190, 191, 198, 199, 200, 201, 202 }

C grade: { }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 96, 100, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 182, 183, 184, 185, 186, 193, 194, 195, 196, 197, 208

B grade: { 53, 54, 57, 58, 59, 60, 87, 88, 89, 90, 99, 181, 192 }

C grade: { }

F grade: { 91, 92, 93, 94, 95, 97, 98, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 176, 177, 178, 179, 180, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 15, 16, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 91, 92, 95, 96, 99, 100, 103, 107, 111, 112, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 183, 184, 187, 188, 198, 199, 208 }

B grade: { 10, 11, 12, 13, 17, 24, 30, 57, 58, 59, 60, 74, 84, 88, 89, 90, 93, 94, 97, 98, 101, 102, 104, 105, 106, 108, 109, 110, 113, 114, 147, 148, 149, 181, 182, 185, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202 }

C grade: { }

F grade: { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

2.1.6 Sympy

A grade: { 22, 32, 40, 56 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208 }

2.1.7 Giac

A grade: { 4, 5, 6, 7, 12, 13, 17, 18, 22, 28, 32, 33, 37, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 142, 143, 144, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202, 208 }

B grade: { 3, 10, 11, 16, 21, 23, 24, 42, 53, 57, 58, 59, 60, 93, 94, 97, 98, 101, 102, 103, 104, 105, 106, 109, 110, 114, 141, 146, 147, 151, 156, 167, 168, 181, 193 }

C grade: { }

F grade: { 1, 2, 8, 9, 14, 15, 19, 20, 25, 26, 27, 29, 30, 31, 34, 35, 36, 38, 39, 91, 92, 95, 96, 99, 100, 107, 108, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 145, 150, 155, 160, 161, 165, 166, 203, 204, 205, 206, 207 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	123	147	128	410	0	0
normalized size	1	1.	1.07	1.28	1.11	3.57	0.	0.
time (sec)	N/A	0.072	0.446	0.064	1.049	1.552	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	77	96	69	224	0	0
normalized size	1	1.	1.08	1.35	0.97	3.15	0.	0.
time (sec)	N/A	0.048	0.108	0.056	0.992	1.532	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	38	29	34	65	0	1966
normalized size	1	1.	1.27	0.97	1.13	2.17	0.	65.53
time (sec)	N/A	0.021	0.021	0.031	1.076	1.371	0.	1.611

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	26	25	30	62	0	31
normalized size	1	1.	1.08	1.04	1.25	2.58	0.	1.29
time (sec)	N/A	0.02	0.033	0.016	1.092	1.573	0.	1.268

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	60	83	61	167	0	81
normalized size	1	1.	1.11	1.54	1.13	3.09	0.	1.5
time (sec)	N/A	0.037	0.118	0.037	1.09	1.496	0.	1.325

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	93	292	0	111
normalized size	1	1.	1.07	1.68	1.15	3.6	0.	1.37
time (sec)	N/A	0.048	0.198	0.036	1.081	1.573	0.	1.389

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	111	195	123	412	0	140
normalized size	1	1.	0.97	1.7	1.07	3.58	0.	1.22
time (sec)	N/A	0.059	0.399	0.039	1.153	1.599	0.	1.387

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	110	135	117	300	0	0
normalized size	1	1.	1.09	1.34	1.16	2.97	0.	0.
time (sec)	N/A	0.092	0.065	0.071	1.659	1.467	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	88	224	0	0
normalized size	1	1.	1.12	1.36	1.22	3.11	0.	0.
time (sec)	N/A	0.074	0.048	0.057	1.825	1.476	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	47	59	53	194	0	1361
normalized size	1	1.	1.21	1.51	1.36	4.97	0.	34.9
time (sec)	N/A	0.104	0.045	0.039	1.688	1.421	0.	3.472

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	73	236	0	146
normalized size	1	1.	1.83	1.39	1.78	5.76	0.	3.56
time (sec)	N/A	0.052	0.048	0.029	2.303	1.438	0.	1.25

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	124	425	0	190
normalized size	1	1.	1.52	1.29	1.51	5.18	0.	2.32
time (sec)	N/A	0.08	0.053	0.034	1.691	1.504	0.	1.278

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	164	159	169	620	0	269
normalized size	1	1.	1.34	1.3	1.39	5.08	0.	2.2
time (sec)	N/A	0.096	0.071	0.038	1.63	1.538	0.	1.293

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	75	261	130	412	0	0
normalized size	1	1.	0.63	2.19	1.09	3.46	0.	0.
time (sec)	N/A	0.082	0.245	0.085	1.129	1.557	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	54	162	78	212	0	0
normalized size	1	1.	0.75	2.25	1.08	2.94	0.	0.
time (sec)	N/A	0.062	0.103	0.071	1.112	1.482	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	40	69	58	108	0	9038
normalized size	1	1.	0.77	1.33	1.12	2.08	0.	173.81
time (sec)	N/A	0.038	0.038	0.042	1.125	1.418	0.	4.572

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	94	72	173	0	63
normalized size	1	1.	0.93	3.13	2.4	5.77	0.	2.1
time (sec)	N/A	0.039	0.041	0.049	1.113	1.44	0.	1.371

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	86	313	144	508	0	163
normalized size	1	1.	0.65	2.37	1.09	3.85	0.	1.23
time (sec)	N/A	0.075	0.216	0.049	1.086	1.768	0.	1.496

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	174	251	205	382	0	0
normalized size	1	1.	1.17	1.68	1.38	2.56	0.	0.
time (sec)	N/A	0.165	0.838	0.078	1.602	1.701	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	159	186	162	468	0	0
normalized size	1	1.	1.32	1.55	1.35	3.9	0.	0.
time (sec)	N/A	0.203	1.306	0.071	1.625	1.471	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	145	117	113	296	0	7250
normalized size	1	1.	2.04	1.65	1.59	4.17	0.	102.11
time (sec)	N/A	0.089	0.438	0.049	1.615	1.537	0.	64.982

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	52	63	97	78	51
normalized size	1	1.	0.76	1.16	1.4	2.16	1.73	1.13
time (sec)	N/A	0.014	0.193	0.023	1.085	1.558	0.697	1.853

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	94	89	107	281	0	193
normalized size	1	1.	1.27	1.2	1.45	3.8	0.	2.61
time (sec)	N/A	0.102	0.58	0.036	1.64	1.524	0.	1.732

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	191	190	188	474	0	282
normalized size	1	1.	1.95	1.94	1.92	4.84	0.	2.88
time (sec)	N/A	0.161	5.581	0.044	1.647	1.703	0.	1.762

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	99	445	180	594	0	0
normalized size	1	1.	0.62	2.78	1.12	3.71	0.	0.
time (sec)	N/A	0.109	0.564	0.105	1.077	1.575	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	66	205	97	248	0	0
normalized size	1	1.	0.73	2.25	1.07	2.73	0.	0.
time (sec)	N/A	0.071	0.157	0.078	1.071	1.599	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	52	85	77	147	0	0
normalized size	1	1.	0.74	1.21	1.1	2.1	0.	0.
time (sec)	N/A	0.045	0.045	0.051	1.071	1.516	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	67	109	108	284	0	127
normalized size	1	1.	0.68	1.11	1.1	2.9	0.	1.3
time (sec)	N/A	0.065	0.202	0.053	1.06	1.483	0.	1.372

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	243	359	282	734	0	0
normalized size	1	1.	1.35	1.99	1.57	4.08	0.	0.
time (sec)	N/A	0.357	5.614	0.131	1.648	1.598	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	177	266	223	539	0	0
normalized size	1	1.	1.49	2.24	1.87	4.53	0.	0.
time (sec)	N/A	0.194	2.25	0.085	1.612	1.583	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	115	167	158	378	0	0
normalized size	1	1.	1.29	1.88	1.78	4.25	0.	0.
time (sec)	N/A	0.125	0.49	0.053	1.57	1.42	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	74	97	134	121	74
normalized size	1	1.	0.7	1.17	1.54	2.13	1.92	1.17
time (sec)	N/A	0.054	0.326	0.027	1.074	1.458	0.655	1.64

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	106	105	126	321	0	219
normalized size	1	1.	1.15	1.14	1.37	3.49	0.	2.38
time (sec)	N/A	0.137	1.065	0.043	1.637	1.566	0.	2.059

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	83	387	147	394	0	0
normalized size	1	1.	0.64	3.	1.14	3.05	0.	0.
time (sec)	N/A	0.095	0.469	0.1	1.107	1.672	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	76	245	115	293	0	0
normalized size	1	1.	0.71	2.29	1.07	2.74	0.	0.
time (sec)	N/A	0.079	0.155	0.084	1.065	1.532	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	62	101	95	182	0	0
normalized size	1	1.	0.7	1.15	1.08	2.07	0.	0.
time (sec)	N/A	0.053	0.07	0.052	1.076	1.538	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	78	125	111	324	0	130
normalized size	1	1.	0.76	1.23	1.09	3.18	0.	1.27
time (sec)	N/A	0.067	0.137	0.054	1.118	1.734	0.	1.393

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	252	360	321	620	0	0
normalized size	1	1.	1.76	2.52	2.24	4.34	0.	0.
time (sec)	N/A	0.199	1.651	0.092	1.673	1.563	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	125	231	244	456	0	0
normalized size	1	1.	1.11	2.04	2.16	4.04	0.	0.
time (sec)	N/A	0.161	1.126	0.06	1.589	1.589	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	57	111	146	177	224	97
normalized size	1	1.	0.66	1.28	1.68	2.03	2.57	1.11
time (sec)	N/A	0.082	0.425	0.029	1.114	1.479	2.074	1.381

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	136	127	158	360	0	262
normalized size	1	1.	1.17	1.09	1.36	3.1	0.	2.26
time (sec)	N/A	0.159	1.574	0.045	1.573	1.693	0.	1.396

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	209	190	294	560	0	370
normalized size	1	1.	1.49	1.36	2.1	4.	0.	2.64
time (sec)	N/A	0.225	5.36	0.05	1.527	1.977	0.	1.482

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	283	293	423	770	0	458
normalized size	1	1.	1.43	1.48	2.14	3.89	0.	2.31
time (sec)	N/A	0.428	1.52	0.054	1.692	2.085	0.	1.703

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	101	162	236	464	0	184
normalized size	1	1.	0.78	1.25	1.82	3.57	0.	1.42
time (sec)	N/A	0.161	1.009	0.059	1.028	2.054	0.	8.07

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	84	126	176	398	0	157
normalized size	1	1.	0.79	1.19	1.66	3.75	0.	1.48
time (sec)	N/A	0.136	0.308	0.056	1.	1.836	0.	3.934

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	54	90	120	338	0	130
normalized size	1	1.	0.66	1.1	1.46	4.12	0.	1.59
time (sec)	N/A	0.116	0.165	0.053	1.	1.343	0.	2.115

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	58	28	54	63	163	0	78
normalized size	1	1.57	0.76	1.46	1.7	4.41	0.	2.11
time (sec)	N/A	0.067	0.039	0.058	1.088	1.636	0.	1.341

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	33	42	74	0	45
normalized size	1	1.	1.	1.03	1.31	2.31	0.	1.41
time (sec)	N/A	0.039	0.019	0.023	1.047	1.407	0.	1.283

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	30	35	73	0	35
normalized size	1	1.	0.75	0.94	1.09	2.28	0.	1.09
time (sec)	N/A	0.068	0.033	0.032	1.118	1.37	0.	1.323

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	30	49	62	162	0	62
normalized size	1	1.	0.59	0.96	1.22	3.18	0.	1.22
time (sec)	N/A	0.089	0.049	0.085	1.106	1.4	0.	1.321

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	67	89	248	0	89
normalized size	1	1.	0.9	0.99	1.31	3.65	0.	1.31
time (sec)	N/A	0.093	0.149	0.095	1.128	1.413	0.	1.462

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	77	87	116	343	0	116
normalized size	1	1.	0.92	1.04	1.38	4.08	0.	1.38
time (sec)	N/A	0.1	0.209	0.115	1.108	1.517	0.	1.438

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	146	175	456	248	0	232
normalized size	1	1.	1.74	2.08	5.43	2.95	0.	2.76
time (sec)	N/A	0.097	0.319	0.057	1.095	1.673	0.	5.32

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	106	130	289	194	0	162
normalized size	1	1.	1.54	1.88	4.19	2.81	0.	2.35
time (sec)	N/A	0.092	0.32	0.055	1.081	1.511	0.	2.567

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	106	70	122	127	0	92
normalized size	1	1.	2.12	1.4	2.44	2.54	0.	1.84
time (sec)	N/A	0.088	0.164	0.047	1.095	1.371	0.	1.659

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	48	22	36	108	27	28
normalized size	1	1.	2.09	0.96	1.57	4.7	1.17	1.22
time (sec)	N/A	0.012	0.041	0.022	1.078	1.46	1.726	1.27

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	69	56	95	173	0	88
normalized size	1	1.	2.38	1.93	3.28	5.97	0.	3.03
time (sec)	N/A	0.051	0.218	0.068	1.087	1.539	0.	1.286

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	124	132	209	311	0	171
normalized size	1	1.	2.14	2.28	3.6	5.36	0.	2.95
time (sec)	N/A	0.088	0.491	0.075	1.027	1.416	0.	1.416

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	189	208	316	431	0	252
normalized size	1	1.	2.3	2.54	3.85	5.26	0.	3.07
time (sec)	N/A	0.106	0.743	0.088	1.02	1.396	0.	1.578

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	284	284	425	545	0	329
normalized size	1	1.	2.68	2.68	4.01	5.14	0.	3.1
time (sec)	N/A	0.127	0.914	0.102	1.125	1.579	0.	1.979

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	112	180	273	599	0	197
normalized size	1	1.	0.59	0.95	1.44	3.17	0.	1.04
time (sec)	N/A	0.149	1.693	0.096	1.106	1.728	0.	9.844

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	91	144	225	532	0	170
normalized size	1	1.	0.62	0.99	1.54	3.64	0.	1.16
time (sec)	N/A	0.108	0.45	0.084	1.054	1.528	0.	5.289

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	70	108	149	467	0	138
normalized size	1	1.	0.67	1.04	1.43	4.49	0.	1.33
time (sec)	N/A	0.086	0.311	0.083	1.055	1.531	0.	2.998

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	36	72	95	274	0	122
normalized size	1	1.	0.6	1.2	1.58	4.57	0.	2.03
time (sec)	N/A	0.048	0.081	0.082	1.025	1.456	0.	1.848

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	36	50	62	162	0	61
normalized size	1	1.	0.69	0.96	1.19	3.12	0.	1.17
time (sec)	N/A	0.05	0.055	0.034	1.119	1.509	0.	1.795

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	66	74	204	0	155
normalized size	1	1.	0.75	1.02	1.14	3.14	0.	2.38
time (sec)	N/A	0.059	0.07	0.106	1.384	1.548	0.	2.102

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	39	49	138	0	49
normalized size	1	1.	0.69	0.71	0.89	2.51	0.	0.89
time (sec)	N/A	0.051	0.074	0.092	0.956	1.466	0.	1.962

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	49	62	234	0	62
normalized size	1	1.	1.	0.67	0.85	3.21	0.	0.85
time (sec)	N/A	0.057	0.073	0.108	1.072	1.401	0.	2.364

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	78	79	103	331	0	103
normalized size	1	1.	0.61	0.62	0.81	2.61	0.	0.81
time (sec)	N/A	0.074	0.143	0.125	1.762	1.446	0.	2.225

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	116	431	0	116
normalized size	1	1.	0.61	0.61	0.8	2.97	0.	0.8
time (sec)	N/A	0.081	0.205	0.139	1.837	1.676	0.	1.931

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	118	119	157	541	0	157
normalized size	1	1.	0.59	0.6	0.79	2.72	0.	0.79
time (sec)	N/A	0.102	0.329	0.165	1.835	1.654	0.	2.161

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	102	162	254	657	0	184
normalized size	1	1.	0.6	0.95	1.49	3.84	0.	1.08
time (sec)	N/A	0.123	0.926	0.106	1.961	1.671	0.	4.824

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	82	126	197	586	0	154
normalized size	1	1.	0.65	1.	1.56	4.65	0.	1.22
time (sec)	N/A	0.091	0.363	0.092	1.938	1.581	0.	2.552

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	52	90	132	409	0	109
normalized size	1	1.	0.63	1.1	1.61	4.99	0.	1.33
time (sec)	N/A	0.057	0.145	0.095	2.143	1.553	0.	2.19

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	68	97	284	0	80
normalized size	1	1.	0.7	0.92	1.31	3.84	0.	1.08
time (sec)	N/A	0.058	0.175	0.037	1.769	1.607	0.	1.98

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	61	84	108	393	0	208
normalized size	1	1.	0.71	0.98	1.26	4.57	0.	2.42
time (sec)	N/A	0.07	0.191	0.129	1.951	1.547	0.	1.734

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	69	97	101	348	0	235
normalized size	1	1.	0.72	1.01	1.05	3.62	0.	2.45
time (sec)	N/A	0.071	0.328	0.126	2.056	1.652	0.	2.575

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	48	49	62	208	0	62
normalized size	1	1.	0.66	0.67	0.85	2.85	0.	0.85
time (sec)	N/A	0.057	0.099	0.119	1.539	1.461	0.	1.64

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	68	69	89	302	0	89
normalized size	1	1.	0.62	0.63	0.82	2.77	0.	0.82
time (sec)	N/A	0.069	0.072	0.136	1.212	1.476	0.	1.619

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	116	398	0	116
normalized size	1	1.	0.61	0.61	0.8	2.74	0.	0.8
time (sec)	N/A	0.081	0.113	0.153	1.965	1.591	0.	2.013

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	88	89	116	498	0	116
normalized size	1	1.	0.61	0.61	0.8	3.43	0.	0.8
time (sec)	N/A	0.078	0.118	0.174	2.733	1.669	0.	1.993

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	112	180	288	775	0	197
normalized size	1	1.	0.57	0.92	1.48	3.97	0.	1.01
time (sec)	N/A	0.135	1.489	0.088	3.476	1.734	0.	5.53

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	50	81	128	250	0	103
normalized size	1	1.	0.38	0.61	0.97	1.89	0.	0.78
time (sec)	N/A	0.089	0.115	0.088	2.939	1.354	0.	2.775

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	62	108	163	525	0	123
normalized size	1	1.	0.59	1.03	1.55	5.	0.	1.17
time (sec)	N/A	0.065	0.254	0.092	3.157	1.591	0.	2.141

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	73	101	139	522	0	250
normalized size	1	1.	0.69	0.95	1.31	4.92	0.	2.36
time (sec)	N/A	0.083	0.839	0.139	2.616	1.583	0.	1.819

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	89	130	128	502	0	313
normalized size	1	1.	0.66	0.96	0.95	3.72	0.	2.32
time (sec)	N/A	0.086	0.171	0.148	2.681	1.598	0.	1.93

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	124	158	481	338	0	197
normalized size	1	1.	0.98	1.24	3.79	2.66	0.	1.55
time (sec)	N/A	0.31	0.416	0.076	3.082	1.496	0.	2.539

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	315	161	389	992	0	182
normalized size	1	1.	2.92	1.49	3.6	9.19	0.	1.69
time (sec)	N/A	0.318	0.42	0.125	3.729	1.597	0.	1.945

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	589	195	385	1187	0	242
normalized size	1	1.	4.91	1.62	3.21	9.89	0.	2.02
time (sec)	N/A	0.249	6.089	0.138	3.069	1.581	0.	2.401

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	733	229	377	1191	0	275
normalized size	1	1.	5.51	1.72	2.83	8.95	0.	2.07
time (sec)	N/A	0.249	6.145	0.149	3.619	1.787	0.	1.735

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	195	394	172	0	548	0	0
normalized size	1	1.2	2.43	1.06	0.	3.38	0.	0.
time (sec)	N/A	0.923	5.54	0.464	0.	2.006	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	114	89	0	459	0	0
normalized size	1	1.	1.13	0.88	0.	4.54	0.	0.
time (sec)	N/A	0.185	0.328	0.502	0.	1.822	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	206	125	0	749	0	570
normalized size	1	1.	2.31	1.4	0.	8.42	0.	6.4
time (sec)	N/A	0.192	0.955	0.55	0.	1.936	0.	2.091

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	309	170	0	1025	0	950
normalized size	1	1.	1.9	1.04	0.	6.29	0.	5.83
time (sec)	N/A	0.377	1.568	0.701	0.	1.816	0.	2.115

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	195	141	139	0	648	0	0
normalized size	1	1.17	0.84	0.83	0.	3.88	0.	0.
time (sec)	N/A	0.975	5.562	0.574	0.	1.902	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	46	55	196	123	0	0
normalized size	1	1.	0.52	0.62	2.23	1.4	0.	0.
time (sec)	N/A	0.196	3.992	0.402	1.547	1.754	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	233	144	0	833	0	671
normalized size	1	1.	1.93	1.19	0.	6.88	0.	5.55
time (sec)	N/A	0.321	0.76	0.642	0.	1.846	0.	2.634

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	334	196	0	1123	0	1058
normalized size	1	1.	1.7	0.99	0.	5.7	0.	5.37
time (sec)	N/A	0.496	1.356	0.651	0.	1.723	0.	3.102

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	208	112	87	374	246	0	0
normalized size	1	1.38	0.74	0.58	2.48	1.63	0.	0.
time (sec)	N/A	0.979	5.459	0.404	1.764	1.432	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	60	67	258	173	0	0
normalized size	1	1.	0.51	0.57	2.19	1.47	0.	0.
time (sec)	N/A	0.214	5.463	0.412	1.663	1.423	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	261	162	0	934	0	714
normalized size	1	1.	1.73	1.07	0.	6.19	0.	4.73
time (sec)	N/A	0.429	1.193	0.733	0.	1.571	0.	2.906

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	360	222	0	1249	0	1135
normalized size	1	1.	1.59	0.98	0.	5.5	0.	5.
time (sec)	N/A	0.627	1.899	0.694	0.	1.732	0.	3.194

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	241	118	231	0	624	0	1056
normalized size	1	1.61	0.79	1.54	0.	4.16	0.	7.04
time (sec)	N/A	0.933	0.673	0.766	0.	1.639	0.	5.889

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	118	130	0	549	0	594
normalized size	1	1.	1.1	1.21	0.	5.13	0.	5.55
time (sec)	N/A	0.195	0.269	0.462	0.	1.543	0.	2.823

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	138	103	0	706	0	502
normalized size	1	1.	2.23	1.66	0.	11.39	0.	8.1
time (sec)	N/A	0.11	0.308	0.59	0.	1.609	0.	2.219

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	292	144	0	987	0	819
normalized size	1	1.	2.16	1.07	0.	7.31	0.	6.07
time (sec)	N/A	0.622	0.58	0.638	0.	1.463	0.	2.373

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	195	334	289	0	726	0	0
normalized size	1	1.1	1.89	1.63	0.	4.1	0.	0.
time (sec)	N/A	1.199	0.356	0.619	0.	1.62	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	128	202	0	644	0	0
normalized size	1	1.	0.96	1.51	0.	4.81	0.	0.
time (sec)	N/A	0.223	0.444	0.607	0.	1.635	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	206	134	0	1133	0	656
normalized size	1	1.	1.82	1.19	0.	10.03	0.	5.81
time (sec)	N/A	0.229	2.198	0.591	0.	1.764	0.	2.485

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-1)	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	294	144	0	1003	0	828
normalized size	1	1.	2.04	1.	0.	6.97	0.	5.75
time (sec)	N/A	0.553	0.761	0.621	0.	1.572	0.	3.131

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	260	394	353	0	830	0	0
normalized size	1	1.26	1.9	1.71	0.	4.01	0.	0.
time (sec)	N/A	1.434	0.546	0.843	0.	1.901	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	284	266	0	751	0	0
normalized size	1	1.	1.7	1.59	0.	4.5	0.	0.
time (sec)	N/A	0.303	0.408	0.648	0.	1.697	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	451	219	0	1436	0	0
normalized size	1	1.	3.2	1.55	0.	10.18	0.	0.
time (sec)	N/A	0.346	0.718	0.57	0.	1.859	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	191	191	332	182	0	1504	0	975
normalized size	1	1.	1.74	0.95	0.	7.87	0.	5.1
time (sec)	N/A	0.959	2.389	0.722	0.	1.745	0.	3.16

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	982	982	318	0	0	0	0	0
normalized size	1	1.	0.32	0.	0.	0.	0.	0.
time (sec)	N/A	1.271	3.273	0.128	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	290	0	0	0	0	0
normalized size	1	1.	2.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	2.746	0.104	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	2692	0	0	0	0	0
normalized size	1	1.	33.65	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	23.898	0.095	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	80	80	2796	0	0	0	0	0
normalized size	1	1.	34.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	22.599	0.102	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	128	0	0	0	0	0
normalized size	1	1.	0.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.494	0.796	0.119	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	100	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.513	0.105	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	12.766	0.093	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	9.129	0.105	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	269	269	5199	0	0	0	0	0
normalized size	1	1.	19.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.345	58.895	1.709	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	2054	0	0	0	0	0
normalized size	1	1.	10.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.227	18.053	1.421	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	2.025	0.907	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	108	108	232	0	0	0	0	0
normalized size	1	1.	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	4.033	0.178	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	667	0	0	0	0	0
normalized size	1	1.	4.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.275	14.178	0.41	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	248	248	1276	0	0	0	0	0
normalized size	1	1.	5.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.422	31.232	0.451	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	367	0	0	0	0	0
normalized size	1	1.	3.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	2.198	0.847	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	105	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.149	0.267	0.144	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	0.071	0.76	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.059	0.821	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	68	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.069	0.188	0.268	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	83	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.275	0.352	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	311	311	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.355	1.064	0.152	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	157	157	11184	0	0	0	0	0
normalized size	1	1.	71.24	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	34.458	0.12	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	90	0	0	0	0	0
normalized size	1	1.	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.181	0.286	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	47487	0	0	0	0	0
normalized size	1	1.	533.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	99.938	0.231	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.741	0.294	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	77	96	99	236	0	0
normalized size	1	1.	0.88	1.09	1.12	2.68	0.	0.
time (sec)	N/A	0.077	0.119	0.032	1.901	1.68	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	38	46	58	124	0	1966
normalized size	1	1.	0.69	0.84	1.05	2.25	0.	35.75
time (sec)	N/A	0.038	0.016	0.025	1.943	1.61	0.	2.827

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	43	25	30	62	0	31
normalized size	1	1.	1.79	1.04	1.25	2.58	0.	1.29
time (sec)	N/A	0.021	0.033	0.019	1.698	1.505	0.	2.282

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	60	83	61	167	0	81
normalized size	1	1.	1.11	1.54	1.13	3.09	0.	1.5
time (sec)	N/A	0.041	0.21	0.039	1.44	1.601	0.	2.007

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	87	136	93	292	0	111
normalized size	1	1.	1.07	1.68	1.15	3.6	0.	1.37
time (sec)	N/A	0.052	0.239	0.04	1.281	1.536	0.	2.1

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	88	182	0	0
normalized size	1	1.	1.12	1.36	1.22	2.53	0.	0.
time (sec)	N/A	0.078	0.038	0.032	2.133	1.815	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	47	59	53	108	0	1361
normalized size	1	1.	1.24	1.55	1.39	2.84	0.	35.82
time (sec)	N/A	0.059	0.033	0.026	2.723	1.705	0.	8.562

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	75	57	73	236	0	146
normalized size	1	1.	1.83	1.39	1.78	5.76	0.	3.56
time (sec)	N/A	0.053	0.031	0.029	2.207	1.836	0.	2.308

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	125	106	124	425	0	190
normalized size	1	1.	1.52	1.29	1.51	5.18	0.	2.32
time (sec)	N/A	0.08	0.042	0.036	1.873	1.864	0.	2.177

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	164	159	169	620	0	269
normalized size	1	1.	1.34	1.3	1.39	5.08	0.	2.2
time (sec)	N/A	0.097	0.06	0.039	2.7	1.706	0.	2.224

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	108	172	142	348	0	0
normalized size	1	1.	0.97	1.55	1.28	3.14	0.	0.
time (sec)	N/A	0.173	0.432	0.049	1.676	1.548	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	64	82	95	186	0	10604
normalized size	1	1.	0.82	1.05	1.22	2.38	0.	135.95
time (sec)	N/A	0.071	0.136	0.039	1.631	1.609	0.	5.72

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	45	54	108	0	55
normalized size	1	1.	1.	0.98	1.17	2.35	0.	1.2
time (sec)	N/A	0.039	0.023	0.023	1.893	1.58	0.	2.322

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	70	120	93	271	0	134
normalized size	1	1.	0.83	1.43	1.11	3.23	0.	1.6
time (sec)	N/A	0.073	0.236	0.053	1.568	1.521	0.	2.313

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	107	220	142	437	0	186
normalized size	1	1.	0.85	1.75	1.13	3.47	0.	1.48
time (sec)	N/A	0.104	0.757	0.047	1.403	1.629	0.	2.223

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	176	185	161	281	0	0
normalized size	1	1.	1.18	1.24	1.08	1.89	0.	0.
time (sec)	N/A	0.162	0.687	0.044	2.5	1.464	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	112	190	0	10355
normalized size	1	1.	0.82	1.23	1.19	2.02	0.	110.16
time (sec)	N/A	0.122	0.457	0.039	2.52	1.37	0.	30.612

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	116	102	107	308	0	200
normalized size	1	1.	1.49	1.31	1.37	3.95	0.	2.56
time (sec)	N/A	0.086	0.416	0.039	2.595	1.65	0.	2.237

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	293	199	186	539	0	325
normalized size	1	1.	2.2	1.5	1.4	4.05	0.	2.44
time (sec)	N/A	0.149	6.233	0.049	2.453	1.827	0.	2.059

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	351	302	247	786	0	455
normalized size	1	1.	1.74	1.5	1.22	3.89	0.	2.25
time (sec)	N/A	0.169	1.093	0.052	1.676	1.639	0.	2.779

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	141	279	219	470	0	0
normalized size	1	1.	0.94	1.86	1.46	3.13	0.	0.
time (sec)	N/A	0.242	0.263	0.056	1.546	1.721	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	90	139	153	277	0	0
normalized size	1	1.	0.86	1.32	1.46	2.64	0.	0.
time (sec)	N/A	0.114	0.19	0.046	1.707	1.606	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	64	77	158	0	78
normalized size	1	1.	1.	0.96	1.15	2.36	0.	1.16
time (sec)	N/A	0.045	0.025	0.027	1.567	1.492	0.	2.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	97	165	132	358	0	177
normalized size	1	1.	0.84	1.42	1.14	3.09	0.	1.53
time (sec)	N/A	0.094	0.321	0.065	1.9	1.563	0.	2.022

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	144	316	192	536	0	250
normalized size	1	1.	0.87	1.92	1.16	3.25	0.	1.52
time (sec)	N/A	0.141	1.164	0.062	1.354	1.6	0.	2.141

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	226	268	225	369	0	0
normalized size	1	1.	1.03	1.22	1.02	1.68	0.	0.
time (sec)	N/A	0.224	0.678	0.058	1.824	1.593	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	113	169	161	271	0	0
normalized size	1	1.	0.77	1.16	1.1	1.86	0.	0.
time (sec)	N/A	0.169	0.729	0.047	3.013	1.485	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	125	128	370	0	269
normalized size	1	1.	1.4	1.23	1.25	3.63	0.	2.64
time (sec)	N/A	0.112	1.295	0.052	1.515	1.605	0.	2.628

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	355	264	252	699	0	568
normalized size	1	1.	1.83	1.36	1.3	3.6	0.	2.93
time (sec)	N/A	0.184	6.236	0.062	2.21	1.652	0.	2.652

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	346	415	340	1014	0	636
normalized size	1	1.	1.19	1.43	1.17	3.48	0.	2.19
time (sec)	N/A	0.231	2.578	0.063	2.641	1.832	0.	2.013

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	184	304	389	594	0	463
normalized size	1	1.	0.9	1.49	1.91	2.91	0.	2.27
time (sec)	N/A	0.363	1.462	0.066	1.826	2.474	0.	5.118

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	117	164	192	367	0	239
normalized size	1	1.	0.93	1.3	1.52	2.91	0.	1.9
time (sec)	N/A	0.191	0.484	0.059	1.536	1.961	0.	1.98

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	87	76	88	157	0	96
normalized size	1	1.	1.18	1.03	1.19	2.12	0.	1.3
time (sec)	N/A	0.066	0.086	0.049	1.551	1.611	0.	1.3

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	35	45	80	0	47
normalized size	1	1.	1.	1.03	1.32	2.35	0.	1.38
time (sec)	N/A	0.04	0.02	0.026	1.465	1.57	0.	1.695

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	65	106	104	271	0	154
normalized size	1	1.	0.77	1.26	1.24	3.23	0.	1.83
time (sec)	N/A	0.089	0.161	0.063	1.616	1.596	0.	2.011

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	115	216	188	629	0	271
normalized size	1	1.	0.78	1.46	1.27	4.25	0.	1.83
time (sec)	N/A	0.138	3.847	0.063	1.463	1.663	0.	1.538

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	195	269	0	1054	0	325
normalized size	1	1.	1.1	1.52	0.	5.95	0.	1.84
time (sec)	N/A	0.239	1.419	0.067	0.	1.639	0.	3.306

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	152	117	0	684	0	144
normalized size	1	1.	1.58	1.22	0.	7.12	0.	1.5
time (sec)	N/A	0.099	0.203	0.053	0.	1.67	0.	2.053

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	108	155	0	801	0	174
normalized size	1	1.	1.35	1.94	0.	10.01	0.	2.17
time (sec)	N/A	0.243	0.236	0.06	0.	1.742	0.	1.87

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	350	348	0	1507	0	369
normalized size	1	1.	2.27	2.26	0.	9.79	0.	2.4
time (sec)	N/A	0.426	6.119	0.069	0.	2.929	0.	1.911

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	307	504	629	0	2566	0	662
normalized size	1	1.	1.64	2.05	0.	8.36	0.	2.16
time (sec)	N/A	1.106	1.386	0.073	0.	4.834	0.	2.212

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	240	318	682	1231	0	667
normalized size	1	1.	0.99	1.31	2.82	5.09	0.	2.76
time (sec)	N/A	0.633	6.252	0.093	1.828	3.386	0.	4.672

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	145	182	370	869	0	335
normalized size	1	1.	0.9	1.13	2.3	5.4	0.	2.08
time (sec)	N/A	0.312	0.719	0.087	1.753	2.394	0.	1.843

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	162	132	167	463	0	211
normalized size	1	1.	1.49	1.21	1.53	4.25	0.	1.94
time (sec)	N/A	0.096	0.287	0.078	1.764	1.695	0.	1.284

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	42	54	63	176	0	69
normalized size	1	1.	0.79	1.02	1.19	3.32	0.	1.3
time (sec)	N/A	0.053	0.076	0.038	1.47	1.637	0.	1.499

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	96	150	157	599	0	223
normalized size	1	1.	0.84	1.32	1.38	5.25	0.	1.96
time (sec)	N/A	0.115	0.608	0.089	1.364	1.619	0.	1.976

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	187	282	255	1241	0	375
normalized size	1	1.	0.99	1.5	1.36	6.6	0.	1.99
time (sec)	N/A	0.181	6.144	0.098	1.734	2.294	0.	1.811

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	341	382	0	1790	0	548
normalized size	1	1.	1.02	1.15	0.	5.38	0.	1.65
time (sec)	N/A	0.627	1.967	0.098	0.	2.504	0.	2.784

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	169	282	0	1261	0	339
normalized size	1	1.	0.84	1.41	0.	6.3	0.	1.7
time (sec)	N/A	0.301	1.012	0.088	0.	1.958	0.	2.152

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	139	245	0	1681	0	294
normalized size	1	1.	1.21	2.13	0.	14.62	0.	2.56
time (sec)	N/A	0.453	0.746	0.086	0.	2.816	0.	1.715

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	238	238	403	527	0	2639	0	481
normalized size	1	1.	1.69	2.21	0.	11.09	0.	2.02
time (sec)	N/A	0.695	6.295	0.096	0.	3.101	0.	1.859

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	424	361	897	0	4674	0	805
normalized size	1	1.	0.85	2.12	0.	11.02	0.	1.9
time (sec)	N/A	1.493	1.592	0.108	0.	5.532	0.	1.37

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	304	465	986	2206	0	790
normalized size	1	1.	0.95	1.45	3.07	6.87	0.	2.46
time (sec)	N/A	0.877	6.341	0.116	1.303	6.427	0.	3.708

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	196	323	595	1727	0	626
normalized size	1	1.	0.84	1.39	2.56	7.44	0.	2.7
time (sec)	N/A	0.482	2.189	0.109	1.876	3.964	0.	2.1

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	213	198	308	995	0	347
normalized size	1	1.	1.43	1.33	2.07	6.68	0.	2.33
time (sec)	N/A	0.132	2.02	0.1	1.915	2.649	0.	1.582

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	74	109	365	0	93
normalized size	1	1.	0.8	0.99	1.45	4.87	0.	1.24
time (sec)	N/A	0.06	0.262	0.046	1.547	2.004	0.	1.746

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	121	194	211	911	0	208
normalized size	1	1.	0.83	1.34	1.46	6.28	0.	1.43
time (sec)	N/A	0.133	0.931	0.111	1.921	2.342	0.	1.8

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	195	348	319	1708	0	441
normalized size	1	1.	0.88	1.57	1.44	7.73	0.	2.
time (sec)	N/A	0.211	5.327	0.114	1.612	2.424	0.	2.104

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	351	922	0	2765	0	853
normalized size	1	1.	0.74	1.95	0.	5.83	0.	1.8
time (sec)	N/A	0.872	1.043	0.131	0.	2.853	0.	3.399

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	212	766	0	2067	0	518
normalized size	1	1.	0.61	2.19	0.	5.91	0.	1.48
time (sec)	N/A	0.539	3.291	0.115	0.	2.322	0.	1.956

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	195	729	0	3044	0	458
normalized size	1	1.	0.97	3.61	0.	15.07	0.	2.27
time (sec)	N/A	0.788	5.784	0.116	0.	5.548	0.	1.552

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	289	289	459	780	0	4578	0	609
normalized size	1	1.	1.59	2.7	0.	15.84	0.	2.11
time (sec)	N/A	1.072	6.211	0.134	0.	5.542	0.	2.051

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	492	448	1252	0	6091	0	987
normalized size	1	1.	0.91	2.54	0.	12.38	0.	2.01
time (sec)	N/A	2.127	1.705	0.138	0.	6.517	0.	1.85

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	271	271	4791	0	0	0	0	0
normalized size	1	1.	17.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.38	18.403	1.514	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	186	186	2464	0	0	0	0	0
normalized size	1	1.	13.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.242	14.208	1.446	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	849	0	0	0	0	0
normalized size	1	1.	6.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	8.262	0.904	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	284	0	864	0	0	0	0	0
normalized size	1	0.	3.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	13.498	0.362	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	737	0	908	0	0	0	0	0
normalized size	1	0.	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.045	14.227	0.677	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	2.695	0.862	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [159] had the largest ratio of [0.4762]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	19	0.105
2	A	3	2	1.	19	0.105
3	A	3	2	1.	17	0.118
4	A	3	2	1.	17	0.118
5	A	3	2	1.	19	0.105
6	A	3	2	1.	19	0.105
7	A	3	2	1.	19	0.105
8	A	9	5	1.	19	0.263
9	A	8	5	1.	19	0.263
10	A	5	5	1.	19	0.263
11	A	7	6	1.	19	0.316
12	A	9	7	1.	19	0.368
13	A	11	7	1.	19	0.368
14	A	3	2	1.	21	0.095
15	A	3	2	1.	21	0.095
16	A	3	2	1.	19	0.105
17	A	2	2	1.	21	0.095
18	A	3	2	1.	21	0.095
19	A	14	9	1.	21	0.429
20	A	4	4	1.	21	0.19
21	A	6	5	1.	21	0.238
22	A	1	1	1.	12	0.083
23	A	8	6	1.	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	12	7	1.	21	0.333
25	A	3	2	1.	21	0.095
26	A	3	2	1.	21	0.095
27	A	3	2	1.	19	0.105
28	A	3	2	1.	21	0.095
29	A	9	7	1.	21	0.333
30	A	10	7	1.	21	0.333
31	A	8	6	1.	21	0.286
32	A	7	5	1.	12	0.417
33	A	10	7	1.	21	0.333
34	A	3	2	1.	21	0.095
35	A	3	2	1.	21	0.095
36	A	3	2	1.	19	0.105
37	A	3	2	1.	21	0.095
38	A	13	7	1.	21	0.333
39	A	11	6	1.	21	0.286
40	A	10	5	1.	12	0.417
41	A	12	6	1.	21	0.286
42	A	17	8	1.	21	0.381
43	A	21	8	1.	21	0.381
44	A	8	5	1.	21	0.238
45	A	7	5	1.	21	0.238
46	A	6	5	1.	21	0.238
47	A	5	5	1.57	19	0.263
48	A	4	4	1.	19	0.21
49	A	5	4	1.	21	0.19
50	A	5	4	1.	21	0.19
51	A	6	5	1.	21	0.238
52	A	6	5	1.	21	0.238
53	A	6	5	1.	21	0.238
54	A	6	5	1.	21	0.238
55	A	5	4	1.	21	0.19

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	1	1	1.	12	0.083
57	A	4	4	1.	21	0.19
58	A	5	5	1.	21	0.238
59	A	6	5	1.	21	0.238
60	A	7	5	1.	21	0.238
61	A	4	3	1.	21	0.143
62	A	4	3	1.	21	0.143
63	A	4	3	1.	21	0.143
64	A	4	3	1.	19	0.158
65	A	3	2	1.	19	0.105
66	A	3	2	1.	21	0.095
67	A	3	2	1.	21	0.095
68	A	3	2	1.	21	0.095
69	A	3	2	1.	21	0.095
70	A	3	2	1.	21	0.095
71	A	3	2	1.	21	0.095
72	A	4	3	1.	21	0.143
73	A	4	3	1.	21	0.143
74	A	4	3	1.	19	0.158
75	A	3	2	1.	19	0.105
76	A	3	2	1.	21	0.095
77	A	3	2	1.	21	0.095
78	A	3	2	1.	21	0.095
79	A	3	2	1.	21	0.095
80	A	3	2	1.	21	0.095
81	A	3	2	1.	21	0.095
82	A	4	3	1.	21	0.143
83	A	3	2	1.	21	0.095
84	A	4	3	1.	19	0.158
85	A	3	2	1.	21	0.095
86	A	3	2	1.	21	0.095
87	A	17	5	1.	21	0.238

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	14	8	1.	21	0.381
89	A	14	8	1.	21	0.381
90	A	16	6	1.	21	0.286
91	A	15	10	1.2	23	0.435
92	A	4	4	1.	23	0.174
93	A	4	4	1.	23	0.174
94	A	7	7	1.	23	0.304
95	A	14	9	1.17	23	0.391
96	A	3	3	1.	23	0.13
97	A	5	5	1.	23	0.217
98	A	8	8	1.	23	0.348
99	A	10	7	1.38	23	0.304
100	A	4	4	1.	23	0.174
101	A	6	5	1.	23	0.217
102	A	10	8	1.	23	0.348
103	A	17	9	1.61	23	0.391
104	A	4	4	1.	23	0.174
105	A	4	4	1.	23	0.174
106	A	11	7	1.	23	0.304
107	A	20	9	1.1	23	0.391
108	A	5	5	1.	23	0.217
109	A	6	5	1.	23	0.217
110	A	10	6	1.	23	0.261
111	A	23	9	1.26	23	0.391
112	A	6	6	1.	23	0.261
113	A	7	6	1.	23	0.261
114	A	16	8	1.	23	0.348
115	A	10	9	1.	23	0.391
116	A	4	4	1.	23	0.174
117	A	3	3	1.	23	0.13
118	A	3	3	1.	23	0.13
119	A	8	7	1.	23	0.304

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	4	4	1.	23	0.174
121	A	3	3	1.	23	0.13
122	A	3	3	1.	23	0.13
123	A	10	6	1.	23	0.261
124	A	8	6	1.	23	0.261
125	A	6	5	1.	21	0.238
126	A	4	4	1.	23	0.174
127	A	10	6	1.	23	0.261
128	A	13	6	1.	23	0.261
129	A	4	3	1.	23	0.13
130	A	4	4	1.	21	0.19
131	A	3	3	1.	19	0.158
132	A	2	2	1.	19	0.105
133	A	3	3	1.	21	0.143
134	A	4	4	1.	21	0.19
135	A	6	6	1.	21	0.286
136	A	5	5	1.	21	0.238
137	A	2	2	1.	12	0.167
138	A	3	3	1.	21	0.143
139	A	3	3	1.	21	0.143
140	A	6	5	1.	19	0.263
141	A	5	4	1.	17	0.235
142	A	3	2	1.	17	0.118
143	A	3	2	1.	19	0.105
144	A	3	2	1.	19	0.105
145	A	8	5	1.	19	0.263
146	A	7	5	1.	19	0.263
147	A	7	6	1.	19	0.316
148	A	9	7	1.	19	0.368
149	A	11	7	1.	19	0.368
150	A	7	5	1.	21	0.238
151	A	6	4	1.	19	0.21

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
152	A	3	2	1.	19	0.105
153	A	3	2	1.	21	0.095
154	A	3	2	1.	21	0.095
155	A	13	9	1.	21	0.429
156	A	11	9	1.	21	0.429
157	A	9	7	1.	21	0.333
158	A	13	9	1.	21	0.429
159	A	16	10	1.	21	0.476
160	A	7	5	1.	21	0.238
161	A	6	4	1.	19	0.21
162	A	3	2	1.	19	0.105
163	A	3	2	1.	21	0.095
164	A	3	2	1.	21	0.095
165	A	16	9	1.	21	0.429
166	A	14	10	1.	21	0.476
167	A	11	9	1.	21	0.429
168	A	17	10	1.	21	0.476
169	A	21	10	1.	21	0.476
170	A	5	3	1.	21	0.143
171	A	4	3	1.	21	0.143
172	A	3	2	1.	19	0.105
173	A	4	4	1.	19	0.21
174	A	3	2	1.	21	0.095
175	A	3	2	1.	21	0.095
176	A	13	9	1.	21	0.429
177	A	8	7	1.	21	0.333
178	A	7	7	1.	21	0.333
179	A	7	7	1.	21	0.333
180	A	9	7	1.	21	0.333
181	A	5	3	1.	21	0.143
182	A	4	3	1.	21	0.143
183	A	3	2	1.	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	3	2	1.	19	0.105
185	A	3	2	1.	21	0.095
186	A	3	2	1.	21	0.095
187	A	16	8	1.	21	0.381
188	A	12	7	1.	21	0.333
189	A	8	7	1.	21	0.333
190	A	8	7	1.	21	0.333
191	A	10	7	1.	21	0.333
192	A	5	3	1.	21	0.143
193	A	4	3	1.	21	0.143
194	A	3	2	1.	19	0.105
195	A	3	2	1.	19	0.105
196	A	3	2	1.	21	0.095
197	A	3	2	1.	21	0.095
198	A	22	9	1.	21	0.429
199	A	18	8	1.	21	0.381
200	A	9	8	1.	21	0.381
201	A	9	7	1.	21	0.333
202	A	11	7	1.	21	0.333
203	A	10	6	1.	23	0.261
204	A	8	6	1.	23	0.261
205	A	6	5	1.	21	0.238
206	F	0	0	N/A	0	N/A
207	F	0	0	N/A	0	N/A
208	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int (a + a\sin(c + dx))\tan^5(c + dx)\, dx$

Optimal. Leaf size=115

$$\frac{a^3}{8d(a - a\sin(c + dx))^2} - \frac{a^2}{d(a - a\sin(c + dx))} + \frac{a^2}{8d(a\sin(c + dx) + a)} - \frac{a\sin(c + dx)}{d} - \frac{23a\log(1 - \sin(c + dx))}{16d} + \frac{7a\,\mathrm{lo}}{}$$

```
[Out] (-23*a*Log[1 - Sin[c + d*x]])/(16*d) + (7*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(d*(a - a*Sin[
c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.0716649, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a^3}{8d(a - a\sin(c + dx))^2} - \frac{a^2}{d(a - a\sin(c + dx))} + \frac{a^2}{8d(a\sin(c + dx) + a)} - \frac{a\sin(c + dx)}{d} - \frac{23a\log(1 - \sin(c + dx))}{16d} + \frac{7a\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] (-23*a*Log[1 - Sin[c + d*x]])/(16*d) + (7*a*Log[1 + Sin[c + d*x]])/(16*d) -
 (a*Sin[c + d*x])/d + a^3/(8*d*(a - a*Sin[c + d*x])^2) - a^2/(d*(a - a*Sin[
c + d*x])) + a^2/(8*d*(a + a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a + a\sin(c+dx))\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-1 + \frac{a^3}{4(a-x)^3} - \frac{a^2}{(a-x)^2} + \frac{23a}{16(a-x)} - \frac{a^2}{8(a+x)^2} + \frac{7a}{16(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$

$$= -\frac{23a\log(1-\sin(c+dx))}{16d} + \frac{7a\log(1+\sin(c+dx))}{16d} - \frac{a\sin(c+dx)}{d} + \frac{\cdots}{8d(a-a\,\mathrm{s}\cdots}$$

Mathematica [A] time = 0.446024, size = 123, normalized size = 1.07

$$-\frac{a\sin(c+dx)\tan^4(c+dx)}{d} - \frac{a\left(-\tan^4(c+dx) + 2\tan^2(c+dx) + 4\log(\cos(c+dx))\right)}{4d} - \frac{5a\left(6\tan(c+dx)\sec^3(c+dx)\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

```
[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^4)/d) - (a*(4*Log[Cos[c + d*x]] + 2*Tan[c +
d*x]^2 - Tan[c + d*x]^4))/(4*d) - (5*a*(6*Sec[c + d*x]^3*Tan[c + d*x] - 8*S
ec[c + d*x]*Tan[c + d*x]^3 - 3*(ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c
+ d*x])))/(8*d)
```

Maple [A] time = 0.064, size = 147, normalized size = 1.3

$$\frac{a(\sin(dx+c))^7}{4d(\cos(dx+c))^4} - \frac{3a(\sin(dx+c))^7}{8d(\cos(dx+c))^2} - \frac{3(\sin(dx+c))^5 a}{8d} - \frac{5a(\sin(dx+c))^3}{8d} - \frac{15a\sin(dx+c)}{8d} + \frac{15a\ln(\sec(dx+\cdots}{8\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^5,x)
```

```
[Out] 1/4/d*a*sin(d*x+c)^7/cos(d*x+c)^4-3/8/d*a*sin(d*x+c)^7/cos(d*x+c)^2-3/8/d*s
in(d*x+c)^5*a-5/8/d*a*sin(d*x+c)^3-15/8*a*sin(d*x+c)/d+15/8/d*a*ln(sec(d*x+
c)+tan(d*x+c))+1/4/d*a*tan(d*x+c)^4-1/2/d*a*tan(d*x+c)^2-1/d*a*ln(cos(d*x+c
))
```

Maxima [A] time = 1.04939, size = 128, normalized size = 1.11

$$\frac{7\,a\log\left(\sin\left(dx+c\right)+1\right)-23\,a\log\left(\sin\left(dx+c\right)-1\right)-16\,a\sin\left(dx+c\right)+\frac{2\left(9\,a\sin(dx+c)^{2}-a\sin(dx+c)-6\,a\right)}{\sin(dx+c)^{3}-\sin(dx+c)^{2}-\sin(dx+c)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] 1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) - 16*a*sin(d*x
 + c) + 2*(9*a*sin(d*x + c)^2 - a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - sin
(d*x + c)^2 - sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.55232, size = 410, normalized size = 3.57

$$\frac{16\,a\cos\left(dx+c\right)^{4}+2\,a\cos\left(dx+c\right)^{2}+7\left(a\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)-a\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-23\left(a\cos\right.}{16\left(d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*cos(d*x + c)^4 + 2*a*cos(d*x + c)^2 + 7*(a*cos(d*x + c)^2*sin(d*
x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 23*(a*cos(d*x + c)^2*sin
(d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(8*a*cos(d*x + c)^
2 + a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x + c)^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin(c+dx)\tan^5(c+dx)\,dx + \int \tan^5(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**5,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5, x)
)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.2 $\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=71

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\sin(c+dx)}{d}+\frac{5a\log(1-\sin(c+dx))}{4d}-\frac{a\log(\sin(c+dx)+1)}{4d}$$

```
[Out] (5*a*Log[1 - Sin[c + d*x]])/(4*d) - (a*Log[1 + Sin[c + d*x]])/(4*d) + (a*Si
n[c + d*x])/d + a^2/(2*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.048039, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a^2}{2d(a-a\sin(c+dx))}+\frac{a\sin(c+dx)}{d}+\frac{5a\log(1-\sin(c+dx))}{4d}-\frac{a\log(\sin(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] (5*a*Log[1 - Sin[c + d*x]])/(4*d) - (a*Log[1 + Sin[c + d*x]])/(4*d) + (a*Si
n[c + d*x])/d + a^2/(2*d*(a - a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{2(a-x)^2}-\frac{5a}{4(a-x)}-\frac{a}{4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{5a\log(1-\sin(c+dx))}{4d} - \frac{a\log(1+\sin(c+dx))}{4d} + \frac{a\sin(c+dx)}{d} + \frac{a^2}{2d(a-a\sin(c$$

Mathematica [A] time = 0.108376, size = 77, normalized size = 1.08

$$-\frac{a\sin(c+dx)\tan^2(c+dx)}{d} + \frac{a\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d} - \frac{3a\left(\tanh^{-1}(\sin(c+dx))-\tan(c+dx)\sec(c+dx\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((a*Sin[c + d*x]*Tan[c + d*x]^2)/d) - (3*a*(ArcTanh[Sin[c + d*x]] - Sec[c
+ d*x]*Tan[c + d*x]))/(2*d) + (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2
*d)
```

Maple [A] time = 0.056, size = 96, normalized size = 1.4

$$\frac{a(\sin(dx+c))^5}{2d(\cos(dx+c))^2} + \frac{a(\sin(dx+c))^3}{2d} + \frac{3a\sin(dx+c)}{2d} - \frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{a(\tan(dx+c))^2}{2d} + \frac{a\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*a*sin(d*x+c)^3+3/2*a*sin(d*x+c)/d-3
/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a*tan(d*x+c)^2+1/d*a*ln(cos(d*x+c))
```

Maxima [A] time = 0.99153, size = 69, normalized size = 0.97

$$-\frac{a\log(\sin(dx+c)+1) - 5a\log(\sin(dx+c)-1) - 4a\sin(dx+c) + \frac{2a}{\sin(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*log(sin(d*x + c) + 1) - 5*a*log(sin(d*x + c) - 1) - 4*a*sin(d*x + c
) + 2*a/(sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.53161, size = 224, normalized size = 3.15

$$-\frac{4\,a\cos\left(dx+c\right)^{2}+\left(a\sin\left(dx+c\right)-a\right)\log\left(\sin\left(dx+c\right)+1\right)-5\left(a\sin\left(dx+c\right)-a\right)\log\left(-\sin\left(dx+c\right)+1\right)+4\,a\sin\left(dx\right.}{4\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*cos(d*x + c)^2 + (a*sin(d*x + c) - a)*log(sin(d*x + c) + 1) - 5*(
a*sin(d*x + c) - a)*log(-sin(d*x + c) + 1) + 4*a*sin(d*x + c) - 2*a)/(d*sin
(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin\left(c+dx\right)\tan^{3}\left(c+dx\right)dx+\int\tan^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x)
)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.3 $\int (a + a\sin(c + dx)) \tan(c + dx)\, dx$

Optimal. Leaf size=30

$$-\frac{a\sin(c+dx)}{d}-\frac{a\log(1-\sin(c+dx))}{d}$$

```
[Out] -((a*Log[1 - Sin[c + d*x]])/d) - (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0212762, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2707, 43}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a*Log[1 - Sin[c + d*x]])/d) - (a*Sin[c + d*x])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a + a\sin(c+dx))\tan(c+dx)\,dx = \frac{\operatorname{Subst}\left(\int \frac{x}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-1+\frac{a}{a-x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\log(1-\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0212587, size = 38, normalized size = 1.27

$$-\frac{a\sin(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Maple [A] time = 0.031, size = 29, normalized size = 1.

$$-\frac{a\sin(dx+c)}{d} - \frac{a\ln(\sin(dx+c)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] -a*sin(d*x+c)/d-1/d*a*ln(sin(d*x+c)-1)
```

Maxima [A] time = 1.07582, size = 34, normalized size = 1.13

$$-\frac{a\log(\sin(dx+c)-1) + a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*log(sin(d*x + c) - 1) + a*sin(d*x + c))/d
```

Fricas [A] time = 1.37141, size = 65, normalized size = 2.17

$$-\frac{a\log\left(-\sin\left(dx+c\right)+1\right)+a\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*log(-sin(d*x + c) + 1) + a*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin\left(c+dx\right)\tan\left(c+dx\right)\,dx+\int\tan\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

Giac [B] time = 1.61062, size = 1966, normalized size = 65.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - a*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
```

```
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(4*(tan(c)^2 + 1)/(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(2*(tan(1/2*c)^2 + 1)
/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x
)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 - a*log(2*(tan(1/2*c)^2
 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/
2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 + a*log(4*(tan(c)^
2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2 - 4*a*tan(1/2*d*x)^2*tan
(1/2*c) + a*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*ta
n(1/2*c)^2 - a*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))
*tan(1/2*c)^2 + a*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2
*c)^2 - 4*a*tan(1/2*d*x)*tan(1/2*c)^2 + a*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2
*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 2*tan(1/2*c) + 1)) - a*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)) + a*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x
)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 4
*a*tan(1/2*d*x) + 4*a*tan(1/2*c))/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/
2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

3.4 $\int \cot(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0202604, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2707, 43}

$$\frac{a\sin(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{a+x}{x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{a\log(\sin(c+dx))}{d} + \frac{a\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0331529, size = 26, normalized size = 1.08

$$\frac{a(\sin(c+dx)+\log(\tan(c+dx))+\log(\cos(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]] + Sin[c + d*x]))/d
```

Maple [A] time = 0.016, size = 25, normalized size = 1.

$$\frac{a\ln(\sin(dx+c))}{d} + \frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

Maxima [A] time = 1.09179, size = 30, normalized size = 1.25

$$\frac{a\log(\sin(dx+c))+a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sin(d*x + c)) + a*sin(d*x + c))/d
```

Fricas [A] time = 1.57343, size = 62, normalized size = 2.58

$$\frac{a \log \left(\frac{1}{2} \sin \left(dx+c\right)\right)+a \sin \left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*log(1/2*sin(d*x + c)) + a*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin \left(c+dx\right)\cot \left(c+dx\right)dx+\int \cot \left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x), x) + Integral(cot(c + d*x), x))
```

Giac [A] time = 1.26787, size = 31, normalized size = 1.29

$$\frac{a \log \left(|\sin \left(dx+c\right)|\right)+a \sin \left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(sin(d*x + c))) + a*sin(d*x + c))/d
```

3.5 $\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.037076, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 75}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^2(c+dx)}{2d}-\frac{a\csc(c+dx)}{d}-\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (a*Sin[c + d*x])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^2}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^3}{x^3}+\frac{a^2}{x^2}-\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{a\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\log(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d}$$

Mathematica [A] time = 0.117517, size = 60, normalized size = 1.11

$$-\frac{a\sin(c+dx)}{d} - \frac{a\csc(c+dx)}{d} - \frac{a\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Ta
n[c + d*x]]))/(2*d) - (a*Sin[c + d*x])/d
```

Maple [A] time = 0.037, size = 83, normalized size = 1.5

$$-\frac{a(\cos(dx+c))^4}{d\sin(dx+c)} - \frac{(\cos(dx+c))^2\sin(dx+c)\,a}{d} - 2\frac{a\sin(dx+c)}{d} - \frac{a(\cot(dx+c))^2}{2d} - \frac{a\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d*a/sin(d*x+c)*cos(d*x+c)^4-1/d*cos(d*x+c)^2*sin(d*x+c)*a-2*a*sin(d*x+c)
/d-1/2/d*a*cot(d*x+c)^2-a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.08976, size = 61, normalized size = 1.13

$$-\frac{2a\log(\sin(dx+c)) + 2a\sin(dx+c) + \frac{2a\sin(dx+c)+a}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a)/sin
(d*x + c)^2)/d
```

Fricas [A] time = 1.49611, size = 167, normalized size = 3.09

$$-\frac{2\left(a\cos\left(dx+c\right)^{2}-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+2\left(a\cos\left(dx+c\right)^{2}-2\,a\right)\sin\left(dx+c\right)-a}{2\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2
- 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin\left(c+dx\right)\cot^{3}\left(c+dx\right)dx+\int\cot^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x)
)
```

Giac [A] time = 1.32525, size = 81, normalized size = 1.5

$$-\frac{2\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+2\,a\sin\left(dx+c\right)-\frac{3\,a\sin(dx+c)^{2}-2\,a\sin(dx+c)-a}{\sin(dx+c)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) - (3*a*sin(d*x + c)^2 -
 2*a*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

3.6 $\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

```
[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.048033, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^4(c+dx)}{4d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\csc^2(c+dx)}{d}+\frac{2a\csc(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (a*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (a*Sin[c + d*x])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^3}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^5}{x^5}+\frac{a^4}{x^4}-\frac{2a^3}{x^3}-\frac{2a^2}{x^2}+\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin(}{d}$$

Mathematica [A] time = 0.197611, size = 87, normalized size = 1.07

$$\frac{a\sin(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{2a\csc(c+dx)}{d}+\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

```
[Out] (2*a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (a*Sin[c
 + d*x])/d
```

Maple [A] time = 0.036, size = 136, normalized size = 1.7

$$-\frac{a(\cos(dx+c))^6}{3d(\sin(dx+c))^3}+\frac{a(\cos(dx+c))^6}{d\sin(dx+c)}+\frac{8a\sin(dx+c)}{3d}+\frac{\sin(dx+c)(\cos(dx+c))^4a}{d}+\frac{4(\cos(dx+c))^2\sin(dx+c}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/3/d*a/sin(d*x+c)^3*cos(d*x+c)^6+1/d*a/sin(d*x+c)*cos(d*x+c)^6+8/3*a*sin(
d*x+c)/d+1/d*cos(d*x+c)^4*sin(d*x+c)*a+4/3/d*cos(d*x+c)^2*sin(d*x+c)*a-1/4/
d*a*cot(d*x+c)^4+1/2/d*a*cot(d*x+c)^2+a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.08056, size = 93, normalized size = 1.15

$$\frac{12\,a\log(\sin(dx+c))+12\,a\sin(dx+c)+\frac{24\,a\sin(dx+c)^3+12\,a\sin(dx+c)^2-4\,a\sin(dx+c)-3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*a*sin(d*x + c) + (24*a*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*a*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.57272, size = 292, normalized size = 3.6

$$-\frac{12\,a\cos\left(dx+c\right)^{2}-12\left(a\cos\left(dx+c\right)^{4}-2\,a\cos\left(dx+c\right)^{2}+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\left(3\,a\cos\left(dx+c\right)^{4}-12\,a\cos\left(dx+\right.\right.}{12\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 8*a)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin\left(c+dx\right)\cot^{5}\left(c+dx\right)dx+\int \cot^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**5, x) + Integral(cot(c + d*x)**5, x)
)
```

Giac [A] time = 1.38853, size = 111, normalized size = 1.37

$$\frac{12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a\sin\left(dx+c\right)-\frac{25\,a\sin(dx+c)^{4}-24\,a\sin(dx+c)^{3}-12\,a\sin(dx+c)^{2}+4\,a\sin(dx+c)+3\,a}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*a*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

3.7 $\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

```
[Out] (-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d +
 (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)
/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0594132, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 88}

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\csc^5(c+dx)}{5d}+\frac{3a\csc^4(c+dx)}{4d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}-\frac{3a\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d - (3*a*Csc[c + d*x]^2)/(2*d) + (a*Csc[c + d*x]^3)/d +
 (3*a*Csc[c + d*x]^4)/(4*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^6)
/(6*d) - (a*Log[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^4}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{a^7}{x^7}+\frac{a^6}{x^6}-\frac{3a^5}{x^5}-\frac{3a^4}{x^4}+\frac{3a^3}{x^3}+\frac{3a^2}{x^2}-\frac{a}{x}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^5(}{5}$$

Mathematica [A] time = 0.398755, size = 111, normalized size = 0.97

$$-\frac{a\sin(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}+\frac{a\csc^3(c+dx)}{d}-\frac{3a\csc(c+dx)}{d}-\frac{a\left(2\cot^6(c+dx)-3\cot^4(c+dx)+6\cot^2(c+dx)\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) - (
a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos[c +
d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) - (a*Sin[c + d*x])/d
```

Maple [A] time = 0.039, size = 195, normalized size = 1.7

$$-\frac{a(\cos(dx+c))^8}{5d(\sin(dx+c))^5}+\frac{a(\cos(dx+c))^8}{5d(\sin(dx+c))^3}-\frac{a(\cos(dx+c))^8}{d\sin(dx+c)}-\frac{16a\sin(dx+c)}{5d}-\frac{\sin(dx+c)(\cos(dx+c))^6a}{d}-\frac{6\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/5/d*a/sin(d*x+c)^5*cos(d*x+c)^8+1/5/d*a/sin(d*x+c)^3*cos(d*x+c)^8-1/d*a/
sin(d*x+c)*cos(d*x+c)^8-16/5*a*sin(d*x+c)/d-1/d*cos(d*x+c)^6*sin(d*x+c)*a-6
/5/d*cos(d*x+c)^4*sin(d*x+c)*a-8/5/d*cos(d*x+c)^2*sin(d*x+c)*a-1/6/d*a*cot(
d*x+c)^6+1/4/d*a*cot(d*x+c)^4-1/2/d*a*cot(d*x+c)^2-a*ln(sin(d*x+c))/d
```

Maxima [A] time = 1.15296, size = 123, normalized size = 1.07

$$-\frac{60\,a\log\left(\sin\left(dx+c\right)\right)+60\,a\sin\left(dx+c\right)+\frac{180\,a\sin(dx+c)^5+90\,a\sin(dx+c)^4-60\,a\sin(dx+c)^3-45\,a\sin(dx+c)^2+12\,a\sin(dx+c)+10\,a}{\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + (180*a*sin(d*x + c)^5 +
 90*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 - 45*a*sin(d*x + c)^2 + 12*a*sin
(d*x + c) + 10*a)/sin(d*x + c)^6)/d
```

Fricas [A] time = 1.59933, size = 412, normalized size = 3.58

$$\frac{90\,a\cos\left(dx+c\right)^4-135\,a\cos\left(dx+c\right)^2-60\left(a\cos\left(dx+c\right)^6-3\,a\cos\left(dx+c\right)^4+3\,a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)}{60\left(d\cos\left(dx+c\right)^6-3\,d\cos\left(dx+c\right)^4+3\,d\cos\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/60*(90*a*cos(d*x + c)^4 - 135*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^6 - 3
*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) - 12*(5*a
*cos(d*x + c)^6 - 30*a*cos(d*x + c)^4 + 40*a*cos(d*x + c)^2 - 16*a)*sin(d*x
 + c) + 55*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 -
 d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38698, size = 140, normalized size = 1.22

$$-\frac{60\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a\sin\left(dx+c\right)-\frac{147\,a\sin(dx+c)^{6}-180\,a\sin(dx+c)^{5}-90\,a\sin(dx+c)^{4}+60\,a\sin(dx+c)^{3}+45\,a\sin(dx+c)^{2}-12\,a\sin(dx+c)-10\,a}{\sin(dx+c)^{6}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/60*(60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - (147*a*sin(d*x + c
)^6 - 180*a*sin(d*x + c)^5 - 90*a*sin(d*x + c)^4 + 60*a*sin(d*x + c)^3 + 45
*a*sin(d*x + c)^2 - 12*a*sin(d*x + c) - 10*a)/sin(d*x + c)^6)/d
```

3.8 $\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{a\cos(c+dx)}{d}+\frac{a\tan^5(c+dx)}{5d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}-\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-a$$

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d - (a*Sec[c + d*x]^3)/d +
 (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (a*Tan[c + d*x]^3)/(3*d) +
 (a*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0922464, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2710, 3473, 8, 2590, 270}

$$\frac{a\cos(c+dx)}{d}+\frac{a\tan^5(c+dx)}{5d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}-\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-a$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d - (a*Sec[c + d*x]^3)/d +
 (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (a*Tan[c + d*x]^3)/(3*d) +
 (a*Tan[c + d*x]^5)/(5*d)
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx = \int \left(a\tan^6(c+dx)+a\sin(c+dx)\tan^6(c+dx)\right)dx$$
$$= a\int \tan^6(c+dx)\,dx + a\int \sin(c+dx)\tan^6(c+dx)\,dx$$
$$= \frac{a\tan^5(c+dx)}{5d} - a\int \tan^4(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{a\tan^3(c+dx)}{3d} + \frac{a\tan^5(c+dx)}{5d} + a\int \tan^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \left(-1+\frac{1}{x}\right.\right.}{}$$
$$= \frac{a\cos(c+dx)}{d} + \frac{3a\sec(c+dx)}{d} - \frac{a\sec^3(c+dx)}{d} + \frac{a\sec^5(c+dx)}{5d} + \frac{a\tan(c+d}{d}$$
$$= -ax + \frac{a\cos(c+dx)}{d} + \frac{3a\sec(c+dx)}{d} - \frac{a\sec^3(c+dx)}{d} + \frac{a\sec^5(c+dx)}{5d} + \frac{a\tan}{}$$

Mathematica [A] time = 0.0654162, size = 110, normalized size = 1.09

$$\frac{a\cos(c+dx)}{d} + \frac{a\tan^5(c+dx)}{5d} - \frac{a\tan^3(c+dx)}{3d} - \frac{a\tan^{-1}(\tan(c+dx))}{d} + \frac{a\tan(c+dx)}{d} + \frac{a\sec^5(c+dx)}{5d} - \frac{a\sec^3(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^6,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d -
 (a*Sec[c + d*x]^3)/d + (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (a*
Tan[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.071, size = 135, normalized size = 1.3

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^8}{5(\cos(dx+c))^5}-\frac{(\sin(dx+c))^8}{5(\cos(dx+c))^3}+\frac{(\sin(dx+c))^8}{\cos(dx+c)}+\left(\frac{16}{5}+(\sin(dx+c))^6+\frac{6(\sin(dx+c))^4}{5}+\frac{8(\sin(dx+c)}{5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^6,x)
```

```
[Out] 1/d*(a*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x
+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(
d*x+c))+a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.65939, size = 117, normalized size = 1.16

$$\frac{\left(3\tan(dx+c)^5-5\tan(dx+c)^3-15\,dx-15\,c+15\tan(dx+c)\right)a+3\,a\left(\frac{15\cos(dx+c)^4-5\cos(dx+c)^2+1}{\cos(dx+c)^5}+5\cos(dx+c)\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/15*((3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c
))*a + 3*a*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*c
os(d*x + c)))/d
```

Fricas [A] time = 1.4669, size = 300, normalized size = 2.97

$$\frac{15\,adx\cos(dx+c)^3-38\,a\cos(dx+c)^4-11\,a\cos(dx+c)^2-\left(15\,adx\cos(dx+c)^3-15\,a\cos(dx+c)^4-22\,a\cos(dx+c\right.}{15\left(d\cos(dx+c)^3\sin(dx+c)-d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(15*a*d*x*cos(d*x + c)^3 - 38*a*cos(d*x + c)^4 - 11*a*cos(d*x + c)^2 -
 (15*a*d*x*cos(d*x + c)^3 - 15*a*cos(d*x + c)^4 - 22*a*cos(d*x + c)^2 + 4*a
```

```
)*sin(d*x + c) + a)/(d*cos(d*x + c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**6,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")

[Out] Timed out
```

3.9 $\int (a + a\sin(c + dx))\tan^4(c + dx)\,dx$

Optimal. Leaf size=72

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

```
[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0743084, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2710, 3473, 8, 2590, 270}

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx &= \int \left(a\tan^4(c+dx)+a\sin(c+dx)\tan^4(c+dx)\right)\,dx\\ &= a\int \tan^4(c+dx)\,dx+a\int \sin(c+dx)\tan^4(c+dx)\,dx\\ &= \frac{a\tan^3(c+dx)}{3d}-a\int \tan^2(c+dx)\,dx-\frac{a\,\mathrm{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}+a\int 1\,dx-\frac{a\,\mathrm{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)\,dx,x,c\right.}{d}\\ &= ax-\frac{a\cos(c+dx)}{d}-\frac{2a\sec(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0483714, size = 81, normalized size = 1.12

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan^{-1}(\tan(c+dx))}{d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a
*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.057, size = 98, normalized size = 1.4

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^6}{3(\cos(dx+c))^3}-\frac{(\sin(dx+c))^6}{\cos(dx+c)}-\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4(\sin(dx+c))^2}{3}\right)\cos(dx+c)\right)+a\left(\frac{(\tan(dx+c))^3}{3}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+
c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A] time = 1.82471, size = 88, normalized size = 1.22

$$\frac{\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a-a\left(\frac{6\,\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}+3\,\cos\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - a*((6*cos(d*x + c)
^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A] time = 1.4761, size = 224, normalized size = 3.11

$$-\frac{3\,adx\cos\left(dx+c\right)-7\,a\cos\left(dx+c\right)^{2}-\left(3\,adx\cos\left(dx+c\right)-3\,a\cos\left(dx+c\right)^{2}-2\,a\right)\sin\left(dx+c\right)-a}{3\,\left(d\cos\left(dx+c\right)\sin\left(dx+c\right)-d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*d*x*cos(d*x + c) - 7*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) - 3
*a*cos(d*x + c)^2 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x + c) - d
*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin\left(c+dx\right)\tan^{4}\left(c+dx\right)\,dx+\int\tan^{4}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4, x)
)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.10 $\int (a + a\sin(c + dx))\tan^2(c + dx)\,dx$

Optimal. Leaf size=39

$$\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}-ax$$

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (a*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rubi [A] time = 0.104128, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2708, 2746, 12, 2735, 2648}

$$\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}-ax$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (a*Cos[c + d*x])/d + (a*Cos[c + d*x])/(d*(1 - Sin[c + d*x]))
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int
[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx &= a^2\int \frac{\sin^2(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= \frac{a\cos(c+dx)}{d}+a\int \frac{a\sin(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= \frac{a\cos(c+dx)}{d}+a^2\int \frac{\sin(c+dx)}{a-a\sin(c+dx)}\,dx\\ &= -ax+\frac{a\cos(c+dx)}{d}+a^2\int \frac{1}{a-a\sin(c+dx)}\,dx\\ &= -ax+\frac{a\cos(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d(a-a\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0452538, size = 47, normalized size = 1.21

$$\frac{a\cos(c+dx)}{d}-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

Maple [A] time = 0.039, size = 59, normalized size = 1.5

$$\frac{1}{d}\left(a\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+a\left(\tan(dx+c)-dx-c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a*(tan(d*x+c)-
d*x-c))
```

Maxima [A] time = 1.68792, size = 53, normalized size = 1.36

$$-\frac{(dx+c-\tan(dx+c))a - a\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [B] time = 1.42136, size = 194, normalized size = 4.97

$$-\frac{adx - a\cos(dx+c)^2 + (adx - 2a)\cos(dx+c) - (adx - a\cos(dx+c) + a)\sin(dx+c) - a}{d\cos(dx+c) - d\sin(dx+c) + d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*d*x - a*cos(d*x + c)^2 + (a*d*x - 2*a)*cos(d*x + c) - (a*d*x - a*cos(d*
x + c) + a)*sin(d*x + c) - a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin(c+dx)\tan^2(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] a*(Integral(sin(c + d*x)*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x)
)
```

Giac [B] time = 3.47197, size = 1361, normalized size = 34.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4*
tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a*ta
n(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*tan(1
/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3*tan
(1/2*c)^3 + 2*a*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d*x)^4
*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x*tan(d
*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*
c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x
)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*tan(1/2*d
*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c)
^3 - 8*a*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*a*tan(d*x)*ta
n(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*a
*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*a*tan(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(c) - 8*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*a*ta
n(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a*tan(d*x)*tan(1
/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - a*tan(d*x)*
tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^3*tan(1/2*c)*tan(
c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^4*tan(c) + 2*a*tan
(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*a*tan(1/2*d*x)^3*tan(1/2*
c) + 24*a*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*tan(1/2*d*x)*tan(1/2*c)^3 + 2*a
*tan(1/2*c)^4 + a*d*x*tan(d*x)*tan(c) + 8*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c
)*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*
c)*tan(c) - a*d*x - 8*a*tan(1/2*d*x)*tan(1/2*c) - 2*a*tan(d*x)*tan(c) + a*t
an(d*x) + a*tan(c) + 2*a)/(d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) -
d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*ta
n(c) + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c) -
 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*
tan(1/2*c)^3*tan(c) - d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4 + 4
*d*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)
```

```
^4 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 4*d*tan(1/2*d*x)*tan(1/2
*c) + d*tan(d*x)*tan(c) - d)
```

3.11 $\int \cot^2(c + dx)(a + a\sin(c + dx))\,dx$

Optimal. Leaf size=41

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

```
[Out] -(a*x) - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rubi [A] time = 0.051557, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2710, 2592, 321, 206, 3473, 8}

$$\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx &= \int \left(a\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx \\
&= a\int \cos(c+dx)\cot(c+dx)\,dx + a\int \cot^2(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)}{d} - a\int 1\,dx - \frac{a\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -ax + \frac{a\cos(c+dx)}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d} \\
&= -ax - \frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{a\cos(c+dx)}{d} - \frac{a\cot(c+dx)}{d}
\end{aligned}
$$

Mathematica [C] time = 0.0480103, size = 75, normalized size = 1.83

$$
-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d} + \frac{a\cos(c+dx)}{d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d
```

Maple [A] time = 0.029, size = 57, normalized size = 1.4

$$-ax+\frac{\cos\left(dx+c\right)a}{d}-\frac{a\cot\left(dx+c\right)}{d}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}-\frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c)),x)
```

```
[Out] -a*x+a*cos(d*x+c)/d-a*cot(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a
```

Maxima [A] time = 2.30272, size = 73, normalized size = 1.78

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-a\left(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+c\right)-1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.43844, size = 236, normalized size = 5.76

$$-\frac{a\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-a\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)+2\,(adx-a\cos(dx}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - a*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin(c+dx)\cot^2(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x)
)
```

Giac [B] time = 1.25007, size = 146, normalized size = 3.56

$$-\frac{6\,(dx+c)a - 6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - 3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 3\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*a*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

3.12 $\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

```
[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rubi [A] time = 0.0797018, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3a\tanh^{-1}(\cos(c+dx))}{2d}+ax$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx = \int \left(a\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx$$
$$= a\int \cos(c+dx)\cot^3(c+dx)\,dx + a\int \cot^4(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)}{3d} - a\int \cot^2(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + a\int 1\,dx + \frac{(3a)\,\mathrm{S}}{}$$
$$= ax - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+}{3d}$$
$$= ax + \frac{3a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3a\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{a\cos(c+dx)}{2d}$$

Mathematica [C] time = 0.0531241, size = 125, normalized size = 1.52

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{a\cos(c+dx)}{d} - \frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3a\log\left(\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*a*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2
)/(8*d)
```

Maple [A] time = 0.034, size = 106, normalized size = 1.3

$$-\frac{a(\cos(dx+c))^5}{2d(\sin(dx+c))^2} - \frac{a(\cos(dx+c))^3}{2d} - \frac{3\cos(dx+c)\,a}{2d} - \frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{2d} - \frac{a(\cot(dx+c))^3}{3d} + \frac{a\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a/sin(d*x+c)^2*cos(d*x+c)^5-1/2/d*cos(d*x+c)^3*a-3/2*a*cos(d*x+c)/d-
3/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1
/d*c*a
```

Maxima [A] time = 1.69055, size = 124, normalized size = 1.51

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*a*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.50448, size = 425, normalized size = 5.18

$$\frac{16\,a\cos\left(dx+c\right)^3+9\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\left(a\cos\left(dx+c\right)^2-a\right)\log\left(-\frac{1}{2}\,\cos\left(dx\right.\right.}{12\left(d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*a*cos(d*x
+ c)^3 - 2*a*d*x + 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin\left(c+dx\right)\cot^4\left(c+dx\right)dx+\int\cot^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

```
[Out] a*(Integral(sin(c + d*x)*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x)
)
```

Giac [A] time = 1.27813, size = 190, normalized size = 2.32

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,(dx+c)a-36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{\cdots}{\tan\left(\frac{1}{2}\cdots\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*a*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*a/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.13 $\int \cot^6(c + dx)(a + a\sin(c + dx))\,dx$

Optimal. Leaf size=122

$$\frac{15a\cos(c + dx)}{8d} - \frac{a\cot^5(c + dx)}{5d} + \frac{a\cot^3(c + dx)}{3d} - \frac{a\cot(c + dx)}{d} - \frac{a\cos(c + dx)\cot^4(c + dx)}{4d} + \frac{5a\cos(c + dx)\cot^2(c +}{8d}$$

```
[Out] -(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rubi [A] time = 0.0962739, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2710, 2592, 288, 321, 206, 3473, 8}

$$\frac{15a\cos(c + dx)}{8d} - \frac{a\cot^5(c + dx)}{5d} + \frac{a\cot^3(c + dx)}{3d} - \frac{a\cot(c + dx)}{d} - \frac{a\cos(c + dx)\cot^4(c + dx)}{4d} + \frac{5a\cos(c + dx)\cot^2(c +}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx = \int \left(a\cos(c+dx)\cot^5(c+dx)+a\cot^6(c+dx)\right)\,dx$$
$$= a\int \cos(c+dx)\cot^5(c+dx)\,dx + a\int \cot^6(c+dx)\,dx$$
$$= -\frac{a\cot^5(c+dx)}{5d} - a\int \cot^4(c+dx)\,dx - \frac{a\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c+dx)\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + a\int \cot^2(c+dx)$$
$$= -\frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c+dx)}{3d} - \frac{a\cos(c+dx)\text{c}}{4d}$$
$$= -ax + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c}{3d}$$
$$= -ax - \frac{15a\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15a\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5a\cos(c+}{}$$

Mathematica [C] time = 0.0710632, size = 164, normalized size = 1.34

$$-\frac{a\cot^5(c+dx)\,_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} + \frac{a\cos(c+dx)}{d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{9a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x])/d + (9*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^
4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x
]^2])/(5*d) - (15*a*Log[Cos[(c + d*x)/2]])/(8*d) + (15*a*Log[Sin[(c + d*x)/
2]])/(8*d) - (9*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d
)
```

Maple [A] time = 0.038, size = 159, normalized size = 1.3

$$-\frac{a(\cos(dx+c))^7}{4d(\sin(dx+c))^4} + \frac{3a(\cos(dx+c))^7}{8d(\sin(dx+c))^2} + \frac{3(\cos(dx+c))^5a}{8d} + \frac{5a(\cos(dx+c))^3}{8d} + \frac{15\cos(dx+c)a}{8d} + \frac{15a\ln(\csc(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*a/sin(d*x+c)^2*cos(d*x+c)^7+3/8/d*
cos(d*x+c)^5*a+5/8/d*cos(d*x+c)^3*a+15/8*a*cos(d*x+c)/d+15/8/d*a*ln(csc(d*x
+c)-cot(d*x+c))-1/5*a*cot(d*x+c)^5/d+1/3*a*cot(d*x+c)^3/d-a*cot(d*x+c)/d-a*
x-1/d*c*a
```

Maxima [A] time = 1.63037, size = 169, normalized size = 1.39

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+15\,a\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a + 15*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*
log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.53839, size = 620, normalized size = 5.08

$$-\frac{368\,a\cos\left(dx+c\right)^5-560\,a\cos\left(dx+c\right)^3+225\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(a*cos(d*x + c)^4
 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(
a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*a*cos(d*x +
 c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*a*cos(d*x + c)^3 + 8*a*d*x - 15*a*cos(
```

```
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29258, size = 269, normalized size = 2.2

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-960\,(dx+c)a+1800\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
a*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*a/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*a*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*a*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*a*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.14 $\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx$

Optimal. Leaf size=119

$$-\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\mathrm{lo}}{}$$

```
[Out] (-31*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) -
 (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*Sin[
c + d*x])^2) - (9*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0818173, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^4}{4d(a-a\sin(c+dx))^2}-\frac{9a^3}{4d(a-a\sin(c+dx))}-\frac{2a^2\sin(c+dx)}{d}-\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] (-31*a^2*Log[1 - Sin[c + d*x]])/(8*d) - (a^2*Log[1 + Sin[c + d*x]])/(8*d) -
 (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d) + a^4/(4*d*(a - a*Sin[
c + d*x])^2) - (9*a^3)/(4*d*(a - a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^4}{2(a-x)^3}-\frac{9a^3}{4(a-x)^2}+\frac{31a^2}{8(a-x)}-x-\frac{a^2}{8(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{31a^2\log(1-\sin(c+dx))}{8d}-\frac{a^2\log(1+\sin(c+dx))}{8d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2}{}$$

Mathematica [A] time = 0.244541, size = 75, normalized size = 0.63

$$-\frac{a^2\left(4\sin^2(c+dx)+16\sin(c+dx)-\frac{18}{\sin(c+dx)-1}-\frac{2}{(\sin(c+dx)-1)^2}+31\log(1-\sin(c+dx))+\log(\sin(c+dx)+1)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

```
[Out] -(a^2*(31*Log[1 - Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[c + d
*x])^2 - 18/(-1 + Sin[c + d*x]) + 16*Sin[c + d*x] + 4*Sin[c + d*x]^2))/(8*d
)
```

Maple [B] time = 0.085, size = 261, normalized size = 2.2

$$\frac{a^2(\sin(dx+c))^8}{4d(\cos(dx+c))^4}-\frac{a^2(\sin(dx+c))^8}{2d(\cos(dx+c))^2}-\frac{a^2(\sin(dx+c))^6}{2d}-\frac{3a^2(\sin(dx+c))^4}{4d}-\frac{3a^2(\sin(dx+c))^2}{2d}-4\frac{a^2\ln(\cos(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x)
```

```
[Out] 1/4/d*a^2*sin(d*x+c)^8/cos(d*x+c)^4-1/2/d*a^2*sin(d*x+c)^8/cos(d*x+c)^2-1/2
/d*a^2*sin(d*x+c)^6-3/4/d*a^2*sin(d*x+c)^4-3/2*a^2*sin(d*x+c)^2/d-4/d*a^2*l
n(cos(d*x+c))+1/2/d*a^2*sin(d*x+c)^7/cos(d*x+c)^4-3/4/d*a^2*sin(d*x+c)^7/co
s(d*x+c)^2-3/4/d*a^2*sin(d*x+c)^5-5/4/d*a^2*sin(d*x+c)^3-15/4*a^2*sin(d*x+c
)/d+15/4/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*a^2*tan(d*x+c)^4-1/2/d*a^2*t
an(d*x+c)^2
```

Maxima [A] time = 1.12912, size = 130, normalized size = 1.09

$$-\frac{4\,a^2\sin\left(dx+c\right)^2+a^2\log\left(\sin\left(dx+c\right)+1\right)+31\,a^2\log\left(\sin\left(dx+c\right)-1\right)+16\,a^2\sin\left(dx+c\right)-\frac{2\left(9\,a^2\sin(dx+c)-8\,a^2\right)}{\sin(dx+c)^2-2\,\sin(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(4*a^2*sin(d*x + c)^2 + a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*x
 + c) - 1) + 16*a^2*sin(d*x + c) - 2*(9*a^2*sin(d*x + c) - 8*a^2)/(sin(d*x
+ c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.55703, size = 412, normalized size = 3.46

$$\frac{4\,a^2\cos\left(dx+c\right)^4+22\,a^2\cos\left(dx+c\right)^2-12\,a^2-\left(a^2\cos\left(dx+c\right)^2+2\,a^2\sin\left(dx+c\right)-2\,a^2\right)\log\left(\sin\left(dx+c\right)+1\right)-31\,(}{8\left(d\cos\left(dx+c\right)^2+2\,d\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*cos(d*x + c)^4 + 22*a^2*cos(d*x + c)^2 - 12*a^2 - (a^2*cos(d*x +
 c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(sin(d*x + c) + 1) - 31*(a^2*cos(d*x
 + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*cos
(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.15 $\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^3}{d(a-a\sin(c+dx))}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\log(1-\sin(c+dx))}{d}$$

```
[Out] (3*a^2*Log[1 - Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x
]^2)/(2*d) + a^3/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0621593, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^3}{d(a-a\sin(c+dx))}+\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (3*a^2*Log[1 - Sin[c + d*x]])/d + (2*a^2*Sin[c + d*x])/d + (a^2*Sin[c + d*x
]^2)/(2*d) + a^3/(d*(a - a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2}\,dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^3}{(a-x)^2}-\frac{3a^2}{a-x}+x\right)dx,x,a\sin(c+dx)\right)}{d}$$
$$= \frac{3a^2\log(1-\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^3}{d(a-a\sin(c+a}$$

Mathematica [A] time = 0.102539, size = 54, normalized size = 0.75

$$\frac{a^2\left(\sin^2(c+dx)+4\sin(c+dx)+\frac{2}{1-\sin(c+dx)}+6\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (a^2*(6*Log[1 - Sin[c + d*x]] + 2/(1 - Sin[c + d*x]) + 4*Sin[c + d*x] + Sin
[c + d*x]^2))/(2*d)
```

Maple [B] time = 0.071, size = 162, normalized size = 2.3

$$\frac{a^2(\sin(dx+c))^6}{2d(\cos(dx+c))^2}+\frac{a^2(\sin(dx+c))^4}{2d}+\frac{a^2(\sin(dx+c))^2}{d}+3\frac{a^2\ln(\cos(dx+c))}{d}+\frac{a^2(\sin(dx+c))^5}{d(\cos(dx+c))^2}+\frac{a^2(\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^2*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*a^2*sin(d*x+c)^4+a^2*sin(d*x+c)^2
/d+3/d*a^2*ln(cos(d*x+c))+1/d*a^2*sin(d*x+c)^5/cos(d*x+c)^2+1/d*a^2*sin(d*x
+c)^3+3*a^2*sin(d*x+c)/d-3/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*a^2*tan(d*
x+c)^2
```

Maxima [A] time = 1.11192, size = 78, normalized size = 1.08

$$\frac{a^2 \sin(dx+c)^2 + 6\,a^2 \log(\sin(dx+c)-1) + 4\,a^2 \sin(dx+c) - \frac{2\,a^2}{\sin(dx+c)-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*sin(d*x + c)^2 + 6*a^2*log(sin(d*x + c) - 1) + 4*a^2*sin(d*x + c)
- 2*a^2/(sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.48194, size = 212, normalized size = 2.94

$$-\frac{6\,a^2 \cos(dx+c)^2 - 3\,a^2 - 12\left(a^2 \sin(dx+c) - a^2\right)\log(-\sin(dx+c)+1) + \left(2\,a^2 \cos(dx+c)^2 + 7\,a^2\right)\sin(dx+c)}{4\,(d\sin(dx+c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^2*cos(d*x + c)^2 - 3*a^2 - 12*(a^2*sin(d*x + c) - a^2)*log(-sin(d
*x + c) + 1) + (2*a^2*cos(d*x + c)^2 + 7*a^2)*sin(d*x + c))/(d*sin(d*x + c)
 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sin(c+dx)\tan^3(c+dx)\,dx + \int \sin^2(c+dx)\tan^3(c+dx)\,dx + \int \tan^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.16 $\int (a + a\sin(c + dx))^2 \tan(c + dx)\, dx$

Optimal. Leaf size=52

$$-\frac{a^2 \sin^2(c+dx)}{2d} - \frac{2a^2 \sin(c+dx)}{d} - \frac{2a^2 \log(1-\sin(c+dx))}{d}$$

```
[Out] (-2*a^2*Log[1 - Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*
x]^2)/(2*d)
```

Rubi [A] time = 0.0375399, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 77}

$$-\frac{a^2 \sin^2(c+dx)}{2d} - \frac{2a^2 \sin(c+dx)}{d} - \frac{2a^2 \log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] (-2*a^2*Log[1 - Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*
x]^2)/(2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{2a^2}{a-x}-x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{2a^2\log(1-\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} - \frac{a^2\sin^2(c+dx)}{2d}$$

Mathematica [A] time = 0.038228, size = 40, normalized size = 0.77

$$-\frac{a^2\left(\sin^2(c+dx)+4\sin(c+dx)+4\log(1-\sin(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -(a^2*(4*Log[1 - Sin[c + d*x]] + 4*Sin[c + d*x] + Sin[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.042, size = 69, normalized size = 1.3

$$-\frac{a^2(\sin(dx+c))^2}{2d} - 2\frac{a^2\ln(\cos(dx+c))}{d} - 2\frac{a^2\sin(dx+c)}{d} + 2\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c),x)
```

```
[Out] -1/2*a^2*sin(d*x+c)^2/d-2/d*a^2*ln(cos(d*x+c))-2*a^2*sin(d*x+c)/d+2/d*a^2*l
n(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.12482, size = 58, normalized size = 1.12

$$-\frac{a^2\sin(dx+c)^2+4a^2\log(\sin(dx+c)-1)+4a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*sin(d*x + c)^2 + 4*a^2*log(sin(d*x + c) - 1) + 4*a^2*sin(d*x + c)
)/d
```

Fricas [A] time = 1.41834, size = 108, normalized size = 2.08

$$\frac{a^2\cos(dx+c)^2 - 4a^2\log(-\sin(dx+c)+1) - 4a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cos(d*x + c)^2 - 4*a^2*log(-sin(d*x + c) + 1) - 4*a^2*sin(d*x + c)
)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sin(c+dx)\tan(c+dx)\,dx + \int \sin^2(c+dx)\tan(c+dx)\,dx + \int \tan(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c),x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x), x) + Integral(sin(c + d*x)**2*t
an(c + d*x), x) + Integral(tan(c + d*x), x))
```

Giac [B] time = 4.57216, size = 9038, normalized size = 173.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

[Out] -1/4*(4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 +
 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)
^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(
d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - a^2*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*
tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*log(2*(tan(
1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x
)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan
(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1
/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan
(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 4*a^2*lo
g(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(
1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1
/2*d*x)^2*tan(c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*ta
n(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)
)*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 16*a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan
(1/2*c)*tan(c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2
*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1
)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*
x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*
a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d

```
*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*c)
^2*tan(c)^2 - 16*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + 4*a^2*
log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*ta
n(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*
tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(4*(tan(c)
^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 +
tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 +
 a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a^2*tan(d*x)*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c) - a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - a^2*tan(d*x
)^2*tan(1/2*c)^2*tan(c)^2 + a^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^
2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*
tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*t
an(1/2*d*x)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
) + 1))*tan(d*x)^2*tan(1/2*d*x)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*
tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d
*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2 - 16*a^2*tan(d*x)^2*tan(1/2*d*x)
^2*tan(1/2*c) + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1))*tan(d*x)^2*tan(1/2*c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d
*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2
*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2 + 4*a^2*log(4*(tan(c)^2 +
 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(
d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*c)^2 - 16*a^2*tan(d*x)^
2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^
4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^
2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*t
an(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
 - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*log(2*(tan(1/2*c)
^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(
1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 +
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2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4
*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(1
/2*c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)
)*tan(d*x)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + ta
n(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*t
an(1/2*c) + 1))*tan(d*x)^2*tan(c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1))*tan(d*x)^2*tan(c)^2 + 16*a^2*tan(d*x)^2*tan(1/2*d*x)*tan
(c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2
*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*
tan(1/2*d*x)^2*tan(c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan
(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 -
 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(c)^2 + 16*a^2*tan(d*x)^2*tan(1/
2*c)*tan(c)^2 - 16*a^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 + 4*a^2*log(2*(ta
n(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c)
+ 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
 tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^2*tan(c)^2 -
 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2
*c)^2*tan(c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*ta
n(1/2*c)^2*tan(c)^2 - 16*a^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + a^2*tan(d
*x)^2*tan(1/2*d*x)^2 + a^2*tan(d*x)^2*tan(1/2*c)^2 - a^2*tan(1/2*d*x)^2*tan
(1/2*c)^2 + 4*a^2*tan(d*x)*tan(1/2*d*x)^2*tan(c) + 4*a^2*tan(d*x)*tan(1/2*c
)^2*tan(c) - a^2*tan(d*x)^2*tan(c)^2 + a^2*tan(1/2*d*x)^2*tan(c)^2 + a^2*ta
n(1/2*c)^2*tan(c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/
2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(
1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d
*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan
```

(1/2*c) + 1))*tan(d*x)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
2*tan(1/2*c) + 1))*tan(d*x)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(
c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*
tan(c) + 1))*tan(d*x)^2 + 16*a^2*tan(d*x)^2*tan(1/2*d*x) + 4*a^2*log(2*(tan
(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 4*a^2*l
og(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan
(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2
+ 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2 +
16*a^2*tan(d*x)^2*tan(1/2*c) - 16*a^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a^2*log
(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1
/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*
x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^2 - 4*
a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d
*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan
(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)
^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c)
+ tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*c)^2 -
16*a^2*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2
*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 2*tan(1/2*c) + 1))*tan(c)^2 - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(
1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(c)^2 + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d
*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2
*tan(d*x)*tan(c) + 1))*tan(c)^2 + 16*a^2*tan(1/2*d*x)*tan(c)^2 + 16*a^2*tan
(1/2*c)*tan(c)^2 + a^2*tan(d*x)^2 - a^2*tan(1/2*d*x)^2 - a^2*tan(1/2*c)^2 +
4*a^2*tan(d*x)*tan(c) + a^2*tan(c)^2 + 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan
(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*t
an(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 2*tan(1/2*c) + 1)) - 4*a^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d

```
*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2
*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 -
 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) + 2*tan(1/2*c) + 1)) + 4*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2
 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(
c) + 1)) + 16*a^2*tan(1/2*d*x) + 16*a^2*tan(1/2*c) - a^2)/(d*tan(d*x)^2*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^
2 + d*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*c)^2*tan(c)
^2 + d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2 +
 d*tan(d*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)^2*t
an(c)^2 + d*tan(1/2*d*x)^2*tan(c)^2 + d*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^
2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d*tan(c)^2 + d)
```

3.17 $\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=30

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^4}{2a^2d}$$

```
[Out] -(Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4)/(2*a^2*d)
```

Rubi [A] time = 0.0392531, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 74}

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^4}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4)/(2*a^2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^3}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^2(c+dx)(a+a\sin(c+dx))^4}{2a^2d}$$

Mathematica [A] time = 0.0412311, size = 28, normalized size = 0.93

$$-\frac{a^2(\sin(c+dx)+1)^4\csc^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[c + d*x]^2*(1 + Sin[c + d*x])^4)/(2*d)
```

Maple [B] time = 0.049, size = 94, normalized size = 3.1

$$\frac{a^2(\cos(dx+c))^2}{2d} - 2\frac{a^2(\cos(dx+c))^4}{d\sin(dx+c)} - 2\frac{a^2(\cos(dx+c))^2\sin(dx+c)}{d} - 4\frac{a^2\sin(dx+c)}{d} - \frac{a^2(\cot(dx+c))^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d*a^2*cos(d*x+c)^2-2/d*a^2/sin(d*x+c)*cos(d*x+c)^4-2/d*a^2*cos(d*x+c)^2
*sin(d*x+c)-4*a^2*sin(d*x+c)/d-1/2/d*a^2*cot(d*x+c)^2
```

Maxima [A] time = 1.11304, size = 72, normalized size = 2.4

$$-\frac{a^2\sin(dx+c)^2 + 4a^2\sin(dx+c) + \frac{4a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*sin(d*x + c)^2 + 4*a^2*sin(d*x + c) + (4*a^2*sin(d*x + c) + a^2)/
sin(d*x + c)^2)/d
```

Fricas [B] time = 1.44029, size = 173, normalized size = 5.77

$$\frac{2\,a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right)^2+3\,a^2-8\left(a^2\cos\left(dx+c\right)^2-2\,a^2\right)\sin\left(dx+c\right)}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 + 3*a^2 - 8*(a^2*cos(d*x +
 c)^2 - 2*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sin\left(c+dx\right)\cot^3\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\cot^3\left(c+dx\right)dx+\int\cot^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x))
```

Giac [A] time = 1.37063, size = 63, normalized size = 2.1

$$-\frac{a^2\left(\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)\right)^2+4\,a^2\left(\frac{1}{\sin(dx+c)}+\sin\left(dx+c\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^2*(1/sin(d*x + c) + sin(d*x + c))^2 + 4*a^2*(1/sin(d*x + c) + sin(d
*x + c)))/d
```

3.18 $\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=132

$$-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{2d}+\frac{2a^2\csc^3(c+dx)}{d}-\frac{6a^2\csc(}{d}$$

```
[Out] (-6*a^2*Csc[c + d*x])/d + (2*a^2*Csc[c + d*x]^3)/d + (a^2*Csc[c + d*x]^4)/(
2*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d*x]^6)/(6*d) + (2*a^2*L
og[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0754306, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{a^2\sin^2(c+dx)}{2d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\csc^6(c+dx)}{6d}-\frac{2a^2\csc^5(c+dx)}{5d}+\frac{a^2\csc^4(c+dx)}{2d}+\frac{2a^2\csc^3(c+dx)}{d}-\frac{6a^2\csc(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-6*a^2*Csc[c + d*x])/d + (2*a^2*Csc[c + d*x]^3)/d + (a^2*Csc[c + d*x]^4)/(
2*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (a^2*Csc[c + d*x]^6)/(6*d) + (2*a^2*L
og[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^2)/(2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)^5}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^8}{x^7}+\frac{2a^7}{x^6}-\frac{2a^6}{x^5}-\frac{6a^5}{x^4}+\frac{6a^3}{x^2}+\frac{2a^2}{x}-x\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{6a^2\csc(c+dx)}{d}+\frac{2a^2\csc^3(c+dx)}{d}+\frac{a^2\csc^4(c+dx)}{2d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{a^2}{}$$

Mathematica [A] time = 0.215678, size = 86, normalized size = 0.65

$$-\frac{a^2\left(15\sin^2(c+dx)+60\sin(c+dx)+5\csc^6(c+dx)+12\csc^5(c+dx)-15\csc^4(c+dx)-60\csc^3(c+dx)+180\csc(c\right.}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(180*Csc[c + d*x] - 60*Csc[c + d*x]^3 - 15*Csc[c + d*x]^4 + 12*Csc[c
+ d*x]^5 + 5*Csc[c + d*x]^6 - 60*Log[Sin[c + d*x]] + 60*Sin[c + d*x] + 15*S
in[c + d*x]^2))/(30*d)
```

Maple [B] time = 0.049, size = 313, normalized size = 2.4

$$-\frac{a^2(\cos(dx+c))^8}{4d(\sin(dx+c))^4}+\frac{a^2(\cos(dx+c))^8}{2d(\sin(dx+c))^2}+\frac{a^2(\cos(dx+c))^6}{2d}+\frac{3a^2(\cos(dx+c))^4}{4d}+\frac{3a^2(\cos(dx+c))^2}{2d}+2\frac{a^2\ln(\sin}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2/sin(d*x+c)^4*cos(d*x+c)^8+1/2/d*a^2/sin(d*x+c)^2*cos(d*x+c)^8+1/
2/d*a^2*cos(d*x+c)^6+3/4/d*a^2*cos(d*x+c)^4+3/2/d*a^2*cos(d*x+c)^2+2*a^2*ln
(sin(d*x+c))/d-2/5/d*a^2/sin(d*x+c)^5*cos(d*x+c)^8+2/5/d*a^2/sin(d*x+c)^3*c
os(d*x+c)^8-2/d*a^2/sin(d*x+c)*cos(d*x+c)^8-32/5*a^2*sin(d*x+c)/d-2/d*a^2*s
in(d*x+c)*cos(d*x+c)^6-12/5/d*a^2*sin(d*x+c)*cos(d*x+c)^4-16/5/d*a^2*cos(d*
x+c)^2*sin(d*x+c)-1/6/d*a^2*cot(d*x+c)^6+1/4/d*a^2*cot(d*x+c)^4-1/2/d*a^2*c
ot(d*x+c)^2
```

Maxima [A] time = 1.08562, size = 144, normalized size = 1.09

$$-\frac{15\,a^2\sin\left(dx+c\right)^2-60\,a^2\log\left(\sin\left(dx+c\right)\right)+60\,a^2\sin\left(dx+c\right)+\frac{180\,a^2\sin(dx+c)^5-60\,a^2\sin(dx+c)^3-15\,a^2\sin(dx+c)^2+12\,a^2\sin(dx+c)}{\sin(dx+c)^6}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*a^2*sin(d*x + c)^2 - 60*a^2*log(sin(d*x + c)) + 60*a^2*sin(d*x +
c) + (180*a^2*sin(d*x + c)^5 - 60*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*x + c)^
2 + 12*a^2*sin(d*x + c) + 5*a^2)/sin(d*x + c)^6)/d
```

Fricas [A] time = 1.76805, size = 508, normalized size = 3.85

$$\frac{30\,a^2\cos\left(dx+c\right)^8-105\,a^2\cos\left(dx+c\right)^6+135\,a^2\cos\left(dx+c\right)^4-45\,a^2\cos\left(dx+c\right)^2-5\,a^2+120\left(a^2\cos\left(dx+c\right)^6-3\,a^2\right.}{60\left(d\cos\left(dx+c\right)^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(30*a^2*cos(d*x + c)^8 - 105*a^2*cos(d*x + c)^6 + 135*a^2*cos(d*x + c)
^4 - 45*a^2*cos(d*x + c)^2 - 5*a^2 + 120*(a^2*cos(d*x + c)^6 - 3*a^2*cos(d*
x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x + c)) - 24*(5*a^2*co
s(d*x + c)^6 - 30*a^2*cos(d*x + c)^4 + 40*a^2*cos(d*x + c)^2 - 16*a^2)*sin(
d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

[Out] Timed out

Giac [A] time = 1.49559, size = 163, normalized size = 1.23

$$-\frac{15\,a^2\sin\left(dx+c\right)^2-60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a^2\sin\left(dx+c\right)+\frac{147\,a^2\sin(dx+c)^6+180\,a^2\sin(dx+c)^5-60\,a^2\sin(dx+c)^3-15\,a^2\sin(d}{\sin(dx+c)^6}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/30*(15*a^2*sin(d*x + c)^2 - 60*a^2*log(abs(sin(d*x + c))) + 60*a^2*sin(d
*x + c) + (147*a^2*sin(d*x + c)^6 + 180*a^2*sin(d*x + c)^5 - 60*a^2*sin(d*x
 + c)^3 - 15*a^2*sin(d*x + c)^2 + 12*a^2*sin(d*x + c) + 5*a^2)/sin(d*x + c)
^6)/d
```

3.19 $\int (a + a\sin(c + dx))^2 \tan^6(c + dx)\,dx$

Optimal. Leaf size=149

$$\frac{2a^2\cos(c+dx)}{d} + \frac{9a^2\tan^5(c+dx)}{10d} - \frac{3a^2\tan^3(c+dx)}{2d} + \frac{9a^2\tan(c+dx)}{2d} + \frac{2a^2\sec^5(c+dx)}{5d} - \frac{2a^2\sec^3(c+dx)}{d} + 6a^2\mathrm{s}$$

```
[Out] (-9*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (6*a^2*Sec[c + d*x])/d - (2*a^2*Sec
[c + d*x]^3)/d + (2*a^2*Sec[c + d*x]^5)/(5*d) + (9*a^2*Tan[c + d*x])/(2*d)
- (3*a^2*Tan[c + d*x]^3)/(2*d) + (9*a^2*Tan[c + d*x]^5)/(10*d) - (a^2*Sin[c
 + d*x]^2*Tan[c + d*x]^5)/(2*d)
```

Rubi [A] time = 0.165139, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2710, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$\frac{2a^2\cos(c+dx)}{d} + \frac{9a^2\tan^5(c+dx)}{10d} - \frac{3a^2\tan^3(c+dx)}{2d} + \frac{9a^2\tan(c+dx)}{2d} + \frac{2a^2\sec^5(c+dx)}{5d} - \frac{2a^2\sec^3(c+dx)}{d} + 6a^2\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] (-9*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (6*a^2*Sec[c + d*x])/d - (2*a^2*Sec
[c + d*x]^3)/d + (2*a^2*Sec[c + d*x]^5)/(5*d) + (9*a^2*Tan[c + d*x])/(2*d)
- (3*a^2*Tan[c + d*x]^3)/(2*d) + (9*a^2*Tan[c + d*x]^5)/(10*d) - (a^2*Sin[c
 + d*x]^2*Tan[c + d*x]^5)/(2*d)
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\tan^6(c+dx)\,dx = \int \left(a^2\tan^6(c+dx)+2a^2\sin(c+dx)\tan^6(c+dx)+a^2\sin^2(c+dx)\tan^6(c+dx\right.$$

$$= a^2\int \tan^6(c+dx)\,dx + a^2\int \sin^2(c+dx)\tan^6(c+dx)\,dx + \left(2a^2\right)\int \sin(c+dx$$

$$= \frac{a^2\tan^5(c+dx)}{5d} - a^2\int \tan^4(c+dx)\,dx + \frac{a^2\,\mathrm{Subst}\left(\int \frac{x^8}{\left(1+x^2\right)^2}\,dx, x, \tan(c+dx)\right.}{d}$$

$$= -\frac{a^2\tan^3(c+dx)}{3d} + \frac{a^2\tan^5(c+dx)}{5d} - \frac{a^2\sin^2(c+dx)\tan^5(c+dx)}{2d} + a^2\int \tan^2$$

$$= \frac{2a^2\cos(c+dx)}{d} + \frac{6a^2\sec(c+dx)}{d} - \frac{2a^2\sec^3(c+dx)}{d} + \frac{2a^2\sec^5(c+dx)}{5d} + \frac{a^2\,\mathrm{t}}{}$$

$$= -a^2x + \frac{2a^2\cos(c+dx)}{d} + \frac{6a^2\sec(c+dx)}{d} - \frac{2a^2\sec^3(c+dx)}{d} + \frac{2a^2\sec^5(c+dx}{5d}$$

$$= -\frac{9a^2x}{2} + \frac{2a^2\cos(c+dx)}{d} + \frac{6a^2\sec(c+dx)}{d} - \frac{2a^2\sec^3(c+dx)}{d} + \frac{2a^2\sec^5(c+d}{5d}$$

Mathematica [A] time = 0.837593, size = 174, normalized size = 1.17

$$-\frac{a^2\sec^5(c+dx)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4(250\sin(c+dx)-720c\sin(2(c+dx))-720dx\sin(2(c+dx))-824}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

```
[Out] -(a^2*Sec[c + d*x]^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-500 + 10*(10
3 + 90*c + 90*d*x)*Cos[c + d*x] - 544*Cos[2*(c + d*x)] - 206*Cos[3*(c + d*x
)] - 180*c*Cos[3*(c + d*x)] - 180*d*x*Cos[3*(c + d*x)] + 20*Cos[4*(c + d*x)
] + 250*Sin[c + d*x] - 824*Sin[2*(c + d*x)] - 720*c*Sin[2*(c + d*x)] - 720*
d*x*Sin[2*(c + d*x)] + 351*Sin[3*(c + d*x)] + 5*Sin[5*(c + d*x)]))/(160*d)
```

Maple [A] time = 0.078, size = 251, normalized size = 1.7

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^9}{5(\cos(dx+c))^5} - \frac{4(\sin(dx+c))^9}{15(\cos(dx+c))^3} + \frac{8(\sin(dx+c))^9}{5\cos(dx+c)} + \frac{8\cos(dx+c)}{5}\left((\sin(dx+c))^7 + \frac{7(\sin(dx+c))^5}{6} + \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x)
```

```
[Out] 1/d*(a^2*(1/5*sin(d*x+c)^9/cos(d*x+c)^5-4/15*sin(d*x+c)^9/cos(d*x+c)^3+8/5*
sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)
^3+35/16*sin(d*x+c))*cos(d*x+c)-7/2*d*x-7/2*c)+2*a^2*(1/5*sin(d*x+c)^8/cos(
d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*
x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/5*tan(d*x+c)^5
-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.60183, size = 205, normalized size = 1.38

$$\frac{\left(6\tan\left(dx+c\right)^5-20\tan\left(dx+c\right)^3-105\,dx-105\,c+\frac{15\tan(dx+c)}{\tan(dx+c)^2+1}+90\tan\left(dx+c\right)\right)a^2+2\left(3\tan\left(dx+c\right)^5-5\tan(d\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/30*((6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*x
+ c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^2 + 2*(3*tan(d*x + c)^5 - 5*
tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^2 + 12*a^2*((15*cos(d*x
 + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*cos(d*x + c)))/d
```

Fricas [A] time = 1.70126, size = 382, normalized size = 2.56

$$-\frac{45\,a^2dx\cos\left(dx+c\right)^3-10\,a^2\cos\left(dx+c\right)^4-90\,a^2dx\cos\left(dx+c\right)+78\,a^2\cos\left(dx+c\right)^2-4\,a^2-\left(5\,a^2\cos\left(dx+c\right)^4-90\right.}{10\left(d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/10*(45*a^2*d*x*cos(d*x + c)^3 - 10*a^2*cos(d*x + c)^4 - 90*a^2*d*x*cos(d
*x + c) + 78*a^2*cos(d*x + c)^2 - 4*a^2 - (5*a^2*cos(d*x + c)^4 - 90*a^2*d*
x*cos(d*x + c) + 84*a^2*cos(d*x + c)^2 - 6*a^2)*sin(d*x + c))/(d*cos(d*x +
c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

3.20 $\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=120

$$-\frac{16a^2\cos(c+dx)}{3d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{8a^2\sin^2(c+dx)\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{7a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^2x}{2}$$

```
[Out] (7*a^2*x)/2 - (16*a^2*Cos[c + d*x])/(3*d) - (7*a^2*Cos[c + d*x]*Sin[c + d*x
])/(2*d) - (8*a^2*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(1 - Sin[c + d*x])) + (
a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.203143, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2708, 2765, 2977, 2734}

$$-\frac{16a^2\cos(c+dx)}{3d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{8a^2\sin^2(c+dx)\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{7a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{7a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] (7*a^2*x)/2 - (16*a^2*Cos[c + d*x])/(3*d) - (7*a^2*Cos[c + d*x]*Sin[c + d*x
])/(2*d) - (8*a^2*Cos[c + d*x]*Sin[c + d*x]^2)/(3*d*(1 - Sin[c + d*x])) + (
a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2)
```

Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
```

```
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx &= a^4\int \frac{\sin^4(c+dx)}{(a-a\sin(c+dx))^2}\,dx \\
&= \frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2}+\frac{1}{3}a^2\int \frac{\sin^2(c+dx)(-3a-5a\sin(c+dx))}{a-a\sin(c+dx)}\,dx \\
&= -\frac{8a^2\cos(c+dx)\sin^2(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{1}{3}\int \sin(c+dx)\Big(\\
&= \frac{7a^2x}{2}-\frac{16a^2\cos(c+dx)}{3d}-\frac{7a^2\cos(c+dx)\sin(c+dx)}{2d}-\frac{8a^2\cos(c+dx)\sin^2(c}{3d(1-\sin(c+dx)}
\end{aligned}
$$

Mathematica [A] time = 1.3057, size = 159, normalized size = 1.32

$$
-\frac{a^2\left(-21(12c+12dx+7)\cos\left(\frac{1}{2}(c+dx)\right)+(84c+84dx+239)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-5\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\frac{7}{2}(c+dx\right.\right.\right.}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\mathrm{si}\right.}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -(a^2*(-21*(7 + 12*c + 12*d*x)*Cos[(c + d*x)/2] + (239 + 84*c + 84*d*x)*Cos
[(3*(c + d*x))/2] + 3*(-5*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2] + 2*(
50 + 56*c + 56*d*x + (-27 + 28*c + 28*d*x)*Cos[c + d*x] - 6*Cos[2*(c + d*x)
] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(48*d*(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])^3)
```

Maple [A] time = 0.071, size = 186, normalized size = 1.6

$$\frac{1}{d}\left(a^2\left(\frac{(\sin(dx+c))^7}{3\,(\cos(dx+c))^3}-\frac{4\,(\sin(dx+c))^7}{3\cos(dx+c)}-\frac{4\cos(dx+c)}{3}\left((\sin(dx+c))^5+\frac{5\,(\sin(dx+c))^3}{4}+\frac{15\sin(dx+c)}{8}\right)+\frac{5}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(si
n(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+2*a^
2*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+
4/3*sin(d*x+c)^2)*cos(d*x+c))+a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A] time = 1.625, size = 162, normalized size = 1.35

$$\frac{\left(2\tan(dx+c)^3+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan(dx+c)\right)a^2+2\left(\tan(dx+c)^3+3\,dx+3\,c-3\tan(dx+c)\right)a^2-}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 12*tan(d*x + c))*a^2 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c)
)*a^2 - 4*a^2*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A] time = 1.47114, size = 468, normalized size = 3.9

$$\frac{3\,a^2\cos\left(dx+c\right)^4-6\,a^2\cos\left(dx+c\right)^3-42\,a^2dx+\left(21\,a^2dx+31\,a^2\right)\cos\left(dx+c\right)^2-2\,a^2-\left(21\,a^2dx-38\,a^2\right)\cos\left(dx+c\right)-}{6\left(d\cos\left(dx+c\right)^2-d\cos\left(dx+c\right)+\left(d\cos\left(dx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^3 - 42*a^2*d*x + (21*a^2*d*x
 + 31*a^2)*cos(d*x + c)^2 - 2*a^2 - (21*a^2*d*x - 38*a^2)*cos(d*x + c) - (3
*a^2*cos(d*x + c)^3 - 42*a^2*d*x + 9*a^2*cos(d*x + c)^2 + 2*a^2 - (21*a^2*d
*x - 40*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c)
 + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.21 $\int (a + a\sin(c + dx))^2 \tan^2(c + dx)\, dx$

Optimal. Leaf size=71

$$\frac{2a^2\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))} - \frac{5a^2x}{2}$$

```
[Out] (-5*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c
+ d*x])) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0889338, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2709, 2648, 2638, 2635, 8}

$$\frac{2a^2\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))} - \frac{5a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-5*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x])/(d*(1 - Sin[c
+ d*x])) + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx &= a^2\int\left(-2-\frac{2}{-1+\sin(c+dx)}-2\sin(c+dx)-\sin^2(c+dx)\right)dx\\ &= -2a^2x-a^2\int\sin^2(c+dx)\,dx-\left(2a^2\right)\int\frac{1}{-1+\sin(c+dx)}\,dx-\left(2a^2\right)\int\sin(c+\\ &= -2a^2x+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}-\frac{1}{2}\\ &= -\frac{5a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [B] time = 0.437947, size = 145, normalized size = 2.04

$$-\frac{a^2(\sin(c+dx)+1)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)(10(c+dx)-\sin(2(c+dx))-8\cos(c+dx))+\sin\left(\frac{1}{2}(c+dx)\right)(-2(5c+5dx+8)\right.}{4d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*(1 + Sin[c + d*x])^2*(Cos[(c + d*x)/2]*(10*(c + d*x) - 8*Cos[c + d*x]
 - Sin[2*(c + d*x)]) + Sin[(c + d*x)/2]*(-2*(8 + 5*c + 5*d*x) + 8*Cos[c + d
*x] + Sin[2*(c + d*x)])))/(4*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [A] time = 0.049, size = 117, normalized size = 1.7

$$\frac{1}{d}\left(a^{2}\left(\frac{(\sin(dx+c))^{5}}{\cos(dx+c)}+\left((\sin(dx+c))^{3}+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^{2}\left(\frac{(\sin(dx+c))^{4}}{\cos(dx+c)}+\left(2+(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-
3/2*d*x-3/2*c)+2*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+
a^2*(tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.61458, size = 113, normalized size = 1.59

$$-\frac{\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^{2}+1}-2\,\tan(dx+c)\right)a^{2}+2\,(dx+c-\tan(dx+c))a^{2}-4\,a^{2}\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^
2 + 2*(d*x + c - tan(d*x + c))*a^2 - 4*a^2*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

Fricas [A] time = 1.53691, size = 296, normalized size = 4.17

$$\frac{a^{2}\cos(dx+c)^{3}-5\,a^{2}dx+4\,a^{2}\cos(dx+c)^{2}+4\,a^{2}-\left(5\,a^{2}dx-7\,a^{2}\right)\cos(dx+c)+\left(5\,a^{2}dx+a^{2}\cos(dx+c)^{2}-3\,a^{2}\cos\right.}{2\,(d\cos(dx+c)-d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*cos(d*x + c)^3 - 5*a^2*d*x + 4*a^2*cos(d*x + c)^2 + 4*a^2 - (5*a^2
*d*x - 7*a^2)*cos(d*x + c) + (5*a^2*d*x + a^2*cos(d*x + c)^2 - 3*a^2*cos(d*
x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sin(c+dx)\tan^2(c+dx)\,dx+\int \sin^2(c+dx)\tan^2(c+dx)\,dx+\int \tan^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(sin(c + d*x)**
2*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))
```

Giac [B] time = 64.9824, size = 7250, normalized size = 102.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 5*a^2*d*x
*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 5*a^2*d*x*tan(d*x)^2*tan(1
/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(
1/2*c)^3*tan(c)^3 + 5*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3
 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 5*a^2*tan(d*x)^3
*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 5*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan
(1/2*c)^4*tan(c)^3 - 5*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 20*
a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 5*a^2*d*x*tan(d*x)*
tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/
2*c)^4*tan(c) + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2
- 5*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(d*x)^2*tan(1/2
*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^
3 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 20*a^2*d*x*t
an(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*
d*x)^3*tan(1/2*c)^3*tan(c)^3 + 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^
3*tan(c)^3 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*a^2*tan(d*x)*ta
n(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 4*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/
2*c)^4 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 20*a^2*tan(d
*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*tan(d*x)*tan(1/2*d*x)^4*
tan(1/2*c)^4*tan(c)^2 - 20*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c
```

)^3 + 4*a^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 20*a^2*d*x*tan(d*x)^2*ta
n(1/2*d*x)^3*tan(1/2*c)^3 - 5*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8*a^2*t
an(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4
*tan(c) - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 20*a^2*d
*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 20*a^2*d*x*tan(d*x)*tan(1/
2*d*x)^3*tan(1/2*c)^3*tan(c) + 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^
3*tan(c) - 5*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)*tan(1/
2*d*x)^4*tan(1/2*c)^4*tan(c) + 5*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2
 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 20*a^2*d*x*ta
n(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 20*a^2*d*x*tan(1/2*d*x)^3*tan
(1/2*c)^3*tan(c)^2 - 32*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2
 + 5*a^2*d*x*tan(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(1/2*d*x)^4*tan(1/
2*c)^4*tan(c)^2 - 5*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*tan(d*
x)^3*tan(1/2*d*x)^4*tan(c)^3 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c
)*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 32*a^
2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 96*a^2*tan(d*x)^3*tan(1/2
*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)
^3*tan(c)^3 - 32*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 + 32*a^2
*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 5*a^2*d*x*tan(d*x)*tan(1/2
*c)^4*tan(c)^3 - 8*a^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 16*a^2*tan(d*x)^3
*tan(1/2*d*x)^3*tan(1/2*c)^3 + 5*a^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4 -
 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 5*a^2*tan(1/2*d*x)^4
*tan(1/2*c)^4*tan(c) - 5*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^2 - 20*a^2*ta
n(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 20*a^2*tan(d*x)^3*tan(1/2*d*x
)*tan(1/2*c)^3*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)
^2 - 5*a^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)^2 - 5*a^2*tan(d*x)^2*tan(1/2*d*x)
^4*tan(c)^3 - 20*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 20*a^2
*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 16*a^2*tan(1/2*d*x)^3*tan(
1/2*c)^3*tan(c)^3 - 5*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c)^3 + 5*a^2*d*x*tan(
d*x)^2*tan(1/2*d*x)^4 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 2
0*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 20*a^2*d*x*tan(1/2*d*x)^3*
tan(1/2*c)^3 - 32*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 5*a^2*d*x*ta
n(d*x)^2*tan(1/2*c)^4 + 8*a^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 5*a^2*d*x*tan(d
*x)*tan(1/2*d*x)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 20*a^2
*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 20*a^2*d*x*tan(d*x)*tan(1/
2*d*x)^3*tan(1/2*c)*tan(c) - 32*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*ta
n(c) - 96*a^2*tan(d*x)^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 20*a^2*d*x*ta
n(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 32*a^2*tan(d*x)^3*tan(1/2*d*x)*ta
n(1/2*c)^3*tan(c) + 32*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 5*
a^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)
 + 5*a^2*d*x*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(
c)^2 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 20*a^2*d*x*
tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 32*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1
/2*c)*tan(c)^2 + 96*a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2
0*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 32*a^2*tan(d*x)^2*tan(1/2*d*

x)*tan(1/2*c)^3*tan(c)^2 - 32*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 5*
a^2*d*x*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 5*
a^2*d*x*tan(d*x)^3*tan(c)^3 - 8*a^2*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 20*a
^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 + 32*a^2*tan(d*x)^3*tan(1/
2*d*x)*tan(1/2*c)*tan(c)^3 - 32*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(
c)^3 - 96*a^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 32*a^2*tan(d*
x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*a^2*tan(d*x)*tan(1/2*c)^4*tan(c)^
3 - 4*a^2*tan(d*x)^3*tan(1/2*d*x)^4 - 16*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(
1/2*c) - 16*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3 - 20*a^2*tan(d*x)*tan(
1/2*d*x)^3*tan(1/2*c)^3 - 4*a^2*tan(d*x)^3*tan(1/2*c)^4 - 2*a^2*tan(d*x)^2*
tan(1/2*d*x)^4*tan(c) - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) -
 8*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 20*a^2*tan(1/2*d*x)^3*
tan(1/2*c)^3*tan(c) - 2*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c) - 2*a^2*tan(d*x)
*tan(1/2*d*x)^4*tan(c)^2 - 20*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)
^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 8*a^2*tan(d*x)*tan
(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 2*a^2*tan(d*x)*tan(1/2*c)^4*tan(c)^2 - 4*
a^2*tan(1/2*d*x)^4*tan(c)^3 - 20*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan
(c)^3 - 16*a^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 16*a^2*tan(1/2*d*x)*tan
(1/2*c)^3*tan(c)^3 - 4*a^2*tan(1/2*c)^4*tan(c)^3 + 5*a^2*d*x*tan(1/2*d*x)^4
 + 8*a^2*tan(d*x)^2*tan(1/2*d*x)^4 + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan
(1/2*c) + 20*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 32*a^2*tan(d*x)^2*tan(1/2*
d*x)^3*tan(1/2*c) + 96*a^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 20*a^2*
d*x*tan(1/2*d*x)*tan(1/2*c)^3 + 32*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3
 - 32*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 5*a^2*d*x*tan(1/2*c)^4 + 8*a^2*tan(
d*x)^2*tan(1/2*c)^4 + 5*a^2*d*x*tan(d*x)^3*tan(c) - 8*a^2*tan(d*x)*tan(1/2*
d*x)^4*tan(c) - 20*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 32*a^2
*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 32*a^2*tan(d*x)*tan(1/2*d*x)^3
*tan(1/2*c)*tan(c) - 96*a^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 3
2*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 8*a^2*tan(d*x)*tan(1/2*c)
^4*tan(c) - 5*a^2*d*x*tan(d*x)^2*tan(c)^2 + 8*a^2*tan(1/2*d*x)^4*tan(c)^2 +
 20*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 - 32*a^2*tan(d*x)^2*tan(1/2*d*
x)*tan(1/2*c)*tan(c)^2 + 32*a^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 96*a^2
*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 32*a^2*tan(1/2*d*x)*tan(1/2*c)^3*ta
n(c)^2 + 8*a^2*tan(1/2*c)^4*tan(c)^2 + 5*a^2*d*x*tan(d*x)*tan(c)^3 - 8*a^2*
tan(d*x)^3*tan(c)^3 + 32*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 5*
a^2*tan(d*x)*tan(1/2*d*x)^4 - 16*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c) - 2
0*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 20*a^2*tan(d*x)*tan(1/2*d*x)*tan
(1/2*c)^3 - 5*a^2*tan(d*x)*tan(1/2*c)^4 - 5*a^2*tan(1/2*d*x)^4*tan(c) - 8*a
^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 20*a^2*tan(1/2*d*x)^3*tan(1/
2*c)*tan(c) - 20*a^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 5*a^2*tan(1/2*c)^4*
tan(c) + 5*a^2*tan(d*x)^3*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)
*tan(c)^2 + 5*a^2*tan(d*x)^2*tan(c)^3 - 16*a^2*tan(1/2*d*x)*tan(1/2*c)*tan(
c)^3 - 5*a^2*d*x*tan(d*x)^2 + 8*a^2*tan(1/2*d*x)^4 + 20*a^2*d*x*tan(1/2*d*x
)*tan(1/2*c) - 32*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 32*a^2*tan(1/2*d
*x)^3*tan(1/2*c) + 96*a^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 32*a^2*tan(1/2*d*x)

```
*tan(1/2*c)^3 + 8*a^2*tan(1/2*c)^4 + 5*a^2*d*x*tan(d*x)*tan(c) - 8*a^2*tan(
d*x)^3*tan(c) + 32*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 5*a^2*d*x*
tan(c)^2 + 8*a^2*tan(d*x)^2*tan(c)^2 - 32*a^2*tan(1/2*d*x)*tan(1/2*c)*tan(c
)^2 - 8*a^2*tan(d*x)*tan(c)^3 + 4*a^2*tan(d*x)^3 - 20*a^2*tan(d*x)*tan(1/2*
d*x)*tan(1/2*c) + 2*a^2*tan(d*x)^2*tan(c) - 20*a^2*tan(1/2*d*x)*tan(1/2*c)*
tan(c) + 2*a^2*tan(d*x)*tan(c)^2 + 4*a^2*tan(c)^3 - 5*a^2*d*x + 8*a^2*tan(d
*x)^2 - 32*a^2*tan(1/2*d*x)*tan(1/2*c) - 8*a^2*tan(d*x)*tan(c) + 8*a^2*tan(
c)^2 + 5*a^2*tan(d*x) + 5*a^2*tan(c) + 8*a^2)/(d*tan(d*x)^3*tan(1/2*d*x)^4*
tan(1/2*c)^4*tan(c)^3 + d*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - d
*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 4*d*tan(d*x)^3*tan(1/2*d
*x)^3*tan(1/2*c)^3*tan(c)^3 + d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)
^3 - d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)^3*tan(1/2*d*x)
^3*tan(1/2*c)^3*tan(c) + d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*
d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - d*tan(1/2*d*x)^4*tan(1/
2*c)^4*tan(c)^2 - d*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 4*d*tan(d*x)^3*tan
(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*
tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - d*tan(d*x)^3
*tan(1/2*c)^4*tan(c)^3 + 4*d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan
(1/2*d*x)^4*tan(1/2*c)^4 - d*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 4*d*tan(d*x
)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*
c)^3*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - d*tan(d*x)^
3*tan(1/2*c)^4*tan(c) + d*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 4*d*tan(d*x)
^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2
*c)^3*tan(c)^2 + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + d*tan(d*x)^2*ta
n(1/2*c)^4*tan(c)^2 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 4*d*tan(d*x)^3*t
an(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*ta
n(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - d*tan(d*x)*tan(1
/2*c)^4*tan(c)^3 + d*tan(d*x)^2*tan(1/2*d*x)^4 + 4*d*tan(d*x)^2*tan(1/2*d*x
)^3*tan(1/2*c) + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 4*d*tan(1/2*d*x
)^3*tan(1/2*c)^3 + d*tan(d*x)^2*tan(1/2*c)^4 - d*tan(d*x)*tan(1/2*d*x)^4*ta
n(c) - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2
*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) -
 d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4*tan(c)^2 + 4*d*tan(d*x)^
2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 4*d*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2
 + 4*d*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + d*tan(1/2*c)^4*tan(c)^2 + d*tan
(d*x)^3*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 + d*tan(1/
2*d*x)^4 + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 4*d*tan(1/2*d*x)^3*tan(
1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)^4 + d*tan(d*x)^3*tan(
c) - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - d*tan(d*x)^2*tan(c)^2 +
4*d*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d*tan(d*x)^2 +
 4*d*tan(1/2*d*x)*tan(1/2*c) + d*tan(d*x)*tan(c) - d*tan(c)^2 - d)
```

3.22 $\int (a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=45

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

```
[Out] (3*a^2*x)/2 - (2*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rubi [A] time = 0.0136893, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2644}

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/2 - (2*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d
)
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (a+a\sin(c+dx))^2\,dx=\frac{3a^2x}{2}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.19256, size = 34, normalized size = 0.76

$$-\frac{a^2(-6(c+dx)+\sin(2(c+dx))+8\cos(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*(-6*(c + d*x) + 8*Cos[c + d*x] + Sin[2*(c + d*x)]))/(4*d)
```

Maple [A] time = 0.023, size = 52, normalized size = 1.2

$$\frac{1}{d}\left(a^{2}\left(-\frac{\cos\left(dx+c\right)\sin\left(dx+c\right)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2\cos\left(dx+c\right)a^{2}+a^{2}\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-2*cos(d*x+c)*a^2+a^2*(d
*x+c))
```

Maxima [A] time = 1.08459, size = 63, normalized size = 1.4

$$a^{2}x+\frac{(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))a^{2}}{4\,d}-\frac{2\,a^{2}\cos\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d - 2*a^2*cos(d*x + c)/d
```

Fricas [A] time = 1.55815, size = 97, normalized size = 2.16

$$\frac{3\,a^{2}dx-a^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)-4\,a^{2}\cos\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*d*x - a^2*cos(d*x + c)*sin(d*x + c) - 4*a^2*cos(d*x + c))/d
```

Sympy [A] time = 0.697112, size = 78, normalized size = 1.73

$$\begin{cases} \frac{a^2 x \sin^2{(c+dx)}}{2} + \frac{a^2 x \cos^2{(c+dx)}}{2} + a^2 x - \frac{a^2 \sin{(c+dx)} \cos{(c+dx)}}{2d} - \frac{2a^2 \cos{(c+dx)}}{d} & \text{for}\ d \neq 0 \\ x\left(a \sin{(c)} + a\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*x - a
**2*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**2*cos(c + d*x)/d, Ne(d, 0)), (x*
(a*sin(c) + a)**2, True))
```

Giac [A] time = 1.85333, size = 51, normalized size = 1.13

$$\frac{3}{2}\, a^2 x - \frac{2\, a^2 \cos\left(dx + c\right)}{d} - \frac{a^2 \sin\left(2\, dx + 2\, c\right)}{4\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 3/2*a^2*x - 2*a^2*cos(d*x + c)/d - 1/4*a^2*sin(2*d*x + 2*c)/d
```

3.23 $\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=74

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

```
[Out] -(a^2*x)/2 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.102471, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 2638, 2635}

$$\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x)/2 - (2*a^2*ArcTanh[Cos[c + d*x]])/d + (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x])/d + (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(2a^4\csc(c+dx)+a^4\csc^2(c+dx)-2a^4\sin(c+dx)-a^4\sin^2(c+dx)\right)dx}{a^2}$$
$$= a^2\int \csc^2(c+dx)\,dx - a^2\int \sin^2(c+dx)\,dx + \left(2a^2\right)\int \csc(c+dx)\,dx - \left(2a^2\right)\int$$
$$= -\frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} + \frac{a^2\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}a^2$$
$$= -\frac{a^2x}{2} - \frac{2a^2\tanh^{-1}(\cos(c+dx))}{d} + \frac{2a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+dx)}{d} + \frac{a^2\cos(c}{}$$

Mathematica [A] time = 0.58033, size = 94, normalized size = 1.27

$$-\frac{a^2\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(7\cos(c+dx)+\cos(3(c+dx))+4\sin(c+dx)\left(-4\cos(c+dx)-4\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(7*Cos[c + d*x] + Cos[3*(c + d*x)]
+ 4*(c + d*x - 4*Cos[c + d*x] + 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*
```

```
x)/2]])*Sin[c + d*x]))/(16*d)
```

Maple [A] time = 0.036, size = 89, normalized size = 1.2

$$\frac{\cos\left(dx+c\right)a^{2}\sin\left(dx+c\right)}{2d}-\frac{a^{2}x}{2}-\frac{a^{2}c}{2d}+2\frac{\cos\left(dx+c\right)a^{2}}{d}+2\frac{a^{2}\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}-\frac{a^{2}\cot\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*cos(d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c+2*a^2*cos(d*x+c)/d+2/
d*a^2*ln(csc(d*x+c)-cot(d*x+c))-a^2*cot(d*x+c)/d
```

Maxima [A] time = 1.64023, size = 107, normalized size = 1.45

$$\frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^{2}-4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^{2}+4\,a^{2}(2\,\cos\left(dx+c\right)-\log\left(\cos\left(dx+c\right)+1\right)+\log\left(\cos\left(dx+\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 4*(d*x + c + 1/tan(d*x + c))*a^
2 + 4*a^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

Fricas [A] time = 1.52358, size = 281, normalized size = 3.8

$$-\frac{a^{2}\cos\left(dx+c\right)^{3}+2\,a^{2}\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2\,a^{2}\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+a^{2}\cos\left(dx+\right.}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*cos(d*x + c)^3 + 2*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + a^2*cos(d*x + c) + (a^2*
d*x - 4*a^2*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int 2\sin\left(c+dx\right)\cot^{2}\left(c+dx\right)dx+\int\sin^{2}\left(c+dx\right)\cot^{2}\left(c+dx\right)dx+\int\cot^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))
```

Giac [B] time = 1.73197, size = 193, normalized size = 2.61

$$-\frac{(dx+c)a^{2}-4\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{4\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{2}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{2\left(a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - a^2*tan(1/2*d*
x + 1/2*c) + (4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) + 2*(a
^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d*x
+ 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.24 $\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

Optimal. Leaf size=98

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}$$

```
[Out] -(a^2*x)/2 + (3*a^2*ArcTanh[Cos[c + d*x]])/d - (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (a^2*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.161405, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635}

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```


```
[Out] -(a^2*x)/2 + (3*a^2*ArcTanh[Cos[c + d*x]])/d - (2*a^2*Cos[c + d*x])/d - (a^
2*Cot[c + d*x]^3)/(3*d) - (a^2*Cot[c + d*x]*Csc[c + d*x])/d - (a^2*Cos[c +
d*x]*Sin[c + d*x])/(2*d)
```


Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{\int \left(-a^6 - 4a^6\csc(c+dx) - a^6\csc^2(c+dx) + 2a^6\csc^3(c+dx) + a^6\csc^4(c+dx)\right.}{a^4}$$
$$= -a^2x - a^2\int \csc^2(c+dx)\,dx + a^2\int \csc^4(c+dx)\,dx + a^2\int \sin^2(c+dx)\,dx + \left(2\right.$$
$$= -a^2x + \frac{4a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^2\cos(c+dx)}{d} - \frac{a^2\cot(c+dx)\csc(c+dx)}{d}$$
$$= -\frac{a^2x}{2} + \frac{3a^2\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^2\cos(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot(}{}$$

Mathematica [A] time = 5.58143, size = 191, normalized size = 1.95

$$\frac{a^2(\sin(c+dx)+1)^2\left(-12(c+dx) - 6\sin(2(c+dx)) - 48\cos(c+dx) - 4\tan\left(\frac{1}{2}(c+dx)\right) + 4\cot\left(\frac{1}{2}(c+dx)\right) - 6\csc^2\left(\frac{1}{2}(\right.\right.}{24d\left(\mathrm{s}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Sin[c + d*x])^2*(-12*(c + d*x) - 48*Cos[c + d*x] + 4*Cot[(c + d*x
)/2] - 6*Csc[(c + d*x)/2]^2 + 72*Log[Cos[(c + d*x)/2]] - 72*Log[Sin[(c + d*
x)/2]] + 6*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (Csc[
(c + d*x)/2]^4*Sin[c + d*x])/2 - 6*Sin[2*(c + d*x)] - 4*Tan[(c + d*x)/2]))/
(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

Maple [B] time = 0.044, size = 190, normalized size = 1.9

$$-\frac{a^2(\cos(dx+c))^5}{d\sin(dx+c)}-\frac{a^2\sin(dx+c)(\cos(dx+c))^3}{d}-\frac{3\cos(dx+c)a^2\sin(dx+c)}{2d}-\frac{a^2x}{2}-\frac{a^2c}{2d}-\frac{a^2(\cos(dx+c))^5}{d(\sin(dx+c))^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^2/sin(d*x+c)*cos(d*x+c)^5-1/d*a^2*sin(d*x+c)*cos(d*x+c)^3-3/2*a^2*co
s(d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c-1/d*a^2/sin(d*x+c)^2*cos(d*x+c)
^5-1/d*a^2*cos(d*x+c)^3-3*a^2*cos(d*x+c)/d-3/d*a^2*ln(csc(d*x+c)-cot(d*x+c)
)-1/3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d
```

Maxima [A] time = 1.64741, size = 188, normalized size = 1.92

$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2-2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+c)+3\,\log(\mathrm{co}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c
)))*a^2 - 2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*a
^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x +
c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.70258, size = 474, normalized size = 4.84

$$\frac{3\,a^2\cos\left(dx+c\right)^5-4\,a^2\cos\left(dx+c\right)^3+3\,a^2\cos\left(dx+c\right)+9\left(a^2\cos\left(dx+c\right)^2-a^2\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-}{6\left(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*a^2*cos(d*x + c)^5 - 4*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c) + 9*(
a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a^2
*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a^2*d
*x*cos(d*x + c)^2 + 4*a^2*cos(d*x + c)^3 - a^2*d*x - 6*a^2*cos(d*x + c))*si
n(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sin\left(c+dx\right)\cot^4\left(c+dx\right)dx+\int\sin^2\left(c+dx\right)\cot^4\left(c+dx\right)dx+\int\cot^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**4, x) + Integral(sin(c + d*x)**
2*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x))
```

Giac [B] time = 1.76195, size = 282, normalized size = 2.88

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\left(dx+c\right)a^2-72\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{24\left(\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x +
 c)*a^2 - 72*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2*c
) + 24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan
(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a^2*tan(1/
2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*tan(1/2*d*x + 1/2*c
) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.25 $\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx$

Optimal. Leaf size=160

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^6}{6d(a-a\sin(c+dx))^3}-\frac{13a^5}{8d(a-a\sin(c+dx))^2}+\frac{71a^4}{8d(a-a\sin(c+dx))}+\frac{7a^3\sin(c}{d}$$

```
[Out] (209*a^3*Log[1 - Sin[c + d*x]])/(16*d) - (a^3*Log[1 + Sin[c + d*x]])/(16*d)
 + (7*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x
]^3)/(3*d) + a^6/(6*d*(a - a*Sin[c + d*x])^3) - (13*a^5)/(8*d*(a - a*Sin[c
+ d*x])^2) + (71*a^4)/(8*d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.109078, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^6}{6d(a-a\sin(c+dx))^3}-\frac{13a^5}{8d(a-a\sin(c+dx))^2}+\frac{71a^4}{8d(a-a\sin(c+dx))}+\frac{7a^3\sin(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (209*a^3*Log[1 - Sin[c + d*x]])/(16*d) - (a^3*Log[1 + Sin[c + d*x]])/(16*d)
 + (7*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d*x]^2)/(2*d) + (a^3*Sin[c + d*x
]^3)/(3*d) + a^6/(6*d*(a - a*Sin[c + d*x])^3) - (13*a^5)/(8*d*(a - a*Sin[c
+ d*x])^2) + (71*a^4)/(8*d*(a - a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{(a-x)^4(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(7a^2+\frac{a^6}{2(a-x)^4}-\frac{13a^5}{4(a-x)^3}+\frac{71a^4}{8(a-x)^2}-\frac{209a^3}{16(a-x)}+3ax+x^2-\frac{a^3}{16(a+x)}\right)dx, x}{d}$$

$$= \frac{209a^3\log(1-\sin(c+dx))}{16d}-\frac{a^3\log(1+\sin(c+dx))}{16d}+\frac{7a^3\sin(c+dx)}{d}+\frac{3a^3}{}$$

Mathematica [A] time = 0.563704, size = 99, normalized size = 0.62

$$\frac{a^3\left(16\sin^3(c+dx)+72\sin^2(c+dx)+336\sin(c+dx)-\frac{426}{\sin(c+dx)-1}-\frac{78}{(\sin(c+dx)-1)^2}-\frac{8}{(\sin(c+dx)-1)^3}+627\log(1-\sin(c+\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]
```

```
[Out] (a^3*(627*Log[1 - Sin[c + d*x]] - 3*Log[1 + Sin[c + d*x]] - 8/(-1 + Sin[c +
 d*x])^3 - 78/(-1 + Sin[c + d*x])^2 - 426/(-1 + Sin[c + d*x]) + 336*Sin[c +
 d*x] + 72*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3))/(48*d)
```

Maple [B] time = 0.105, size = 445, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x)
```

```
[Out] 35/48/d*a^3*sin(d*x+c)^9+3/2/d*a^3*sin(d*x+c)^8+15/8/d*a^3*sin(d*x+c)^7+21/
8/d*a^3*sin(d*x+c)^5+35/8*a^3*sin(d*x+c)^3/d+105/8*a^3*sin(d*x+c)/d-105/8/d
*a^3*ln(sec(d*x+c)+tan(d*x+c))+2/d*a^3*sin(d*x+c)^6+3/d*a^3*sin(d*x+c)^4+6*
a^3*sin(d*x+c)^2/d+13/d*a^3*ln(cos(d*x+c))+1/6/d*a^3*sin(d*x+c)^11/cos(d*x+
c)^6-5/24/d*a^3*sin(d*x+c)^11/cos(d*x+c)^4+35/48/d*a^3*sin(d*x+c)^11/cos(d*
x+c)^2+1/2/d*a^3*sin(d*x+c)^10/cos(d*x+c)^6-1/2/d*a^3*sin(d*x+c)^10/cos(d*x
+c)^4+3/2/d*a^3*sin(d*x+c)^10/cos(d*x+c)^2+1/2/d*a^3*sin(d*x+c)^9/cos(d*x+c
```

```
)^6-3/8/d*a^3*sin(d*x+c)^9/cos(d*x+c)^4+15/16/d*a^3*sin(d*x+c)^9/cos(d*x+c)
^2+1/6/d*a^3*tan(d*x+c)^6-1/4/d*a^3*tan(d*x+c)^4+1/2/d*a^3*tan(d*x+c)^2
```

Maxima [A] time = 1.07702, size = 180, normalized size = 1.12

$$\frac{16\,a^3\sin\left(dx+c\right)^3+72\,a^3\sin\left(dx+c\right)^2-3\,a^3\log\left(\sin\left(dx+c\right)+1\right)+627\,a^3\log\left(\sin\left(dx+c\right)-1\right)+336\,a^3\sin\left(dx+c\right)-}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/48*(16*a^3*sin(d*x + c)^3 + 72*a^3*sin(d*x + c)^2 - 3*a^3*log(sin(d*x + c
) + 1) + 627*a^3*log(sin(d*x + c) - 1) + 336*a^3*sin(d*x + c) - 2*(213*a^3*
sin(d*x + c)^2 - 387*a^3*sin(d*x + c) + 178*a^3)/(sin(d*x + c)^3 - 3*sin(d*
x + c)^2 + 3*sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.57514, size = 594, normalized size = 3.71

$$-\frac{16\,a^3\cos\left(dx+c\right)^6-216\,a^3\cos\left(dx+c\right)^4+1002\,a^3\cos\left(dx+c\right)^2-482\,a^3+3\left(3\,a^3\cos\left(dx+c\right)^2-4\,a^3-\left(a^3\cos\left(dx+c\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] -1/48*(16*a^3*cos(d*x + c)^6 - 216*a^3*cos(d*x + c)^4 + 1002*a^3*cos(d*x +
c)^2 - 482*a^3 + 3*(3*a^3*cos(d*x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 - 4*
a^3)*sin(d*x + c))*log(sin(d*x + c) + 1) - 627*(3*a^3*cos(d*x + c)^2 - 4*a^
3 - (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 2*(
12*a^3*cos(d*x + c)^4 + 398*a^3*cos(d*x + c)^2 - 245*a^3)*sin(d*x + c))/(3*
d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")
```

```
[Out] Timed out
```

3.26 $\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{5a^3\sin(c+dx)}{d}+\frac{7a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] (7*a^3*Log[1 - Sin[c + d*x]])/d + (5*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d
*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d) + (2*a^4)/(d*(a - a*Sin[c + d*x])
)
```

Rubi [A] time = 0.0707841, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{2a^4}{d(a-a\sin(c+dx))}+\frac{5a^3\sin(c+dx)}{d}+\frac{7a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (7*a^3*Log[1 - Sin[c + d*x]])/d + (5*a^3*Sin[c + d*x])/d + (3*a^3*Sin[c + d
*x]^2)/(2*d) + (a^3*Sin[c + d*x]^3)/(3*d) + (2*a^4)/(d*(a - a*Sin[c + d*x])
)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(5a^2+\frac{2a^4}{(a-x)^2}-\frac{7a^3}{a-x}+3ax+x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{7a^3\log(1-\sin(c+dx))}{d}+\frac{5a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3(c+d}{3d}$$

Mathematica [A] time = 0.156734, size = 66, normalized size = 0.73

$$\frac{a^3\left(2\sin^3(c+dx)+9\sin^2(c+dx)+30\sin(c+dx)+\frac{12}{1-\sin(c+dx)}+42\log(1-\sin(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (a^3*(42*Log[1 - Sin[c + d*x]] + 12/(1 - Sin[c + d*x]) + 30*Sin[c + d*x] +
9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/(6*d)
```

Maple [B] time = 0.078, size = 205, normalized size = 2.3

$$\frac{a^3(\sin(dx+c))^7}{2d(\cos(dx+c))^2}+\frac{a^3(\sin(dx+c))^5}{2d}+\frac{7a^3(\sin(dx+c))^3}{3d}+7\frac{a^3\sin(dx+c)}{d}-7\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^3*sin(d*x+c)^7/cos(d*x+c)^2+1/2/d*a^3*sin(d*x+c)^5+7/3*a^3*sin(d*x+
c)^3/d+7*a^3*sin(d*x+c)/d-7/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/2/d*a^3*sin(d
*x+c)^6/cos(d*x+c)^2+3/2/d*a^3*sin(d*x+c)^4+3*a^3*sin(d*x+c)^2/d+7/d*a^3*ln
(cos(d*x+c))+3/2/d*a^3*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*a^3*tan(d*x+c)^2
```

Maxima [A] time = 1.07083, size = 97, normalized size = 1.07

$$\frac{2\,a^{3}\sin\left(dx+c\right)^{3}+9\,a^{3}\sin\left(dx+c\right)^{2}+42\,a^{3}\log\left(\sin\left(dx+c\right)-1\right)+30\,a^{3}\sin\left(dx+c\right)-\frac{12\,a^{3}}{\sin(dx+c)-1}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 42*a^3*log(sin(d*x + c)
- 1) + 30*a^3*sin(d*x + c) - 12*a^3/(sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.59931, size = 248, normalized size = 2.73

$$\frac{4\,a^{3}\cos\left(dx+c\right)^{4}-50\,a^{3}\cos\left(dx+c\right)^{2}+31\,a^{3}+84\left(a^{3}\sin\left(dx+c\right)-a^{3}\right)\log\left(-\sin\left(dx+c\right)+1\right)-\left(14\,a^{3}\cos\left(dx+c\right)^{2}+\right.}{12\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^4 - 50*a^3*cos(d*x + c)^2 + 31*a^3 + 84*(a^3*sin(d
*x + c) - a^3)*log(-sin(d*x + c) + 1) - (14*a^3*cos(d*x + c)^2 + 55*a^3)*si
n(d*x + c))/(d*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.27 $\int (a+a\sin(c+dx))^3 \tan(c+dx)\,dx$

Optimal. Leaf size=70

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{4a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

```
[Out] (-4*a^3*Log[1 - Sin[c + d*x]])/d - (4*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c +
d*x]^2)/(2*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0449772, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 77}

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{4a^3\sin(c+dx)}{d}-\frac{4a^3\log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] (-4*a^3*Log[1 - Sin[c + d*x]])/d - (4*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c +
d*x]^2)/(2*d) - (a^3*Sin[c + d*x]^3)/(3*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int (a + a\sin(c+dx))^3 \tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^2 + \frac{4a^3}{a-x} - 3ax - x^2\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{4a^3\log(1-\sin(c+dx))}{d} - \frac{4a^3\sin(c+dx)}{d} - \frac{3a^3\sin^2(c+dx)}{2d} - \frac{a^3\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.045258, size = 52, normalized size = 0.74

$$-\frac{a^3\left(2\sin^3(c+dx) + 9\sin^2(c+dx) + 24\sin(c+dx) + 24\log(1-\sin(c+dx))\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -(a^3*(24*Log[1 - Sin[c + d*x]] + 24*Sin[c + d*x] + 9*Sin[c + d*x]^2 + 2*Si
n[c + d*x]^3))/(6*d)
```

Maple [A] time = 0.051, size = 85, normalized size = 1.2

$$-\frac{a^3(\sin(dx+c))^3}{3d} - 4\frac{a^3\sin(dx+c)}{d} + 4\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{3a^3(\sin(dx+c))^2}{2d} - 4\frac{a^3\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c),x)
```

```
[Out] -1/3*a^3*sin(d*x+c)^3/d-4*a^3*sin(d*x+c)/d+4/d*a^3*ln(sec(d*x+c)+tan(d*x+c)
)-3/2*a^3*sin(d*x+c)^2/d-4/d*a^3*ln(cos(d*x+c))
```

Maxima [A] time = 1.07111, size = 77, normalized size = 1.1

$$-\frac{2a^3\sin(dx+c)^3 + 9a^3\sin(dx+c)^2 + 24a^3\log(\sin(dx+c)-1) + 24a^3\sin(dx+c)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 24*a^3*log(sin(d*x + c)
 - 1) + 24*a^3*sin(d*x + c))/d
```

Fricas [A] time = 1.51562, size = 147, normalized size = 2.1

$$\frac{9\,a^{3}\cos\left(dx+c\right)^{2}-24\,a^{3}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(a^{3}\cos\left(dx+c\right)^{2}-13\,a^{3}\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(9*a^3*cos(d*x + c)^2 - 24*a^3*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x
+ c)^2 - 13*a^3)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int 3\sin\left(c+dx\right)\tan\left(c+dx\right)dx+\int 3\sin^{2}\left(c+dx\right)\tan\left(c+dx\right)dx+\int\sin^{3}\left(c+dx\right)\tan\left(c+dx\right)dx+\int\tan\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x), x) + Integral(3*sin(c + d*x)**2
*tan(c + d*x), x) + Integral(sin(c + d*x)**3*tan(c + d*x), x) + Integral(ta
n(c + d*x), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.28 $\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=98

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{2a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{2a^3\log(\sin(c+dx))}{d}$$

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (2*a^3*Log[Sin[c + d
*x]])/d - (2*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c + d*x]^2)/(2*d) - (a^3*Sin[
c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0648456, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 75}

$$-\frac{a^3\sin^3(c+dx)}{3d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{2a^3\sin(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3a^3\csc(c+dx)}{d}+\frac{2a^3\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^3*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) + (2*a^3*Log[Sin[c + d
*x]])/d - (2*a^3*Sin[c + d*x])/d - (3*a^3*Sin[c + d*x]^2)/(2*d) - (a^3*Sin[
c + d*x]^3)/(3*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^4}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2+\frac{a^5}{x^3}+\frac{3a^4}{x^2}+\frac{2a^3}{x}-3ax-x^2\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{2a^3\log(\sin(c+dx))}{d}-\frac{2a^3\sin(c+dx)}{d}-$$

Mathematica [A] time = 0.201659, size = 67, normalized size = 0.68

$$-\frac{a^3\left(2\sin^3(c+dx)+9\sin^2(c+dx)+12\sin(c+dx)+3\csc^2(c+dx)+18\csc(c+dx)-12\log(\sin(c+dx))+30\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*(30 + 18*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] + 12*
Sin[c + d*x] + 9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/(6*d)
```

Maple [A] time = 0.053, size = 109, normalized size = 1.1

$$-\frac{8\,a^3\sin(dx+c)(\cos(dx+c))^2}{3\,d}-\frac{16\,a^3\sin(dx+c)}{3\,d}+\frac{3\,a^3(\cos(dx+c))^2}{2\,d}+2\,\frac{a^3\ln(\sin(dx+c))}{d}-3\,\frac{a^3(\cos(dx+c))}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -8/3/d*a^3*sin(d*x+c)*cos(d*x+c)^2-16/3*a^3*sin(d*x+c)/d+3/2/d*a^3*cos(d*x+
c)^2+2*a^3*ln(sin(d*x+c))/d-3/d*a^3/sin(d*x+c)*cos(d*x+c)^4-1/2/d*a^3*cot(d
*x+c)^2
```

Maxima [A] time = 1.05985, size = 108, normalized size = 1.1

$$-\frac{2\,a^3\sin(dx+c)^3+9\,a^3\sin(dx+c)^2-12\,a^3\log(\sin(dx+c))+12\,a^3\sin(dx+c)+\frac{3\left(6\,a^3\sin(dx+c)+a^3\right)}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 - 12*a^3*log(sin(d*x + c)
) + 12*a^3*sin(d*x + c) + 3*(6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.48336, size = 284, normalized size = 2.9

$$\frac{18\,a^3\cos\left(dx+c\right)^4-27\,a^3\cos\left(dx+c\right)^2+15\,a^3+24\left(a^3\cos\left(dx+c\right)^2-a^3\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+4\left(a^3\cos\left(dx+c\right)^4-8\right.}{12\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a^3*cos(d*x + c)^4 - 27*a^3*cos(d*x + c)^2 + 15*a^3 + 24*(a^3*cos(
d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 4*(a^3*cos(d*x + c)^4 - 8*a^3*cos
(d*x + c)^2 + 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 3\sin\left(c+dx\right)\cot^3\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\cot^3\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\cot^3\left(c+dx\right)dx+\int \cot^3\left(c+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(3*sin(c + d*x)
**2*cot(c + d*x)**3, x) + Integral(sin(c + d*x)**3*cot(c + d*x)**3, x) + In
tegral(cot(c + d*x)**3, x))
```

Giac [A] time = 1.37156, size = 127, normalized size = 1.3

$$-\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2-12\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a^3\sin\left(dx+c\right)+\frac{3\left(6\,a^3\sin(dx+c)^2+6\,a^3\sin(dx+c)+a^3\right)}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 - 12*a^3*log(abs(sin(d*x
+ c))) + 12*a^3*sin(d*x + c) + 3*(6*a^3*sin(d*x + c)^2 + 6*a^3*sin(d*x + c)
 + a^3)/sin(d*x + c)^2)/d
```

3.29 $\int (a + a\sin(c + dx))^3 \tan^6(c + dx)\,dx$

Optimal. Leaf size=180

$$-\frac{136a^3\cos^3(c+dx)}{15d}+\frac{136a^3\cos(c+dx)}{5d}+\frac{23a^6\sin^3(c+dx)\cos(c+dx)}{3d\left(a^3-a^3\sin(c+dx)\right)}+\frac{a^6\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{13a^5\sin^4(c+}{15d(a-a\,\mathrm{s}}$$

```
[Out] (-23*a^3*x)/2 + (136*a^3*Cos[c + d*x])/(5*d) - (136*a^3*Cos[c + d*x]^3)/(15
*d) + (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^6*Cos[c + d*x]*Sin[c +
d*x]^5)/(5*d*(a - a*Sin[c + d*x])^3) - (13*a^5*Cos[c + d*x]*Sin[c + d*x]^4)
/(15*d*(a - a*Sin[c + d*x])^2) + (23*a^6*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*
(a^3 - a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.356664, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2708, 2765, 2977, 2748, 2635, 8, 2633}

$$-\frac{136a^3\cos^3(c+dx)}{15d}+\frac{136a^3\cos(c+dx)}{5d}+\frac{23a^6\sin^3(c+dx)\cos(c+dx)}{3d\left(a^3-a^3\sin(c+dx)\right)}+\frac{a^6\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{13a^5\sin^4(c+}{15d(a-a\,\mathrm{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^6,x]
```


```
[Out] (-23*a^3*x)/2 + (136*a^3*Cos[c + d*x])/(5*d) - (136*a^3*Cos[c + d*x]^3)/(15
*d) + (23*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^6*Cos[c + d*x]*Sin[c +
d*x]^5)/(5*d*(a - a*Sin[c + d*x])^3) - (13*a^5*Cos[c + d*x]*Sin[c + d*x]^4)
/(15*d*(a - a*Sin[c + d*x])^2) + (23*a^6*Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*
(a^3 - a^3*Sin[c + d*x]))
```


Rule 2708

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2765

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
```

```
 + f*x])^m*(c + d*Sin[e + f*x])^(n - 1))/(a*f*(2*m + 1)), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx = a^6\int \frac{\sin^6(c+dx)}{(a-a\sin(c+dx))^3}\,dx$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3}+\frac{1}{5}a^4\int \frac{\sin^4(c+dx)(-5a-8a\sin(c+dx))}{(a-a\sin(c+dx))^2}\,dx$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2}-\frac{1}{15}a^2\int \frac{\sin^3(c+}{}$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{23a^4\cos(c+dx)\,s}{3d(a-a\sin(}$$
$$= \frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2}+\frac{23a^4\cos(c+dx)\,s}{3d(a-a\sin(}$$
$$= \frac{23a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{13a^5\cos(c+dx)\,si}{15d(a-a\sin(c}$$
$$= -\frac{23a^3x}{2}+\frac{136a^3\cos(c+dx)}{5d}-\frac{136a^3\cos^3(c+dx)}{15d}+\frac{23a^3\cos(c+dx)\sin(c+d}{2d}$$

Mathematica [A] time = 5.61378, size = 243, normalized size = 1.35

$$\frac{(a\sin(c+dx)+a)^3\left(-690(c+dx)+45\sin(2(c+dx))+405\cos(c+dx)-5\cos(3(c+dx))+\frac{1576\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\right.}{60d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^6,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-690*(c + d*x) + 405*Cos[c + d*x] - 5*Cos[3*(c + d
*x)] + 12/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 - 112/(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^2 + (24*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^5 - (224*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3
+ (1576*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 45*Sin[2*
(c + d*x)]))/(60*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [B] time = 0.131, size = 359, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x)
```

```
[Out] 1/d*(a^3*(1/5*sin(d*x+c)^10/cos(d*x+c)^5-1/3*sin(d*x+c)^10/cos(d*x+c)^3+7/3
*sin(d*x+c)^10/cos(d*x+c)+7/3*(128/35+sin(d*x+c)^8+8/7*sin(d*x+c)^6+48/35*s
in(d*x+c)^4+64/35*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/5*sin(d*x+c)^9/cos(d*x
+c)^5-4/15*sin(d*x+c)^9/cos(d*x+c)^3+8/5*sin(d*x+c)^9/cos(d*x+c)+8/5*(sin(d
*x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)-7/
2*d*x-7/2*c)+3*a^3*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+
c)^3+sin(d*x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*
x+c)^2)*cos(d*x+c))+a^3*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c
))
```

Maxima [A] time = 1.648, size = 282, normalized size = 1.57

$$\frac{3\left(6\,\tan\left(dx+c\right)^5-20\,\tan\left(dx+c\right)^3-105\,dx-105\,c+\frac{15\,\tan(dx+c)}{\tan(dx+c)^2+1}+90\,\tan\left(dx+c\right)\right)a^3+2\left(3\,\tan\left(dx+c\right)^5-5\,\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/30*(3*(6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*
x + c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^3 + 2*(3*tan(d*x + c)^5 -
5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 - 2*(5*cos(d*x + c)
^3 - (90*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 3)/cos(d*x + c)^5 - 60*cos(d*
x + c))*a^3 + 18*a^3*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x +
c)^5 + 5*cos(d*x + c)))/d
```

Fricas [A] time = 1.59798, size = 734, normalized size = 4.08

$$-\frac{10\,a^3\cos\left(dx+c\right)^6-15\,a^3\cos\left(dx+c\right)^5-140\,a^3\cos\left(dx+c\right)^4-1380\,a^3dx+\left(345\,a^3dx-839\,a^3\right)\cos\left(dx+c\right)^3+6\,a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*a^3*cos(d*x + c)^6 - 15*a^3*cos(d*x + c)^5 - 140*a^3*cos(d*x + c)
^4 - 1380*a^3*d*x + (345*a^3*d*x - 839*a^3)*cos(d*x + c)^3 + 6*a^3 + (1035*
a^3*d*x + 668*a^3)*cos(d*x + c)^2 - 6*(115*a^3*d*x - 233*a^3)*cos(d*x + c)
- (10*a^3*cos(d*x + c)^5 + 25*a^3*cos(d*x + c)^4 - 115*a^3*cos(d*x + c)^3 -
 1380*a^3*d*x - 6*a^3 + (345*a^3*d*x + 724*a^3)*cos(d*x + c)^2 - 6*(115*a^3
*d*x - 232*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + 3*d*cos(d*x
 + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2 - 2*d*cos(d*x + c) - 4*d)*si
n(d*x + c) - 4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")
```

```
[Out] Timed out
```

3.30 $\int (a + a\sin(c + dx))^3 \tan^4(c + dx)\,dx$

Optimal. Leaf size=119

$$\frac{a^3\cos^3(c+dx)}{3d} - \frac{6a^3\cos(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} - \frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} + \frac{17a^3x}{2}$$

```
[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3
*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 -
 Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.194183, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^3(c+dx)}{3d} - \frac{6a^3\cos(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} - \frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))} + \frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2} + \frac{17a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (17*a^3*x)/2 - (6*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) + (2*a^3
*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (25*a^3*Cos[c + d*x])/(3*d*(1 -
 Sin[c + d*x])) - (3*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx = a^4\int\left(\frac{7}{a}+\frac{2}{a(-1+\sin(c+dx))^2}+\frac{9}{a(-1+\sin(c+dx))}+\frac{5\sin(c+dx)}{a}+\frac{3\sin^2\right.$$

$$= 7a^3x+a^3\int\sin^3(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\left(3a^3\right)\int\sin^2(c$$

$$= 7a^3x-\frac{5a^3\cos(c+dx)}{d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{9a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{3a^3\cos(c$$

$$= \frac{17a^3x}{2}-\frac{6a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{25a^3\cos(}{3d(1-\sin}$$

Mathematica [A] time = 2.25007, size = 177, normalized size = 1.49

$$\frac{(a\sin(c+dx)+a)^3\left(102(c+dx)-9\sin(2(c+dx))-69\cos(c+dx)+\cos(3(c+dx))-\frac{200\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{\dots}{\left(\cos\left(\frac{1}{2}(\dots\right.\right.}\right.}{12d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(102*(c + d*x) - 69*Cos[c + d*x] + Cos[3*(c + d*x)]
 + 8/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] - Sin[(c + d*x)/2])^3 - (200*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]) - 9*Sin[2*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^6)
```

Maple [B] time = 0.085, size = 266, normalized size = 2.2

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^8}{3(\cos(dx+c))^3}-\frac{5(\sin(dx+c))^8}{3\cos(dx+c)}-\frac{5\cos(dx+c)}{3}\left(\frac{16}{5}+(\sin(dx+c))^6+\frac{6(\sin(dx+c))^4}{5}+\frac{8(\sin(dx+c)}{5}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16
/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+3*a^3*(1/3*s
in(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*x+c)^5+5/4*
sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+3*a^3*(1/3*sin(d*x+
c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^
2)*cos(d*x+c))+a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A] time = 1.61188, size = 223, normalized size = 1.87

$$\frac{2\left(\cos(dx+c)^3-\frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3}-9\cos(dx+c)\right)a^3+3\left(2\tan(dx+c)^3+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan(dx+c)\right)\dots}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
+ c))*a^3 + 3*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x +
 c)^2 + 1) - 12*tan(d*x + c))*a^3 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan
(d*x + c))*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x +
 c)))/d
```

Fricas [B] time = 1.5826, size = 539, normalized size = 4.53

$$\frac{2\,a^3\cos(dx+c)^5 + 7\,a^3\cos(dx+c)^4 - 22\,a^3\cos(dx+c)^3 - 102\,a^3dx - 4\,a^3 + \left(51\,a^3dx + 77\,a^3\right)\cos(dx+c)^2 - \left(51\,a^3dx\right.}{6\left(d\cos(dx+c)^2 - d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^5 + 7*a^3*cos(d*x + c)^4 - 22*a^3*cos(d*x + c)^3 -
102*a^3*d*x - 4*a^3 + (51*a^3*d*x + 77*a^3)*cos(d*x + c)^2 - (51*a^3*d*x -
100*a^3)*cos(d*x + c) + (2*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + 102*
a^3*d*x - 27*a^3*cos(d*x + c)^2 - 4*a^3 + (51*a^3*d*x - 104*a^3)*cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*
d)*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.31 $\int (a + a\sin(c + dx))^3 \tan^2(c + dx)\, dx$

Optimal. Leaf size=89

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

```
[Out] (-11*a^3*x)/2 + (5*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) + (4*a^
3*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/
(2*d)
```

Rubi [A] time = 0.125111, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 2648, 2638, 2635, 8, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (-11*a^3*x)/2 + (5*a^3*Cos[c + d*x])/d - (a^3*Cos[c + d*x]^3)/(3*d) + (4*a^
3*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (3*a^3*Cos[c + d*x]*Sin[c + d*x])/
(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = a^2\int\left(-4a-\frac{4a}{-1+\sin(c+dx)}-4a\sin(c+dx)-3a\sin^2(c+dx)-a\sin^3(c+
$$
$$= -4a^3x-a^3\int\sin^3(c+dx)\,dx-\left(3a^3\right)\int\sin^2(c+dx)\,dx-\left(4a^3\right)\int\frac{1}{-1+\sin(c
$$
$$= -4a^3x+\frac{4a^3\cos(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}-
$$
$$= -\frac{11a^3x}{2}+\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3\cos(c
$$

Mathematica [A] time = 0.490405, size = 115, normalized size = 1.29

$$\frac{(a\sin(c+dx)+a)^3\left(-66(c+dx)+9\sin(2(c+dx))+57\cos(c+dx)-\cos(3(c+dx))+\frac{96\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{12d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^3*(-66*(c + d*x) + 57*Cos[c + d*x] - Cos[3*(c + d*x)]
 + (96*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 9*Sin[2*(c
 + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

Maple [A] time = 0.053, size = 167, normalized size = 1.9

$$\frac{1}{d}\left(a^3\left(\frac{(\sin(dx+c))^6}{\cos(dx+c)}+\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4\,(\sin(dx+c))^2}{3}\right)\cos(dx+c)\right)+3\,a^3\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\sin(dx+c))^3+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d
*x+c))+3*a^3*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x
+c)-3/2*d*x-3/2*c)+3*a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+
c))+a^3*(tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.57021, size = 158, normalized size = 1.78

$$-\frac{2\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\cos(dx+c)\right)a^3+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan(dx+c)\right)a^3+6\,(dx+c-\tan(dx+}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + 9*(3*d*x +
 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^3 + 6*(d*x + c
 - tan(d*x + c))*a^3 - 18*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.42002, size = 378, normalized size = 4.25

$$-\frac{2\,a^3\cos(dx+c)^4-7\,a^3\cos(dx+c)^3+33\,a^3dx-30\,a^3\cos(dx+c)^2-24\,a^3+3\left(11\,a^3dx-15\,a^3\right)\cos(dx+c)-\left(2\,a^3\,\mathrm{co}\right.}{6\,(d\cos(dx+c)-d\sin(dx+c)+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*a^3*cos(d*x + c)^4 - 7*a^3*cos(d*x + c)^3 + 33*a^3*d*x - 30*a^3*cos
(d*x + c)^2 - 24*a^3 + 3*(11*a^3*d*x - 15*a^3)*cos(d*x + c) - (2*a^3*cos(d*
x + c)^3 + 33*a^3*d*x + 9*a^3*cos(d*x + c)^2 - 21*a^3*cos(d*x + c) + 24*a^3
)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 3\sin\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int 3\sin^2\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\tan^2\left(c+dx\right)dx+\int \tan^2\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(3*sin(c + d*x)
**2*tan(c + d*x)**2, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**2, x) + In
tegral(tan(c + d*x)**2, x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.32 $\int (a + a\sin(c + dx))^3 \, dx$

Optimal. Leaf size=63

$$\frac{a^3\cos^3(c+dx)}{3d} - \frac{4a^3\cos(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{5a^3x}{2}$$

```
[Out] (5*a^3*x)/2 - (4*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) - (3*a^3*
Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0544642, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2638, 2635, 8, 2633}

$$\frac{a^3\cos^3(c+dx)}{3d} - \frac{4a^3\cos(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} + \frac{5a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/2 - (4*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) - (3*a^3*
Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^3\,dx = \int \left(a^3+3a^3\sin(c+dx)+3a^3\sin^2(c+dx)+a^3\sin^3(c+dx)\right)dx$$
$$= a^3x+a^3\int \sin^3(c+dx)\,dx+\left(3a^3\right)\int \sin(c+dx)\,dx+\left(3a^3\right)\int \sin^2(c+dx)\,dx$$
$$= a^3x-\frac{3a^3\cos(c+dx)}{d}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}\left(3a^3\right)\int 1\,dx-\frac{a^3\,\text{Subst}\left(\int \left(1-x\right.\right.}{}$$
$$= \frac{5a^3x}{2}-\frac{4a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

Mathematica [A] time = 0.326201, size = 44, normalized size = 0.7

$$\frac{a^3(-9\sin(2(c+dx))-45\cos(c+dx)+\cos(3(c+dx))+30c+30dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*(30*c + 30*d*x - 45*Cos[c + d*x] + Cos[3*(c + d*x)] - 9*Sin[2*(c + d*x
)]))/(12*d)
```

Maple [A] time = 0.027, size = 74, normalized size = 1.2

$$\frac{1}{d}\left(-\frac{a^3\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}+3\,a^3\left(-1/2\,\cos(dx+c)\sin(dx+c)+1/2\,dx+c/2\right)-3\,a^3\cos(dx+c)+a^3\,(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^3*(-1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)-3*a^3*cos(d*x+c)+a^3*(d*x+c))
```

Maxima [A] time = 1.07352, size = 97, normalized size = 1.54

$$a^3x + \frac{\left(\cos\left(dx+c\right)^3 - 3\,\cos\left(dx+c\right)\right)a^3}{3\,d} + \frac{3\left(2\,dx + 2\,c - \sin\left(2\,dx+2\,c\right)\right)a^3}{4\,d} - \frac{3\,a^3\cos\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3/d + 3/4*(2*d*x + 2*c - si
n(2*d*x + 2*c))*a^3/d - 3*a^3*cos(d*x + c)/d
```

Fricas [A] time = 1.45802, size = 134, normalized size = 2.13

$$\frac{2\,a^3\cos\left(dx+c\right)^3 + 15\,a^3dx - 9\,a^3\cos\left(dx+c\right)\sin\left(dx+c\right) - 24\,a^3\cos\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 15*a^3*d*x - 9*a^3*cos(d*x + c)*sin(d*x + c) -
24*a^3*cos(d*x + c))/d
```

Sympy [A] time = 0.655117, size = 121, normalized size = 1.92

$$\begin{cases} \frac{3a^3x\sin^2\left(c+dx\right)}{2} + \frac{3a^3x\cos^2\left(c+dx\right)}{2} + a^3x - \frac{a^3\sin^2\left(c+dx\right)\cos\left(c+dx\right)}{d} - \frac{3a^3\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} - \frac{2a^3\cos^3\left(c+dx\right)}{3d} - \frac{3a^3\cos\left(c+dx\right)}{d} & \text{for } d \neq (\\ x\left(a\sin\left(c\right)+a\right)^3 & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**2/2 + a**3*x
 - a**3*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**3*sin(c + d*x)*cos(c + d*x)/(
2*d) - 2*a**3*cos(c + d*x)**3/(3*d) - 3*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*
(a*sin(c) + a)**3, True))
```

Giac [A] time = 1.64017, size = 74, normalized size = 1.17

$$\frac{5}{2}\,a^3x + \frac{a^3\cos\left(3\,dx + 3\,c\right)}{12\,d} - \frac{15\,a^3\cos\left(dx + c\right)}{4\,d} - \frac{3\,a^3\sin\left(2\,dx + 2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/2*a^3*x + 1/12*a^3*cos(3*d*x + 3*c)/d - 15/4*a^3*cos(d*x + c)/d - 3/4*a^3
*sin(2*d*x + 2*c)/d
```

3.33 $\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

Optimal. Leaf size=92

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

```
[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3
*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c +
 d*x])/(2*d)
```

Rubi [A] time = 0.136857, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 3770, 3767, 8, 2638, 2635, 2633}

$$-\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*x)/2 - (3*a^3*ArcTanh[Cos[c + d*x]])/d + (3*a^3*Cos[c + d*x])/d - (a^3
*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a^3*Cos[c + d*x]*Sin[c +
 d*x])/(2*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{\int \left(2a^5+3a^5\csc(c+dx)+a^5\csc^2(c+dx)-2a^5\sin(c+dx)-3a^5\sin^2(c+dx)\right.}{a^2}$$

$$= 2a^3x+a^3\int \csc^2(c+dx)\,dx-a^3\int \sin^3(c+dx)\,dx-\left(2a^3\right)\int \sin(c+dx)\,dx+$$

$$= 2a^3x-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+d}{2d}$$

$$= \frac{a^3x}{2}-\frac{3a^3\tanh^{-1}(\cos(c+dx))}{d}+\frac{3a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}-\frac{a^3\cot(}{d}$$

Mathematica [A] time = 1.06475, size = 106, normalized size = 1.15

$$-\frac{a^3\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left((15-66\sin(c+dx))\cos(c+dx)+(2\sin(c+dx)+9)\cos(3(c+dx))-12\sin(c+\right.}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x]*(15 - 66*Sin[c + d*x]
) - 12*(c + d*x - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]])*Sin[c
+ d*x] + Cos[3*(c + d*x)]*(9 + 2*Sin[c + d*x])))/(48*d)
```

Maple [A] time = 0.043, size = 105, normalized size = 1.1

$$-\frac{a^3(\cos(dx+c))^3}{3d}+\frac{3a^3\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^3x}{2}+\frac{a^3c}{2d}+3\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{d}+3\frac{a^3\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cos(d*x+c)^3/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3
*c+3/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3*a^3*cos(d*x+c)/d-a^3*cot(d*x+c)/d
```

Maxima [A] time = 1.63734, size = 126, normalized size = 1.37

$$-\frac{4a^3\cos(dx+c)^3-9(2dx+2c+\sin(2dx+2c))a^3+12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3-18a^3(2\cos(dx+c)-\log(\cos(dx+$$

$$12d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*a^3*cos(d*x + c)^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*(
d*x + c + 1/tan(d*x + c))*a^3 - 18*a^3*(2*cos(d*x + c) - log(cos(d*x + c) +
 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.56613, size = 321, normalized size = 3.49

$$-\frac{9a^3\cos(dx+c)^3+9a^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9a^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-3a^3\cos(dx$$

$$6d\sin(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a^3*cos(d*x + c)^3 + 9*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
 - 9*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^3*cos(d*x + c) + (
2*a^3*cos(d*x + c)^3 - 3*a^3*d*x - 18*a^3*cos(d*x + c))*sin(d*x + c))/(d*si
n(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^3\left(\int 3\sin(c+dx)\cot^2(c+dx)\,dx+\int 3\sin^2(c+dx)\cot^2(c+dx)\,dx+\int \sin^3(c+dx)\cot^2(c+dx)\,dx+\int \cot^2(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(3*sin(c + d*x)
**2*cot(c + d*x)**2, x) + Integral(sin(c + d*x)**3*cot(c + d*x)**2, x) + In
tegral(cot(c + d*x)**2, x))
```

Giac [A] time = 2.05943, size = 219, normalized size = 2.38

$$\frac{3(dx+c)a^3+18a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{3\left(6a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^3\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-\frac{2\left(9a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12a^3\tan}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(d*x + c)*a^3 + 18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/
2*d*x + 1/2*c) - 3*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c)
- 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^3*
tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 16*a^3)/(tan(1/2*d*x
+ 1/2*c)^2 + 1)^3)/d
```

3.34 $\int (a+a\sin(c+dx))^4 \tan^5(c+dx)\,dx$

Optimal. Leaf size=129

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{9a^4\sin^2(c+dx)}{2d}+\frac{a^6}{d(a-a\sin(c+dx))^2}-\frac{11a^5}{d(a-a\sin(c+dx))}-\frac{16a^4\sin(c+dx)}{d}$$

```
[Out] (-25*a^4*Log[1 - Sin[c + d*x]])/d - (16*a^4*Sin[c + d*x])/d - (9*a^4*Sin[c
+ d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^4*Sin[c + d*x]^4)/(4*d)
 + a^6/(d*(a - a*Sin[c + d*x])^2) - (11*a^5)/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0947062, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{9a^4\sin^2(c+dx)}{2d}+\frac{a^6}{d(a-a\sin(c+dx))^2}-\frac{11a^5}{d(a-a\sin(c+dx))}-\frac{16a^4\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^5,x]
```


```
[Out] (-25*a^4*Log[1 - Sin[c + d*x]])/d - (16*a^4*Sin[c + d*x])/d - (9*a^4*Sin[c
+ d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^4*Sin[c + d*x]^4)/(4*d)
 + a^6/(d*(a - a*Sin[c + d*x])^2) - (11*a^5)/(d*(a - a*Sin[c + d*x]))
```


Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan^5(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^5(a+x)}{(a-x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-16a^3+\frac{2a^6}{(a-x)^3}-\frac{11a^5}{(a-x)^2}+\frac{25a^4}{a-x}-9a^2x-4ax^2-x^3\right)dx, x, a\sin(c+d\right.}{d}$$
$$= -\frac{25a^4\log(1-\sin(c+dx))}{d}-\frac{16a^4\sin(c+dx)}{d}-\frac{9a^4\sin^2(c+dx)}{2d}-\frac{4a^4\sin^3(c}{3d}$$

Mathematica [A] time = 0.469245, size = 83, normalized size = 0.64

$$-\frac{a^4\left(3\sin^4(c+dx)+16\sin^3(c+dx)+54\sin^2(c+dx)+192\sin(c+dx)+\frac{120-132\sin(c+dx)}{(\sin(c+dx)-1)^2}+300\log(1-\sin(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^5,x]
```

```
[Out] -(a^4*(300*Log[1 - Sin[c + d*x]] + (120 - 132*Sin[c + d*x])/(-1 + Sin[c + d
*x])^2 + 192*Sin[c + d*x] + 54*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*Sin[c
 + d*x]^4))/(12*d)
```

Maple [B] time = 0.1, size = 387, normalized size = 3.

$$-\frac{5a^4(\sin(dx+c))^7}{2d}-4\frac{a^4(\sin(dx+c))^6}{d}-5\frac{a^4(\sin(dx+c))^5}{d}-\frac{3a^4(\sin(dx+c))^8}{4d}+\frac{a^4(\sin(dx+c))^{10}}{4d(\cos(dx+c))^4}-\frac{3a^4(\sin(}{4d(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x)
```

```
[Out] -5/2/d*a^4*sin(d*x+c)^7-4/d*a^4*sin(d*x+c)^6-5/d*a^4*sin(d*x+c)^5-3/4/d*a^4
*sin(d*x+c)^8+1/4/d*a^4*sin(d*x+c)^10/cos(d*x+c)^4-3/4/d*a^4*sin(d*x+c)^10/
cos(d*x+c)^2+1/d*a^4*sin(d*x+c)^9/cos(d*x+c)^4-5/2/d*a^4*sin(d*x+c)^9/cos(d
*x+c)^2+3/2/d*a^4*sin(d*x+c)^8/cos(d*x+c)^4-3/d*a^4*sin(d*x+c)^8/cos(d*x+c)
^2+1/d*a^4*sin(d*x+c)^7/cos(d*x+c)^4-3/2/d*a^4*sin(d*x+c)^7/cos(d*x+c)^2+1/
4/d*a^4*tan(d*x+c)^4-1/2/d*a^4*tan(d*x+c)^2-25/3*a^4*sin(d*x+c)^3/d-25*a^4*
```

```
sin(d*x+c)/d+25/d*a^4*ln(sec(d*x+c)+tan(d*x+c))-6*a^4*sin(d*x+c)^4/d-12*a^4
*sin(d*x+c)^2/d-25/d*a^4*ln(cos(d*x+c))
```

Maxima [A] time = 1.10668, size = 147, normalized size = 1.14

$$-\frac{3\,a^4\sin(dx+c)^4+16\,a^4\sin(dx+c)^3+54\,a^4\sin(dx+c)^2+300\,a^4\log(\sin(dx+c)-1)+192\,a^4\sin(dx+c)-\frac{12\left(11\,a^4\ldots\right)}{\sin(dx+c\ldots}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 54*a^4*sin(d*x + c)^2
 + 300*a^4*log(sin(d*x + c) - 1) + 192*a^4*sin(d*x + c) - 12*(11*a^4*sin(d*
x + c) - 10*a^4)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A] time = 1.6719, size = 394, normalized size = 3.05

$$-\frac{24\,a^4\cos(dx+c)^6-272\,a^4\cos(dx+c)^4-2393\,a^4\cos(dx+c)^2+1906\,a^4+2400\left(a^4\cos(dx+c)^2+2\,a^4\sin(dx+c)-\ldots\right.}{96\left(d\cos(dx+c)^2+2\,d\sin(dx+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/96*(24*a^4*cos(d*x + c)^6 - 272*a^4*cos(d*x + c)^4 - 2393*a^4*cos(d*x +
c)^2 + 1906*a^4 + 2400*(a^4*cos(d*x + c)^2 + 2*a^4*sin(d*x + c) - 2*a^4)*lo
g(-sin(d*x + c) + 1) - 10*(8*a^4*cos(d*x + c)^4 - 96*a^4*cos(d*x + c)^2 + 1
81*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="giac")
```

```
[Out] Timed out
```

3.35 $\int (a+a\sin(c+dx))^4\tan^3(c+dx)\,dx$

Optimal. Leaf size=107

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{4a^4\sin^2(c+dx)}{d}+\frac{4a^5}{d(a-a\sin(c+dx))}+\frac{12a^4\sin(c+dx)}{d}+\frac{16a^4\log(1-\sin(c+d$$

```
[Out] (16*a^4*Log[1 - Sin[c + d*x]])/d + (12*a^4*Sin[c + d*x])/d + (4*a^4*Sin[c +
 d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d) + (4
*a^5)/(d*(a - a*Sin[c + d*x]))
```

Rubi [A] time = 0.0786493, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{4a^4\sin^2(c+dx)}{d}+\frac{4a^5}{d(a-a\sin(c+dx))}+\frac{12a^4\sin(c+dx)}{d}+\frac{16a^4\log(1-\sin(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]
```

```
[Out] (16*a^4*Log[1 - Sin[c + d*x]])/d + (12*a^4*Sin[c + d*x])/d + (4*a^4*Sin[c +
 d*x]^2)/d + (4*a^4*Sin[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x]^4)/(4*d) + (4
*a^5)/(d*(a - a*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^2}{(a-x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(12a^3+\frac{4a^5}{(a-x)^2}-\frac{16a^4}{a-x}+8a^2x+4ax^2+x^3\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{16a^4\log(1-\sin(c+dx))}{d}+\frac{12a^4\sin(c+dx)}{d}+\frac{4a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin^3(c-}{3d}$$

Mathematica [A] time = 0.155479, size = 76, normalized size = 0.71

$$\frac{a^4\left(3\sin^4(c+dx)+16\sin^3(c+dx)+48\sin^2(c+dx)+144\sin(c+dx)+\frac{48}{1-\sin(c+dx)}+192\log(1-\sin(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]
```

```
[Out] (a^4*(192*Log[1 - Sin[c + d*x]] + 48/(1 - Sin[c + d*x]) + 144*Sin[c + d*x]
+ 48*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)
```

Maple [B] time = 0.084, size = 245, normalized size = 2.3

$$\frac{a^4(\sin(dx+c))^8}{2d(\cos(dx+c))^2}+\frac{a^4(\sin(dx+c))^6}{2d}+\frac{15a^4(\sin(dx+c))^4}{4d}+\frac{15a^4(\sin(dx+c))^2}{2d}+16\frac{a^4\ln(\cos(dx+c))}{d}+2\frac{a^4(\text{si}}{d(\text{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^4*sin(d*x+c)^8/cos(d*x+c)^2+1/2/d*a^4*sin(d*x+c)^6+15/4*a^4*sin(d*x
+c)^4/d+15/2*a^4*sin(d*x+c)^2/d+16/d*a^4*ln(cos(d*x+c))+2/d*a^4*sin(d*x+c)^
7/cos(d*x+c)^2+2/d*a^4*sin(d*x+c)^5+16/3*a^4*sin(d*x+c)^3/d+16*a^4*sin(d*x+
c)/d-16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^4*sin(d*x+c)^6/cos(d*x+c)^2+2
/d*a^4*sin(d*x+c)^5/cos(d*x+c)^2+1/2/d*a^4*tan(d*x+c)^2
```

Maxima [A] time = 1.06484, size = 115, normalized size = 1.07

$$\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+48\,a^4\sin\left(dx+c\right)^2+192\,a^4\log\left(\sin\left(dx+c\right)-1\right)+144\,a^4\sin\left(dx+c\right)-\frac{48\,a^4}{\sin(dx+c)-}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 48*a^4*sin(d*x + c)^2
+ 192*a^4*log(sin(d*x + c) - 1) + 144*a^4*sin(d*x + c) - 48*a^4/(sin(d*x +
c) - 1))/d
```

Fricas [A] time = 1.5324, size = 293, normalized size = 2.74

$$\frac{104\,a^4\cos\left(dx+c\right)^4-976\,a^4\cos\left(dx+c\right)^2+689\,a^4+1536\left(a^4\sin\left(dx+c\right)-a^4\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(24\,a^4\cos\left(dx\right.\right.}{96\left(d\sin\left(dx+c\right)-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(104*a^4*cos(d*x + c)^4 - 976*a^4*cos(d*x + c)^2 + 689*a^4 + 1536*(a^4
*sin(d*x + c) - a^4)*log(-sin(d*x + c) + 1) + (24*a^4*cos(d*x + c)^4 - 304*
a^4*cos(d*x + c)^2 - 1073*a^4)*sin(d*x + c))/(d*sin(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.36 $\int (a + a\sin(c + dx))^4 \tan(c + dx)\, dx$

Optimal. Leaf size=88

$$-\frac{a^4 \sin^4(c+dx)}{4d} - \frac{4a^4 \sin^3(c+dx)}{3d} - \frac{7a^4 \sin^2(c+dx)}{2d} - \frac{8a^4 \sin(c+dx)}{d} - \frac{8a^4 \log(1-\sin(c+dx))}{d}$$

```
[Out] (-8*a^4*Log[1 - Sin[c + d*x]])/d - (8*a^4*Sin[c + d*x])/d - (7*a^4*Sin[c +
d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^4*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0526452, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 77}

$$-\frac{a^4 \sin^4(c+dx)}{4d} - \frac{4a^4 \sin^3(c+dx)}{3d} - \frac{7a^4 \sin^2(c+dx)}{2d} - \frac{8a^4 \sin(c+dx)}{d} - \frac{8a^4 \log(1-\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x],x]
```

```
[Out] (-8*a^4*Log[1 - Sin[c + d*x]])/d - (8*a^4*Sin[c + d*x])/d - (7*a^4*Sin[c +
d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^4*Sin[c + d*x]^4)/(4*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{a-x}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-8a^3+\frac{8a^4}{a-x}-7a^2x-4ax^2-x^3\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{8a^4\log(1-\sin(c+dx))}{d}-\frac{8a^4\sin(c+dx)}{d}-\frac{7a^4\sin^2(c+dx)}{2d}-\frac{4a^4\sin^3(c+\ldots}{3d}$$

Mathematica [A] time = 0.0699065, size = 62, normalized size = 0.7

$$-\frac{a^4\left(3\sin^4(c+dx)+16\sin^3(c+dx)+42\sin^2(c+dx)+96\sin(c+dx)+96\log(1-\sin(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x],x]
```

```
[Out] -(a^4*(96*Log[1 - Sin[c + d*x]] + 96*Sin[c + d*x] + 42*Sin[c + d*x]^2 + 16*
Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)
```

Maple [A] time = 0.052, size = 101, normalized size = 1.2

$$-\frac{a^4(\sin(dx+c))^4}{4d}-\frac{7a^4(\sin(dx+c))^2}{2d}-8\frac{a^4\ln(\cos(dx+c))}{d}-\frac{4a^4(\sin(dx+c))^3}{3d}-8\frac{a^4\sin(dx+c)}{d}+8\frac{a^4\ln(\sec\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c),x)
```

```
[Out] -1/4*a^4*sin(d*x+c)^4/d-7/2*a^4*sin(d*x+c)^2/d-8/d*a^4*ln(cos(d*x+c))-4/3*a
^4*sin(d*x+c)^3/d-8*a^4*sin(d*x+c)/d+8/d*a^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.07582, size = 95, normalized size = 1.08

$$-\frac{3a^4\sin(dx+c)^4+16a^4\sin(dx+c)^3+42a^4\sin(dx+c)^2+96a^4\log(\sin(dx+c)-1)+96a^4\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 42*a^4*sin(d*x + c)^2
 + 96*a^4*log(sin(d*x + c) - 1) + 96*a^4*sin(d*x + c))/d
```

Fricas [A] time = 1.53773, size = 182, normalized size = 2.07

$$-\frac{3\,a^4\cos\left(dx+c\right)^4-48\,a^4\cos\left(dx+c\right)^2+96\,a^4\log\left(-\sin\left(dx+c\right)+1\right)-16\left(a^4\cos\left(dx+c\right)^2-7\,a^4\right)\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a^4*cos(d*x + c)^4 - 48*a^4*cos(d*x + c)^2 + 96*a^4*log(-sin(d*x +
 c) + 1) - 16*(a^4*cos(d*x + c)^2 - 7*a^4)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int 4\sin\left(c+dx\right)\tan\left(c+dx\right)dx+\int 6\sin^2\left(c+dx\right)\tan\left(c+dx\right)dx+\int 4\sin^3\left(c+dx\right)\tan\left(c+dx\right)dx+\int\sin^4\left(c+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c),x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*tan(c + d*x), x) + Integral(6*sin(c + d*x)**2
*tan(c + d*x), x) + Integral(4*sin(c + d*x)**3*tan(c + d*x), x) + Integral(
sin(c + d*x)**4*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.37 $\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=102

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{5a^4\sin^2(c+dx)}{2d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{5a^4\log(\sin(c+dx))}{d}$$

```
[Out] (-4*a^4*Csc[c + d*x])/d - (a^4*Csc[c + d*x]^2)/(2*d) + (5*a^4*Log[Sin[c + d
*x]])/d - (5*a^4*Sin[c + d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^
4*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0667044, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 75}

$$-\frac{a^4\sin^4(c+dx)}{4d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{5a^4\sin^2(c+dx)}{2d}-\frac{a^4\csc^2(c+dx)}{2d}-\frac{4a^4\csc(c+dx)}{d}+\frac{5a^4\log(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a^4*Csc[c + d*x])/d - (a^4*Csc[c + d*x]^2)/(2*d) + (5*a^4*Log[Sin[c + d
*x]])/d - (5*a^4*Sin[c + d*x]^2)/(2*d) - (4*a^4*Sin[c + d*x]^3)/(3*d) - (a^
4*Sin[c + d*x]^4)/(4*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^5}{x^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^6}{x^3}+\frac{4a^5}{x^2}+\frac{5a^4}{x}-5a^2x-4ax^2-x^3\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{4a^4\csc(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}+\frac{5a^4\log(\sin(c+dx))}{d}-\frac{5a^4\sin^2(c+dx)}{2d}$$

Mathematica [A] time = 0.137296, size = 78, normalized size = 0.76

$$-\frac{a^4\sin^4(c+dx)\left(6\csc^6(c+dx)+48\csc^5(c+dx)+30\csc^2(c+dx)+16\csc(c+dx)+\csc^4(c+dx)(90-60\log(\sin(c\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(a^4*(3 + 16*Csc[c + d*x] + 30*Csc[c + d*x]^2 + 48*Csc[c + d*x]^5 + 6*Csc[
c + d*x]^6 + Csc[c + d*x]^4*(90 - 60*Log[Sin[c + d*x]]))*Sin[c + d*x]^4)/(1
2*d)
```

Maple [A] time = 0.054, size = 125, normalized size = 1.2

$$-\frac{a^4(\cos(dx+c))^4}{4d}-\frac{8a^4\sin(dx+c)(\cos(dx+c))^2}{3d}-\frac{16a^4\sin(dx+c)}{3d}+3\frac{a^4(\cos(dx+c))^2}{d}+5\frac{a^4\ln(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/4/d*a^4*cos(d*x+c)^4-8/3/d*a^4*sin(d*x+c)*cos(d*x+c)^2-16/3*a^4*sin(d*x+
c)/d+3/d*a^4*cos(d*x+c)^2+5*a^4*ln(sin(d*x+c))/d-4/d*a^4/sin(d*x+c)*cos(d*x
+c)^4-1/2/d*a^4*cot(d*x+c)^2
```

Maxima [A] time = 1.11802, size = 111, normalized size = 1.09

$$-\frac{3a^4\sin(dx+c)^4+16a^4\sin(dx+c)^3+30a^4\sin(dx+c)^2-60a^4\log(\sin(dx+c))+\frac{6\left(8a^4\sin(dx+c)+a^4\right)}{\sin(dx+c)^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2
 - 60*a^4*log(sin(d*x + c)) + 6*(8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2)/
d
```

Fricas [A] time = 1.73429, size = 324, normalized size = 3.18

$$-\frac{24\,a^4\cos\left(dx+c\right)^6-312\,a^4\cos\left(dx+c\right)^4+423\,a^4\cos\left(dx+c\right)^2-183\,a^4-480\left(a^4\cos\left(dx+c\right)^2-a^4\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right.\right.}{96\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/96*(24*a^4*cos(d*x + c)^6 - 312*a^4*cos(d*x + c)^4 + 423*a^4*cos(d*x + c
)^2 - 183*a^4 - 480*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*sin(d*x + c)) - 128*
(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + 4*a^4)*sin(d*x + c))/(d*cos(d*
x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39327, size = 130, normalized size = 1.27

$$-\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+30\,a^4\sin\left(dx+c\right)^2-60\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)+\frac{6\left(15\,a^4\sin(dx+c)^2+8\,a^4\sin(dx+c)+a^4\right)}{\sin(dx+c)^2}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^2
 - 60*a^4*log(abs(sin(d*x + c))) + 6*(15*a^4*sin(d*x + c)^2 + 8*a^4*sin(d*x
 + c) + a^4)/sin(d*x + c)^2)/d
```

3.38 $\int (a+a\sin(c+dx))^4 \tan^4(c+dx)\,dx$

Optimal. Leaf size=143

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{16a^4\cos(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{35a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{56a^4\cos(c+dx)}{3d(1-\sin(c+dx))}-$$

```
[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4
*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*
(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c
 + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.198685, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2709, 2650, 2648, 2638, 2635, 8, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{16a^4\cos(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{35a^4\sin(c+dx)\cos(c+dx)}{8d}-\frac{56a^4\cos(c+dx)}{3d(1-\sin(c+dx))}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (163*a^4*x)/8 - (16*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) + (4
*a^4*Cos[c + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*a^4*Cos[c + d*x])/(3*d*
(1 - Sin[c + d*x])) - (35*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c
 + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
```

```
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx = a^4\int\left(16+\frac{4}{(-1+\sin(c+dx))^2}+\frac{20}{-1+\sin(c+dx)}+12\sin(c+dx)+8\sin^2(c\right.$$

$$= 16a^4x+a^4\int\sin^4(c+dx)\,dx+\left(4a^4\right)\int\frac{1}{(-1+\sin(c+dx))^2}\,dx+\left(4a^4\right)\int\sin^3($$

$$= 16a^4x-\frac{12a^4\cos(c+dx)}{d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{20a^4\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{4a^4\cos}{}$$

$$= 20a^4x-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{56a^4\mathrm{co}}{3d(1-\mathrm{si}}$$

$$= \frac{163a^4x}{8}-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{56a^4}{3d(1-}$$

Mathematica [A] time = 1.65143, size = 252, normalized size = 1.76

$$\frac{a^4\left(-11736c\sin\left(\frac{1}{2}(c+dx)\right)-11736dx\sin\left(\frac{1}{2}(c+dx)\right)-16488\sin\left(\frac{1}{2}(c+dx)\right)-3912c\sin\left(\frac{3}{2}(c+dx)\right)-3912dx\sin\left(\frac{3}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

```
[Out] (a^4*(24*(209 + 489*c + 489*d*x)*Cos[(c + d*x)/2] - 24*(453 + 163*c + 163*d
*x)*Cos[(3*(c + d*x))/2] + 885*Cos[(5*(c + d*x))/2] - 129*Cos[(7*(c + d*x))
/2] - 23*Cos[(9*(c + d*x))/2] + 3*Cos[(11*(c + d*x))/2] - 16488*Sin[(c + d*
x)/2] - 11736*c*Sin[(c + d*x)/2] - 11736*d*x*Sin[(c + d*x)/2] + 3704*Sin[(3
*(c + d*x))/2] - 3912*c*Sin[(3*(c + d*x))/2] - 3912*d*x*Sin[(3*(c + d*x))/2
] + 885*Sin[(5*(c + d*x))/2] + 129*Sin[(7*(c + d*x))/2] - 23*Sin[(9*(c + d*
x))/2] - 3*Sin[(11*(c + d*x))/2]))/(384*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)
/2])^3)
```

Maple [B] time = 0.092, size = 360, normalized size = 2.5

$$\frac{1}{d}\left(a^4\left(\frac{(\sin(dx+c))^9}{3(\cos(dx+c))^3}-2\frac{(\sin(dx+c))^9}{\cos(dx+c)}-2\left((\sin(dx+c))^7+7/6(\sin(dx+c))^5+\frac{35(\sin(dx+c))^3}{24}+\frac{35\sin(dx+}{16}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^4*(1/3*sin(d*x+c)^9/cos(d*x+c)^3-2*sin(d*x+c)^9/cos(d*x+c)-2*(sin(d*
x+c)^7+7/6*sin(d*x+c)^5+35/24*sin(d*x+c)^3+35/16*sin(d*x+c))*cos(d*x+c)+35/
8*d*x+35/8*c)+4*a^4*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x
+c)-5/3*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c))+6
*a^4*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin(d*
x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)+4*a^4*(1
/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*
sin(d*x+c)^2)*cos(d*x+c))+a^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

Maxima [A] time = 1.67263, size = 321, normalized size = 2.24

$$\frac{32\left(\cos(dx+c)^3 - \frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3} - 9\cos(dx+c)\right)a^4 + \left(8\tan(dx+c)^3 + 105\,dx + 105\,c - \frac{3\left(13\tan(dx+c)^3+11\tan(dx+c)\right)}{\tan(dx+c)^4+2\tan(dx+c)^2+1} - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/24*(32*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*
x + c))*a^4 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*tan(d*x + c)^3 +
11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 72*tan(d*x + c))
*a^4 + 24*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^
2 + 1) - 12*tan(d*x + c))*a^4 + 8*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x
 + c))*a^4 - 32*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)
))/d
```

Fricas [A] time = 1.56258, size = 620, normalized size = 4.34

$$-\frac{6\,a^4\cos(dx+c)^6 - 20\,a^4\cos(dx+c)^5 - 85\,a^4\cos(dx+c)^4 + 214\,a^4\cos(dx+c)^3 + 978\,a^4dx + 32\,a^4 - \left(489\,a^4dx + 7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^6 - 20*a^4*cos(d*x + c)^5 - 85*a^4*cos(d*x + c)^4
 + 214*a^4*cos(d*x + c)^3 + 978*a^4*d*x + 32*a^4 - (489*a^4*d*x + 721*a^4)*
```

```
cos(d*x + c)^2 + (489*a^4*d*x - 962*a^4)*cos(d*x + c) - (6*a^4*cos(d*x + c)
^5 + 26*a^4*cos(d*x + c)^4 - 59*a^4*cos(d*x + c)^3 + 978*a^4*d*x - 273*a^4*
cos(d*x + c)^2 - 32*a^4 + (489*a^4*d*x - 994*a^4)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c)
 - 2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.39 $\int (a + a\sin(c + dx))^4 \tan^2(c + dx)\,dx$

Optimal. Leaf size=113

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{12a^4\cos(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{31a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx)}$$

```
[Out] (-95*a^4*x)/8 + (12*a^4*Cos[c + d*x])/d - (4*a^4*Cos[c + d*x]^3)/(3*d) + (8
*a^4*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (31*a^4*Cos[c + d*x]*Sin[c + d*
x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.160916, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 2648, 2638, 2635, 8, 2633}

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{12a^4\cos(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{31a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] (-95*a^4*x)/8 + (12*a^4*Cos[c + d*x])/d - (4*a^4*Cos[c + d*x]^3)/(3*d) + (8
*a^4*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (31*a^4*Cos[c + d*x]*Sin[c + d*
x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\tan^2(c+dx)\,dx = a^2\int\Bigg(-8a^2-\frac{8a^2}{-1+\sin(c+dx)}-8a^2\sin(c+dx)-7a^2\sin^2(c+dx)-4a^2\sin^3(c$$

$$= -8a^4x-a^4\int\sin^4(c+dx)\,dx-\left(4a^4\right)\int\sin^3(c+dx)\,dx-\left(7a^4\right)\int\sin^2(c+dx)$$

$$= -8a^4x+\frac{8a^4\cos(c+dx)}{d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{7a^4\cos(c+dx)\sin(c+dx)}{2d}+$$

$$= -\frac{23a^4x}{2}+\frac{12a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{31a^4\cos}{}$$

$$= -\frac{95a^4x}{8}+\frac{12a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{31a^4\cos}{}$$

Mathematica [A] time = 1.12581, size = 125, normalized size = 1.11

$$\frac{(a\sin(c+dx)+a)^4\Bigg(-1140(c+dx)+192\sin(2(c+dx))-3\sin(4(c+dx))+1056\cos(c+dx)-32\cos(3(c+dx))+\frac{}{\cos\left(\right.}}{96d\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

```
[Out] ((a + a*Sin[c + d*x])^4*(-1140*(c + d*x) + 1056*Cos[c + d*x] - 32*Cos[3*(c
+ d*x)] + (1536*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 1
92*Sin[2*(c + d*x)] - 3*Sin[4*(c + d*x)]))/(96*d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])^8)
```

Maple [B] time = 0.06, size = 231, normalized size = 2.

$$\frac{1}{d}\left(a^4\left(\frac{(\sin(dx+c))^7}{\cos(dx+c)}+\left((\sin(dx+c))^5+\frac{5\,(\sin(dx+c))^3}{4}+\frac{15\,\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15\,dx}{8}-\frac{15\,c}{8}\right)+4\,a^4\left(\frac{(\sin}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^4*(sin(d*x+c)^7/cos(d*x+c)+(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d
*x+c))*cos(d*x+c)-15/8*d*x-15/8*c)+4*a^4*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(
d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+6*a^4*(sin(d*x+c)^5/cos(d*x+c)+(sin(
d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+4*a^4*(sin(d*x+c)^4/cos(
d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+a^4*(tan(d*x+c)-d*x-c))
```

Maxima [A] time = 1.58886, size = 244, normalized size = 2.16

$$-\frac{32\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\,\cos(dx+c)\right)a^4+3\left(15\,dx+15\,c-\frac{9\,\tan(dx+c)^3+7\,\tan(dx+c)}{\tan(dx+c)^4+2\,\tan(dx+c)^2+1}-8\,\tan(dx+c)\right)a^4+72\,(\cdots}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(32*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^4 + 3*(15*d*
x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x
+ c)^2 + 1) - 8*tan(d*x + c))*a^4 + 72*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 2*tan(d*x + c))*a^4 + 24*(d*x + c - tan(d*x + c))*a^4 - 96*a
^4*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.5892, size = 456, normalized size = 4.04

$$-\frac{6\,a^4\cos\left(dx+c\right)^5+32\,a^4\cos\left(dx+c\right)^4-73\,a^4\cos\left(dx+c\right)^3+285\,a^4dx-288\,a^4\cos\left(dx+c\right)^2-192\,a^4+3\left(95\,a^4dx-1\right.}{24\left(d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^5 + 32*a^4*cos(d*x + c)^4 - 73*a^4*cos(d*x + c)^3
 + 285*a^4*d*x - 288*a^4*cos(d*x + c)^2 - 192*a^4 + 3*(95*a^4*d*x - 127*a^4
)*cos(d*x + c) + (6*a^4*cos(d*x + c)^4 - 26*a^4*cos(d*x + c)^3 - 285*a^4*d*
x - 99*a^4*cos(d*x + c)^2 + 189*a^4*cos(d*x + c) - 192*a^4)*sin(d*x + c))/(
d*cos(d*x + c) - d*sin(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.40 $\int (a + a\sin(c + dx))^4 \, dx$

Optimal. Leaf size=87

$$\frac{4a^4\cos^3(c+dx)}{3d} - \frac{8a^4\cos(c+dx)}{d} - \frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{27a^4\sin(c+dx)\cos(c+dx)}{8d} + \frac{35a^4x}{8}$$

```
[Out] (35*a^4*x)/8 - (8*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) - (27*
a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4
*d)
```

Rubi [A] time = 0.0817973, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2645, 2638, 2635, 8, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d} - \frac{8a^4\cos(c+dx)}{d} - \frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{27a^4\sin(c+dx)\cos(c+dx)}{8d} + \frac{35a^4x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (35*a^4*x)/8 - (8*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) - (27*
a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4
*d)
```

Rule 2645

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int (a+a\sin(c+dx))^4\,dx = \int \left(a^4+4a^4\sin(c+dx)+6a^4\sin^2(c+dx)+4a^4\sin^3(c+dx)+a^4\sin^4(c+dx)\right)dx$$
$$= a^4x+a^4\int \sin^4(c+dx)\,dx+\left(4a^4\right)\int \sin(c+dx)\,dx+\left(4a^4\right)\int \sin^3(c+dx)\,dx+\left(6a^4\right)\int \sin$$
$$= a^4x-\frac{4a^4\cos(c+dx)}{d}-\frac{3a^4\cos(c+dx)\sin(c+dx)}{d}-\frac{a^4\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}\left(3a^4\right)$$
$$= 4a^4x-\frac{8a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^4\cos(c+dx)\sin}{4d}$$
$$= \frac{35a^4x}{8}-\frac{8a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^4\cos(c+dx)}{4d}$$

Mathematica [A] time = 0.425065, size = 57, normalized size = 0.66

$$\frac{a^4(3(-56\sin(2(c+dx))+\sin(4(c+dx))+140c+140dx)-672\cos(c+dx)+32\cos(3(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(-672*Cos[c + d*x] + 32*Cos[3*(c + d*x)] + 3*(140*c + 140*d*x - 56*Sin
[2*(c + d*x)] + Sin[4*(c + d*x)])))/(96*d)
```

Maple [A] time = 0.029, size = 111, normalized size = 1.3

$$\frac{1}{d}\left(a^4\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{4a^4\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}+6a^4(-1/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-4/3*
a^4*(2+sin(d*x+c)^2)*cos(d*x+c)+6*a^4*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1
/2*c)-4*a^4*cos(d*x+c)+a^4*(d*x+c))
```

Maxima [A] time = 1.11432, size = 146, normalized size = 1.68

$$a^4x + \frac{4\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)a^4}{3d} + \frac{(12dx + 12c + \sin(4dx+4c) - 8\sin(2dx+2c))a^4}{32d} + \frac{3(2dx + 2c - \sin}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x + 4/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^4/d + 1/32*(12*d*x + 12*c +
 sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^4/d + 3/2*(2*d*x + 2*c - sin(2*d*
x + 2*c))*a^4/d - 4*a^4*cos(d*x + c)/d
```

Fricas [A] time = 1.47868, size = 177, normalized size = 2.03

$$\frac{32a^4\cos(dx+c)^3 + 105a^4dx - 192a^4\cos(dx+c) + 3\left(2a^4\cos(dx+c)^3 - 29a^4\cos(dx+c)\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(32*a^4*cos(d*x + c)^3 + 105*a^4*d*x - 192*a^4*cos(d*x + c) + 3*(2*a^4
*cos(d*x + c)^3 - 29*a^4*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 2.07397, size = 224, normalized size = 2.57

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + 3a^4x\sin^2(c+dx) + \frac{3a^4x\cos^4(c+dx)}{8} + 3a^4x\cos^2(c+dx) + a^4x - \frac{5a^4\sin^3(c+dx)\cos}{8d} \\ x(a\sin(c)+a)^4 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*x*
cos(c + d*x)**2 + a**4*x - 5*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 4*a*
*4*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*
d) - 3*a**4*sin(c + d*x)*cos(c + d*x)/d - 8*a**4*cos(c + d*x)**3/(3*d) - 4*
a**4*cos(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**4, True))
```

Giac [A] time = 1.38113, size = 97, normalized size = 1.11

$$\frac{35}{8}a^4x + \frac{a^4\cos(3\,dx + 3\,c)}{3\,d} - \frac{7\,a^4\cos(dx + c)}{d} + \frac{a^4\sin(4\,dx + 4\,c)}{32\,d} - \frac{7\,a^4\sin(2\,dx + 2\,c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 35/8*a^4*x + 1/3*a^4*cos(3*d*x + 3*c)/d - 7*a^4*cos(d*x + c)/d + 1/32*a^4*s
in(4*d*x + 4*c)/d - 7/4*a^4*sin(2*d*x + 2*c)/d
```

3.41 $\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=116

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}-$$

```
[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (
4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*S
in[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.159022, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 2635, 2633}

$$-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^4\sin(c+dx)\cos(c+dx)}{8d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (17*a^4*x)/8 - (4*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x])/d - (
4*a^4*Cos[c + d*x]^3)/(3*d) - (a^4*Cot[c + d*x])/d + (23*a^4*Cos[c + d*x]*S
in[c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(5a^6+4a^6\csc(c+dx)+a^6\csc^2(c+dx)-5a^6\sin^2(c+dx)-4a^6\sin^3(c+dx)\right.}{a^2}$$
$$= 5a^4x+a^4\int\csc^2(c+dx)\,dx-a^4\int\sin^4(c+dx)\,dx+\left(4a^4\right)\int\csc(c+dx)\,dx-$$
$$= 5a^4x-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{5a^4\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^4\cos(c+dx)}{4d}$$
$$= \frac{5a^4x}{2}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot}{}$$
$$= \frac{17a^4x}{8}-\frac{4a^4\tanh^{-1}(\cos(c+dx))}{d}+\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot}{}$$

Mathematica [A] time = 1.57393, size = 136, normalized size = 1.17

$$\frac{a^4\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(408c\sin(c+dx)+408dx\sin(c+dx)+320\sin(2(c+dx))-32\sin(4(c+dx))-48\,\mathrm{co}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-48*Cos[c + d*x] - 147*Cos[3*(c + d
*x)] + 3*Cos[5*(c + d*x)] + 408*c*Sin[c + d*x] + 408*d*x*Sin[c + d*x] - 768
*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 768*Log[Sin[(c + d*x)/2]]*Sin[c + d*x
] + 320*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(384*d)
```

Maple [A] time = 0.045, size = 127, normalized size = 1.1

$$-\frac{a^4(\cos(dx+c))^3\sin(dx+c)}{4d}+\frac{25a^4\cos(dx+c)\sin(dx+c)}{8d}+\frac{17a^4x}{8}+\frac{17a^4c}{8d}-\frac{4a^4(\cos(dx+c))^3}{3d}+4\frac{a^4\ln(\csc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/4/d*a^4*cos(d*x+c)^3*sin(d*x+c)+25/8*a^4*cos(d*x+c)*sin(d*x+c)/d+17/8*a^
4*x+17/8/d*a^4*c-4/3*a^4*cos(d*x+c)^3/d+4/d*a^4*ln(csc(d*x+c)-cot(d*x+c))+4
*a^4*cos(d*x+c)/d-a^4*cot(d*x+c)/d
```

Maxima [A] time = 1.57315, size = 158, normalized size = 1.36

$$-\frac{128a^4\cos(dx+c)^3-3(4dx+4c-\sin(4dx+4c))a^4-144(2dx+2c+\sin(2dx+2c))a^4+96\left(dx+c+\frac{1}{\tan(dx+c)}\right)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(128*a^4*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 14
4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 96*(d*x + c + 1/tan(d*x + c))*a^4
- 192*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)))
/d
```

Fricas [A] time = 1.69257, size = 360, normalized size = 3.1

$$\frac{6a^4\cos(dx+c)^5-81a^4\cos(dx+c)^3-48a^4\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+48a^4\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin}{24d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(6*a^4*cos(d*x + c)^5 - 81*a^4*cos(d*x + c)^3 - 48*a^4*log(1/2*cos(d*x
 + c) + 1/2)*sin(d*x + c) + 48*a^4*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 51*a^4*cos(d*x + c) - (32*a^4*cos(d*x + c)^3 - 51*a^4*d*x - 96*a^4*cos(
d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^4\left(\int 4\sin(c+dx)\cot^2(c+dx)\,dx+\int 6\sin^2(c+dx)\cot^2(c+dx)\,dx+\int 4\sin^3(c+dx)\cot^2(c+dx)\,dx+\int \sin^4(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

```
[Out] a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(6*sin(c + d*x)
**2*cot(c + d*x)**2, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**2, x) +
Integral(sin(c + d*x)**4*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x)
)
```

Giac [A] time = 1.39615, size = 262, normalized size = 2.26

$$\frac{51(dx+c)a^4+96a^4\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+12a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\frac{12\left(8a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^4\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-\frac{2\left(69a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+93a^4\tan\right.}{}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(51*(d*x + c)*a^4 + 96*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^4*tan
(1/2*d*x + 1/2*c) - 12*(8*a^4*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1/2
*c) - 2*(69*a^4*tan(1/2*d*x + 1/2*c)^7 + 93*a^4*tan(1/2*d*x + 1/2*c)^5 - 19
2*a^4*tan(1/2*d*x + 1/2*c)^4 - 93*a^4*tan(1/2*d*x + 1/2*c)^3 - 256*a^4*tan(
1/2*d*x + 1/2*c)^2 - 69*a^4*tan(1/2*d*x + 1/2*c) - 64*a^4)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^4)/d
```

3.42 $\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=140

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{2a}{}$$

```
[Out] (-61*a^4*x)/8 + (2*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x]^3)/(3
*d) - (5*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]^3)/(3*d) - (2*a^4*Cot[c +
d*x]*Csc[c + d*x])/d - (19*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.224943, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3770, 3767, 8, 3768, 2638, 2635, 2633}

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-61*a^4*x)/8 + (2*a^4*ArcTanh[Cos[c + d*x]])/d + (4*a^4*Cos[c + d*x]^3)/(3
*d) - (5*a^4*Cot[c + d*x])/d - (a^4*Cot[c + d*x]^3)/(3*d) - (2*a^4*Cot[c +
d*x]*Csc[c + d*x])/d - (19*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(-10a^8 - 4a^8\csc(c+dx) + 4a^8\csc^2(c+dx) + 4a^8\csc^3(c+dx) + a^8\csc^4(c+\right.}{}$$

$$= -10a^4x + a^4\int \csc^4(c+dx)\,dx + a^4\int \sin^4(c+dx)\,dx - \left(4a^4\right)\int \csc(c+dx)\,d$$

$$= -10a^4x + \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos(c+dx)}{d} - \frac{2a^4\cot(c+dx)\csc(c}{d}$$

$$= -8a^4x + \frac{2a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos^3(c+dx)}{3d} - \frac{5a^4\cot(c+dx)}{d} - \frac{a^4 c}{}$$

$$= -\frac{61a^4x}{8} + \frac{2a^4\tanh^{-1}(\cos(c+dx))}{d} + \frac{4a^4\cos^3(c+dx)}{3d} - \frac{5a^4\cot(c+dx)}{d} - \frac{a^4}{}$$

Mathematica [A] time = 5.35963, size = 209, normalized size = 1.49

$$\frac{a^4(\sin(c+dx)+1)^4\left(-732(c+dx) - 120\sin(2(c+dx)) + 3\sin(4(c+dx)) + 96\cos(c+dx) + 32\cos(3(c+dx)) + 224\,\mathrm{t}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(-732*(c + d*x) + 96*Cos[c + d*x] + 32*Cos[3*(c +
 d*x)] - 224*Cot[(c + d*x)/2] - 48*Csc[(c + d*x)/2]^2 + 192*Log[Cos[(c + d*
x)/2]] - 192*Log[Sin[(c + d*x)/2]] + 48*Sec[(c + d*x)/2]^2 + 32*Csc[c + d*x
]^3*Sin[(c + d*x)/2]^4 - 2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 120*Sin[2*(c +
 d*x)] + 3*Sin[4*(c + d*x)] + 224*Tan[(c + d*x)/2]))/(96*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^8)
```

Maple [A] time = 0.05, size = 190, normalized size = 1.4

$$-\frac{23\,a^4(\cos(dx+c))^3\sin(dx+c)}{4\,d} - \frac{69\,a^4\cos(dx+c)\sin(dx+c)}{8\,d} - \frac{61\,a^4x}{8} - \frac{61\,a^4c}{8\,d} - \frac{2\,a^4(\cos(dx+c))^3}{3\,d} - 2\,\frac{a^4\cos(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -23/4/d*a^4*cos(d*x+c)^3*sin(d*x+c)-69/8*a^4*cos(d*x+c)*sin(d*x+c)/d-61/8*a
^4*x-61/8/d*a^4*c-2/3*a^4*cos(d*x+c)^3/d-2*a^4*cos(d*x+c)/d-2/d*a^4*ln(csc(
d*x+c)-cot(d*x+c))-6/d*a^4/sin(d*x+c)*cos(d*x+c)^5-2/d*a^4/sin(d*x+c)^2*cos
(d*x+c)^5-1/3*a^4*cot(d*x+c)^3/d+a^4*cot(d*x+c)/d
```

MaximA [A] time = 1.52732, size = 294, normalized size = 2.1

$$\frac{64\left(2\cos(dx+c)^3+6\cos(dx+c)-3\log(\cos(dx+c)+1)+3\log(\cos(dx+c)-1)\right)a^4+3(12dx+12c+\sin(4dx+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/96*(64*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*l
og(cos(d*x + c) - 1))*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d
*x + 2*c))*a^4 - 288*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3
+ tan(d*x + c)))*a^4 + 32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c
)^3)*a^4 + 96*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3
*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.97689, size = 560, normalized size = 4.

$$-\frac{6a^4\cos(dx+c)^7-75a^4\cos(dx+c)^5+244a^4\cos(dx+c)^3-183a^4\cos(dx+c)-24\left(a^4\cos(dx+c)^2-a^4\right)\log\left(\frac{1}{2}\cos\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/24*(6*a^4*cos(d*x + c)^7 - 75*a^4*cos(d*x + c)^5 + 244*a^4*cos(d*x + c)^
3 - 183*a^4*cos(d*x + c) - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*cos(d*x +
c) + 1/2)*sin(d*x + c) + 24*(a^4*cos(d*x + c)^2 - a^4)*log(-1/2*cos(d*x + c
) + 1/2)*sin(d*x + c) - (32*a^4*cos(d*x + c)^5 - 183*a^4*d*x*cos(d*x + c)^2
 - 32*a^4*cos(d*x + c)^3 + 183*a^4*d*x + 48*a^4*cos(d*x + c))*sin(d*x + c))
/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.48155, size = 370, normalized size = 2.64

$$a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 12\,a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 183\,(dx + c)a^4 - 48\,a^4 \log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) + 57\,a^4 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(a^4*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*tan(1/2*d*x + 1/2*c)^2 - 183*(d*x
 + c)*a^4 - 48*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 57*a^4*tan(1/2*d*x + 1/
2*c) + (88*a^4*tan(1/2*d*x + 1/2*c)^3 - 57*a^4*tan(1/2*d*x + 1/2*c)^2 - 12*
a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(57*a^4*tan(1/2*
d*x + 1/2*c)^7 + 96*a^4*tan(1/2*d*x + 1/2*c)^6 + 81*a^4*tan(1/2*d*x + 1/2*c
)^5 + 96*a^4*tan(1/2*d*x + 1/2*c)^4 - 81*a^4*tan(1/2*d*x + 1/2*c)^3 + 32*a^
4*tan(1/2*d*x + 1/2*c)^2 - 57*a^4*tan(1/2*d*x + 1/2*c) + 32*a^4)/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

3.43 $\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx$

Optimal. Leaf size=198

$$-\frac{4a^4\cos^3(c+dx)}{3d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^5(c+dx)}{5d}-\frac{5a^4\cot^3(c+dx)}{3d}+\frac{10a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}$$

```
[Out] (97*a^4*x)/8 + (5*a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (4*a^4*Cos[c + d*x])/d
 - (4*a^4*Cos[c + d*x]^3)/(3*d) + (10*a^4*Cot[c + d*x])/d - (5*a^4*Cot[c +
d*x]^3)/(3*d) - (a^4*Cot[c + d*x]^5)/(5*d) + (5*a^4*Cot[c + d*x]*Csc[c + d*
x])/(2*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/d + (15*a^4*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.427749, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3767, 8, 3768, 3770, 2638, 2635, 2633}

$$-\frac{4a^4\cos^3(c+dx)}{3d}-\frac{4a^4\cos(c+dx)}{d}-\frac{a^4\cot^5(c+dx)}{5d}-\frac{5a^4\cot^3(c+dx)}{3d}+\frac{10a^4\cot(c+dx)}{d}+\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (97*a^4*x)/8 + (5*a^4*ArcTanh[Cos[c + d*x]])/(2*d) - (4*a^4*Cos[c + d*x])/d
 - (4*a^4*Cos[c + d*x]^3)/(3*d) + (10*a^4*Cot[c + d*x])/d - (5*a^4*Cot[c +
d*x]^3)/(3*d) - (a^4*Cot[c + d*x]^5)/(5*d) + (5*a^4*Cot[c + d*x]*Csc[c + d*
x])/(2*d) - (a^4*Cot[c + d*x]*Csc[c + d*x]^3)/d + (15*a^4*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d)
```


Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{\int \left(14a^{10} - 14a^{10}\csc^2(c+dx) - 8a^{10}\csc^3(c+dx) + 3a^{10}\csc^4(c+dx) + 4a^{10}\csc\right.}{}$$

$$= 14a^4x + a^4\int \csc^6(c+dx)\,dx - a^4\int \sin^4(c+dx)\,dx + \left(3a^4\right)\int \csc^4(c+dx)\,dx$$

$$= 14a^4x - \frac{8a^4\cos(c+dx)}{d} + \frac{4a^4\cot(c+dx)\csc(c+dx)}{d} - \frac{a^4\cot(c+dx)\csc^3(c+}{d}$$

$$= \frac{25a^4x}{2} + \frac{4a^4\tanh^{-1}(\cos(c+dx))}{d} - \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+dx)}{3d} + \frac{10a^4}{}$$

$$= \frac{97a^4x}{8} + \frac{5a^4\tanh^{-1}(\cos(c+dx))}{2d} - \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+dx)}{3d} + \frac{10a^4}{}$$

Mathematica [A] time = 1.5202, size = 283, normalized size = 1.43

$$\frac{a^4(\sin(c+dx)+1)^4\left(5820(c+dx) + 480\sin(2(c+dx)) - 15\sin(4(c+dx)) - 2400\cos(c+dx) - 160\cos(3(c+dx)) - 275\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*(1 + Sin[c + d*x])^4*(5820*(c + d*x) - 2400*Cos[c + d*x] - 160*Cos[3*(
c + d*x)] + 2752*Cot[(c + d*x)/2] + 300*Csc[(c + d*x)/2]^2 - 30*Csc[(c + d*
x)/2]^4 + 1200*Log[Cos[(c + d*x)/2]] - 1200*Log[Sin[(c + d*x)/2]] - 300*Sec
[(c + d*x)/2]^2 + 30*Sec[(c + d*x)/2]^4 + 632*Csc[c + d*x]^3*Sin[(c + d*x)/
2]^4 + 96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - (79*Csc[(c + d*x)/2]^4*Sin[c
+ d*x])/2 - (3*Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 + 480*Sin[2*(c + d*x)] -
15*Sin[4*(c + d*x)] - 2752*Tan[(c + d*x)/2]))/(480*d*(Cos[(c + d*x)/2] + Si
n[(c + d*x)/2])^8)
```

Maple [A] time = 0.054, size = 293, normalized size = 1.5

$$-\frac{5a^4(\cos(dx+c))^3}{6d} + 7\frac{a^4\sin(dx+c)(\cos(dx+c))^5}{d} - \frac{5a^4\cos(dx+c)}{2d} - \frac{a^4(\cos(dx+c))^5}{2d} - \frac{a^4(\cos(dx+c))^7}{d(\sin(dx+c))^4} - 2\frac{a^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x)
```

```
[Out] -5/6*a^4*cos(d*x+c)^3/d+7/d*a^4*sin(d*x+c)*cos(d*x+c)^5-5/2*a^4*cos(d*x+c)/
d-1/2/d*a^4*cos(d*x+c)^5-1/d*a^4/sin(d*x+c)^4*cos(d*x+c)^7-2/d*a^4/sin(d*x+
c)^3*cos(d*x+c)^7-1/2/d*a^4/sin(d*x+c)^2*cos(d*x+c)^7+97/8*a^4*x+7/d*a^4/si
n(d*x+c)*cos(d*x+c)^7+35/4/d*a^4*cos(d*x+c)^3*sin(d*x+c)+105/8*a^4*cos(d*x+
c)*sin(d*x+c)/d-1/5*a^4*cot(d*x+c)^5/d+1/3*a^4*cot(d*x+c)^3/d-a^4*cot(d*x+c
)/d+97/8/d*a^4*c-5/2/d*a^4*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.69153, size = 423, normalized size = 2.14

$$-\frac{40\left(4\cos(dx+c)^3-\frac{6\cos(dx+c)}{\cos(dx+c)^2-1}+24\cos(dx+c)-15\log(\cos(dx+c)+1)+15\log(\cos(dx+c)-1)\right)a^4+15\left(15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/120*(40*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos
(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 + 15*(
15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c)^5
 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^4 - 120*(15*d*x + 15*c + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^4 + 8
*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x + c)^5
)*a^4 + 30*a^4*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*c
os(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*log(co
s(d*x + c) - 1)))/d
```

Fricas [A] time = 2.08524, size = 770, normalized size = 3.89

$$\frac{30a^4\cos(dx+c)^9-345a^4\cos(dx+c)^7+2231a^4\cos(dx+c)^5-3395a^4\cos(dx+c)^3+1455a^4\cos(dx+c)+150\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/120*(30*a^4*cos(d*x + c)^9 - 345*a^4*cos(d*x + c)^7 + 2231*a^4*cos(d*x +
c)^5 - 3395*a^4*cos(d*x + c)^3 + 1455*a^4*cos(d*x + c) + 150*(a^4*cos(d*x +
 c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c
```

```
) - 150*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) - 5*(32*a^4*cos(d*x + c)^7 - 291*a^4*d*x*cos(d*x +
 c)^4 + 32*a^4*cos(d*x + c)^5 + 582*a^4*d*x*cos(d*x + c)^2 - 100*a^4*cos(d*
x + c)^3 - 291*a^4*d*x + 60*a^4*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.70284, size = 458, normalized size = 2.31

$$\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+85\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5820\,(dx+c)a^4-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^4*tan(1/2*d*x + 1/2*c)^5 + 30*a^4*tan(1/2*d*x + 1/2*c)^4 + 85*a^
4*tan(1/2*d*x + 1/2*c)^3 - 240*a^4*tan(1/2*d*x + 1/2*c)^2 + 5820*(d*x + c)*
a^4 - 1200*a^4*log(abs(tan(1/2*d*x + 1/2*c))) - 2670*a^4*tan(1/2*d*x + 1/2*
c) - 40*(45*a^4*tan(1/2*d*x + 1/2*c)^7 + 192*a^4*tan(1/2*d*x + 1/2*c)^6 + 6
9*a^4*tan(1/2*d*x + 1/2*c)^5 + 384*a^4*tan(1/2*d*x + 1/2*c)^4 - 69*a^4*tan(
1/2*d*x + 1/2*c)^3 + 320*a^4*tan(1/2*d*x + 1/2*c)^2 - 45*a^4*tan(1/2*d*x +
1/2*c) + 128*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4 + (2740*a^4*tan(1/2*d*x +
1/2*c)^5 + 2670*a^4*tan(1/2*d*x + 1/2*c)^4 + 240*a^4*tan(1/2*d*x + 1/2*c)^3
 - 85*a^4*tan(1/2*d*x + 1/2*c)^2 - 30*a^4*tan(1/2*d*x + 1/2*c) - 3*a^4)/tan
(1/2*d*x + 1/2*c)^5)/d
```

3.44 $\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{\tan^8(c+dx)}{8ad}-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^7(c+dx)\sec(c+dx)}{8ad}+\frac{7\tan^5(c+dx)\sec(c+dx)}{48ad}-\frac{35\tan^3(c+dx)\sec(}{192ad}$$

```
[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128
*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c
+ d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8
/(8*a*d)
```

Rubi [A] time = 0.161036, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^8(c+dx)}{8ad}-\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}-\frac{\tan^7(c+dx)\sec(c+dx)}{8ad}+\frac{7\tan^5(c+dx)\sec(c+dx)}{48ad}-\frac{35\tan^3(c+dx)\sec(}{192ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```


```
[Out] (-35*ArcTanh[Sin[c + d*x]])/(128*a*d) + (35*Sec[c + d*x]*Tan[c + d*x])/(128
*a*d) - (35*Sec[c + d*x]*Tan[c + d*x]^3)/(192*a*d) + (7*Sec[c + d*x]*Tan[c
+ d*x]^5)/(48*a*d) - (Sec[c + d*x]*Tan[c + d*x]^7)/(8*a*d) + Tan[c + d*x]^8
/(8*a*d)
```


Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^7(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^8(c+dx)\,dx}{a}\\
&= -\frac{\sec(c+dx)\tan^7(c+dx)}{8ad}+\frac{7\int \sec(c+dx)\tan^6(c+dx)\,dx}{8a}+\frac{\operatorname{Subst}\left(\int x^7\,dx,x,\tan(c+dx)\right)}{ad}\\
&= \frac{7\sec(c+dx)\tan^5(c+dx)}{48ad}-\frac{\sec(c+dx)\tan^7(c+dx)}{8ad}+\frac{\tan^8(c+dx)}{8ad}-\frac{35\int \sec(c+dx)\tan^4}{48a}\\
&= -\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad}+\frac{7\sec(c+dx)\tan^5(c+dx)}{48ad}-\frac{\sec(c+dx)\tan^7(c+dx)}{8ad}+\frac{\tan^8(}{8}\\
&= \frac{35\sec(c+dx)\tan(c+dx)}{128ad}-\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad}+\frac{7\sec(c+dx)\tan^5(c+dx)}{48ad}-\sec(c\\
&= -\frac{35\tanh^{-1}(\sin(c+dx))}{128ad}+\frac{35\sec(c+dx)\tan(c+dx)}{128ad}-\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad}+7\sec(c
\end{aligned}$$

Mathematica [A] time = 1.00892, size = 101, normalized size = 0.78

$$-\frac{\frac{279\sin^6(c+dx)+87\sin^5(c+dx)-424\sin^4(c+dx)-136\sin^3(c+dx)+249\sin^2(c+dx)+57\sin(c+dx)-48}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^4}+105\tanh^{-1}(\sin(c+dx))}{384ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(105*ArcTanh[Sin[c + d*x]] + (-48 + 57*Sin[c + d*x] + 249*Sin[c + d*x]^2 -
 136*Sin[c + d*x]^3 - 424*Sin[c + d*x]^4 + 87*Sin[c + d*x]^5 + 279*Sin[c +
d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(384*a*d)
```

Maple [A] time = 0.059, size = 162, normalized size = 1.3

$$-\frac{1}{96\,da\,(\sin(dx+c)-1)^{3}}-\frac{9}{128\,da\,(\sin(dx+c)-1)^{2}}-\frac{29}{128\,da\,(\sin(dx+c)-1)}+\frac{35\ln(\sin(dx+c)-1)}{256\,da}+\frac{}{64\,da\,(1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/96/d/a/(sin(d*x+c)-1)^3-9/128/d/a/(sin(d*x+c)-1)^2-29/128/a/d/(sin(d*x+c
)-1)+35/256/a/d*ln(sin(d*x+c)-1)+1/64/d/a/(1+sin(d*x+c))^4-5/48/d/a/(1+sin(
d*x+c))^3+19/64/a/d/(1+sin(d*x+c))^2-1/2/a/d/(1+sin(d*x+c))-35/256*ln(1+sin
(d*x+c))/a/d
```

Maxima [A] time = 1.02832, size = 236, normalized size = 1.82

$$-\frac{\frac{2\left(279\sin(dx+c)^{6}+87\sin(dx+c)^{5}-424\sin(dx+c)^{4}-136\sin(dx+c)^{3}+249\sin(dx+c)^{2}+57\sin(dx+c)-48\right)}{a\sin(dx+c)^{7}+a\sin(dx+c)^{6}-3\,a\sin(dx+c)^{5}-3\,a\sin(dx+c)^{4}+3\,a\sin(dx+c)^{3}+3\,a\sin(dx+c)^{2}-a\sin(dx+c)-a}+\frac{105\log(\sin(dx+c)+1)}{a}-\frac{105\log(\sin(dx+c)-}{a}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/768*(2*(279*sin(d*x + c)^6 + 87*sin(d*x + c)^5 - 424*sin(d*x + c)^4 - 13
6*sin(d*x + c)^3 + 249*sin(d*x + c)^2 + 57*sin(d*x + c) - 48)/(a*sin(d*x +
c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*sin
(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 105*log(sin(d*x +
c) + 1)/a - 105*log(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 2.05396, size = 464, normalized size = 3.57

$$-\frac{558\cos(dx+c)^{6}-826\cos(dx+c)^{4}+476\cos(dx+c)^{2}+105\left(\cos(dx+c)^{6}\sin(dx+c)+\cos(dx+c)^{6}\right)\log(\sin(d}{768\left(ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/768*(558*cos(d*x + c)^6 - 826*cos(d*x + c)^4 + 476*cos(d*x + c)^2 + 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 105*
(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(
87*cos(d*x + c)^4 - 38*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos(d*x
 + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 8.06961, size = 184, normalized size = 1.42

$$-\frac{\frac{420\log(|\sin(dx+c)+1|)}{a}-\frac{420\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(385\sin(dx+c)^3-807\sin(dx+c)^2+567\sin(dx+c)-129\right)}{a(\sin(dx+c)-1)^3}-\frac{875\sin(dx+c)^4+1964\sin(dx+c)^3+1554\,\mathrm{s}}{a(\sin(dx+c)+1}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))/
a + 2*(385*sin(d*x + c)^3 - 807*sin(d*x + c)^2 + 567*sin(d*x + c) - 129)/(a
*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 1964*sin(d*x + c)^3 + 1554*s
in(d*x + c)^2 + 396*sin(d*x + c) - 21)/(a*(sin(d*x + c) + 1)^4))/d
```

3.45 $\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)\sec(c+dx)}{6ad}+\frac{5\tan^3(c+dx)\sec(c+dx)}{24ad}-\frac{5\tan(c+dx)\sec(c+dx)}{16ad}$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(16*a*d)
 + (5*Sec[c + d*x]*Tan[c + d*x]^3)/(24*a*d) - (Sec[c + d*x]*Tan[c + d*x]^5)
/(6*a*d) + Tan[c + d*x]^6/(6*a*d)
```

Rubi [A] time = 0.13628, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^6(c+dx)}{6ad}+\frac{5\tanh^{-1}(\sin(c+dx))}{16ad}-\frac{\tan^5(c+dx)\sec(c+dx)}{6ad}+\frac{5\tan^3(c+dx)\sec(c+dx)}{24ad}-\frac{5\tan(c+dx)\sec(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(16*a*d) - (5*Sec[c + d*x]*Tan[c + d*x])/(16*a*d)
 + (5*Sec[c + d*x]*Tan[c + d*x]^3)/(24*a*d) - (Sec[c + d*x]*Tan[c + d*x]^5)
/(6*a*d) + Tan[c + d*x]^6/(6*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^5(c+dx)\,dx}{a} - \frac{\int \sec(c+dx)\tan^6(c+dx)\,dx}{a}\\
&= -\frac{\sec(c+dx)\tan^5(c+dx)}{6ad} + \frac{5\int \sec(c+dx)\tan^4(c+dx)\,dx}{6a} + \frac{\operatorname{Subst}\left(\int x^5\,dx, x, \tan(c+dx)\right)}{ad}\\
&= \frac{5\sec(c+dx)\tan^3(c+dx)}{24ad} - \frac{\sec(c+dx)\tan^5(c+dx)}{6ad} + \frac{\tan^6(c+dx)}{6ad} - \frac{5\int \sec(c+dx)\tan^2(\ldots}{8a}\\
&= -\frac{5\sec(c+dx)\tan(c+dx)}{16ad} + \frac{5\sec(c+dx)\tan^3(c+dx)}{24ad} - \frac{\sec(c+dx)\tan^5(c+dx)}{6ad} + \frac{\tan^6(c\ldots}{6a\ldots}\\
&= \frac{5\tanh^{-1}(\sin(c+dx))}{16ad} - \frac{5\sec(c+dx)\tan(c+dx)}{16ad} + \frac{5\sec(c+dx)\tan^3(c+dx)}{24ad} - \frac{\sec(c+dx)\ldots}{6\ldots}\end{aligned}$$

Mathematica [A] time = 0.308239, size = 84, normalized size = 0.79

$$\frac{-\frac{18}{1-\sin(c+dx)}+\frac{48}{\sin(c+dx)+1}+\frac{3}{(1-\sin(c+dx))^2}-\frac{21}{(\sin(c+dx)+1)^2}+\frac{4}{(\sin(c+dx)+1)^3}+30\tanh^{-1}(\sin(c+dx))}{96ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] (30*ArcTanh[Sin[c + d*x]] + 3/(1 - Sin[c + d*x])^2 - 18/(1 - Sin[c + d*x])
+ 4/(1 + Sin[c + d*x])^3 - 21/(1 + Sin[c + d*x])^2 + 48/(1 + Sin[c + d*x]))
```

```
/(96*a*d)
```

Maple [A] time = 0.056, size = 126, normalized size = 1.2

$$\frac{1}{32\,da\left(\sin\left(dx+c\right)-1\right)^{2}}+\frac{3}{16\,da\left(\sin\left(dx+c\right)-1\right)}-\frac{5\,\ln\left(\sin\left(dx+c\right)-1\right)}{32\,da}+\frac{1}{24\,da\left(1+\sin\left(dx+c\right)\right)^{3}}-\frac{7}{32\,da\left(1+\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/32/d/a/(sin(d*x+c)-1)^2+3/16/a/d/(sin(d*x+c)-1)-5/32/a/d*ln(sin(d*x+c)-1)
+1/24/d/a/(1+sin(d*x+c))^3-7/32/a/d/(1+sin(d*x+c))^2+1/2/a/d/(1+sin(d*x+c))
+5/32*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 0.999884, size = 176, normalized size = 1.66

$$\frac{\frac{2\left(33\,\sin(dx+c)^{4}+9\,\sin(dx+c)^{3}-31\,\sin(dx+c)^{2}-7\,\sin(dx+c)+8\right)}{a\sin(dx+c)^{5}+a\sin(dx+c)^{4}-2\,a\sin(dx+c)^{3}-2\,a\sin(dx+c)^{2}+a\sin(dx+c)+a}+\frac{15\,\log(\sin(dx+c)+1)}{a}-\frac{15\,\log(\sin(dx+c)-1)}{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(33*sin(d*x + c)^4 + 9*sin(d*x + c)^3 - 31*sin(d*x + c)^2 - 7*sin(d
*x + c) + 8)/(a*sin(d*x + c)^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 - 2*
a*sin(d*x + c)^2 + a*sin(d*x + c) + a) + 15*log(sin(d*x + c) + 1)/a - 15*lo
g(sin(d*x + c) - 1)/a)/d
```

Fricas [A] time = 1.8359, size = 398, normalized size = 3.75

$$\frac{66\,\cos\left(dx+c\right)^{4}-70\,\cos\left(dx+c\right)^{2}+15\left(\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{4}\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\cos\left(dx\right.\right.}{96\left(ad\cos\left(dx+c\right)^{4}\sin\left(dx+c\right)+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(66*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 15*(cos(d*x + c)^4*sin(d*x +
c) + cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4*sin(d*x + c
) + cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*x + c)^2 - 2)*sin(d
*x + c) + 20)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^{5}(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 3.93359, size = 157, normalized size = 1.48

$$\frac{\frac{30\log(|\sin(dx+c)+1|)}{a} - \frac{30\log(|\sin(dx+c)-1|)}{a} + \frac{3\left(15\sin(dx+c)^{2}-18\sin(dx+c)+5\right)}{a(\sin(dx+c)-1)^{2}} - \frac{55\sin(dx+c)^{3}+69\sin(dx+c)^{2}+15\sin(dx+c)-7}{a(\sin(dx+c)+1)^{3}}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/192*(30*log(abs(sin(d*x + c) + 1))/a - 30*log(abs(sin(d*x + c) - 1))/a +
3*(15*sin(d*x + c)^2 - 18*sin(d*x + c) + 5)/(a*(sin(d*x + c) - 1)^2) - (55*
sin(d*x + c)^3 + 69*sin(d*x + c)^2 + 15*sin(d*x + c) - 7)/(a*(sin(d*x + c)
+ 1)^3))/d
```

3.46 $\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{\tan^4(c+dx)}{4ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)\sec(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(8*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(8*a*d)
- (Sec[c + d*x]*Tan[c + d*x]^3)/(4*a*d) + Tan[c + d*x]^4/(4*a*d)
```

Rubi [A] time = 0.115661, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$\frac{\tan^4(c+dx)}{4ad}-\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{\tan^3(c+dx)\sec(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(8*a*d) + (3*Sec[c + d*x]*Tan[c + d*x])/(8*a*d)
- (Sec[c + d*x]*Tan[c + d*x]^3)/(4*a*d) + Tan[c + d*x]^4/(4*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^3(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^4(c+dx)\,dx}{a}\\ &= -\frac{\sec(c+dx)\tan^3(c+dx)}{4ad}+\frac{3\int \sec(c+dx)\tan^2(c+dx)\,dx}{4a}+\frac{\operatorname{Subst}\left(\int x^3\,dx,x,\tan(c+dx)\right)}{ad}\\ &= \frac{3\sec(c+dx)\tan(c+dx)}{8ad}-\frac{\sec(c+dx)\tan^3(c+dx)}{4ad}+\frac{\tan^4(c+dx)}{4ad}-\frac{3\int \sec(c+dx)\,dx}{8a}\\ &= -\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}-\frac{\sec(c+dx)\tan^3(c+dx)}{4ad}+\frac{\tan^4(c+dx)}{4ad}\end{aligned}$$

Mathematica [A] time = 0.164595, size = 54, normalized size = 0.66

$$-\frac{\frac{1}{\sin(c+dx)-1}+\frac{4}{\sin(c+dx)+1}-\frac{1}{(\sin(c+dx)+1)^2}+3\tanh^{-1}(\sin(c+dx))}{8ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(3*ArcTanh[Sin[c + d*x]] + (-1 + Sin[c + d*x])^(-1) - (1 + Sin[c + d*x])^(
-2) + 4/(1 + Sin[c + d*x]))/(8*a*d)
```

Maple [A] time = 0.053, size = 90, normalized size = 1.1

$$-\frac{1}{8da\left(\sin\left(dx+c\right)-1\right)}+\frac{3\ln\left(\sin\left(dx+c\right)-1\right)}{16da}+\frac{1}{8da\left(1+\sin\left(dx+c\right)\right)^{2}}-\frac{1}{2da\left(1+\sin\left(dx+c\right)\right)}-\frac{3\ln\left(1+\sin\left(dx+c\right)\right)}{16da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/8/a/d/(sin(d*x+c)-1)+3/16/a/d*ln(sin(d*x+c)-1)+1/8/a/d/(1+sin(d*x+c))^2-
1/2/a/d/(1+sin(d*x+c))-3/16*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 0.999972, size = 120, normalized size = 1.46

$$-\frac{\frac{2\left(5\sin(dx+c)^{2}+\sin(dx+c)-2\right)}{a\sin(dx+c)^{3}+a\sin(dx+c)^{2}-a\sin(dx+c)-a}+\frac{3\log(\sin(dx+c)+1)}{a}-\frac{3\log(\sin(dx+c)-1)}{a}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*(2*(5*sin(d*x + c)^2 + sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(d*
x + c)^2 - a*sin(d*x + c) - a) + 3*log(sin(d*x + c) + 1)/a - 3*log(sin(d*x
+ c) - 1)/a)/d
```

Fricas [A] time = 1.34329, size = 338, normalized size = 4.12

$$-\frac{10\cos\left(dx+c\right)^{2}+3\left(\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\sin\left(dx+c\right)-6}{16\left(ad\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/16*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)
*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)*l
og(-sin(d*x + c) + 1) - 2*sin(d*x + c) - 6)/(a*d*cos(d*x + c)^2*sin(d*x + c
```

```
) + a*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 2.11468, size = 130, normalized size = 1.59

$$-\frac{\frac{6\log(|\sin(dx+c)+1|)}{a}-\frac{6\log(|\sin(dx+c)-1|)}{a}+\frac{2(3\sin(dx+c)-1)}{a(\sin(dx+c)-1)}-\frac{9\sin(dx+c)^2+2\sin(dx+c)-3}{a(\sin(dx+c)+1)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2*
(3*sin(d*x + c) - 1)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 2*sin(d*x
 + c) - 3)/(a*(sin(d*x + c) + 1)^2))/d
```

3.47 $\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{1}{2d(a\sin(c+dx)+a)}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) + 1/(2*d*(a + a*Sin[c + d*x]))
```

Rubi [A] time = 0.067309, antiderivative size = 58, normalized size of antiderivative = 1.57, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2706, 2606, 30, 2611, 3770}

$$\frac{\sec^2(c+dx)}{2ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) + Sec[c + d*x]^2/(2*a*d) - (Sec[c + d*x]*Tan[
c + d*x])/(2*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= -\frac{\sec(c+dx)\tan(c+dx)}{2ad}+\frac{\int \sec(c+dx)\,dx}{2a}+\frac{\operatorname{Subst}(\int x\,dx, x, \sec(c+dx))}{ad}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{\sec^2(c+dx)}{2ad}-\frac{\sec(c+dx)\tan(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.038978, size = 28, normalized size = 0.76

$$\frac{\frac{1}{\sin(c+dx)+1}+\tanh^{-1}(\sin(c+dx))}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

Maple [A] time = 0.058, size = 54, normalized size = 1.5

$$-\frac{\ln(\sin(dx+c)-1)}{4\,da}+\frac{1}{2\,da\,(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{4\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/4/a/d*ln(sin(d*x+c)-1)+1/2/a/d/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.08759, size = 63, normalized size = 1.7

$$\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} + \frac{2}{a\sin(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a + 2/(a*sin(d*x + c)
+ a))/d
```

Fricas [A] time = 1.63615, size = 163, normalized size = 4.41

$$\frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1) - (\sin(dx+c)+1)\log(-\sin(dx+c)+1) + 2}{4\,(ad\sin(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-sin
(d*x + c) + 1) + 2)/(a*d*sin(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.34095, size = 78, normalized size = 2.11

$$\frac{\frac{\log\left(|\sin(dx+c)+1|\right)}{a} - \frac{\log\left(|\sin(dx+c)-1|\right)}{a} - \frac{\sin(dx+c)-1}{a(\sin(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c) - 1))/a - (sin(d*x
 + c) - 1)/(a*(sin(d*x + c) + 1)))/d
```

3.48 $\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0390786, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2707, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, a\sin(c+dx)\right)}{ad} - \frac{\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, a\sin(c+dx)\right)}{ad}\\ &= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(1+\sin(c+dx))}{ad}\end{aligned}$$

Mathematica [A] time = 0.0193949, size = 32, normalized size = 1.

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(\sin(c+dx)+1)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

Maple [A] time = 0.023, size = 33, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{da} - \frac{\ln(1+\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

Maxima [A] time = 1.04724, size = 42, normalized size = 1.31

$$-\frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a)/d
```

Fricas [A] time = 1.40674, size = 74, normalized size = 2.31

$$\frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right) - \log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (log(1/2*sin(d*x + c)) - log(sin(d*x + c) + 1))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.2827, size = 45, normalized size = 1.41

$$-\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a)/d
```

3.49 $\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.0677649, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2706, 2606, 30, 8}

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}(\int 1\,dx, x, \csc(c+dx))}{ad}-\frac{\text{Subst}(\int x\,dx, x, \csc(c+dx))}{ad}\\ &= \frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.032614, size = 24, normalized size = 0.75

$$-\frac{(\csc(c+dx)-2)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((-2 + Csc[c + d*x])*Csc[c + d*x])/(2*a*d)
```

Maple [A] time = 0.032, size = 30, normalized size = 0.9

$$-\frac{1}{da}\left(-(\sin(dx+c))^{-1}+\frac{1}{2\,(\sin(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c)),x)
```

```
[Out] -1/d/a*(-1/sin(d*x+c)+1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.11797, size = 35, normalized size = 1.09

$$\frac{2\,\sin(dx+c)-1}{2\,ad\sin(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

Fricas [A] time = 1.37048, size = 73, normalized size = 2.28

$$-\frac{2\,\sin\left(dx+c\right)-1}{2\left(ad\cos\left(dx+c\right)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cot^3\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.32292, size = 35, normalized size = 1.09

$$\frac{2\,\sin\left(dx+c\right)-1}{2\,ad\sin\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

3.50 $\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -Cot[c + d*x]^4/(4*a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0889617, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2706, 2607, 30, 2606}

$$-\frac{\cot^4(c+dx)}{4ad}+\frac{\csc^3(c+dx)}{3ad}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^4/(4*a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(3*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^3(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^3(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^3\,dx,x,-\cot(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int\left(-1+x^2\right)dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot^4(c+dx)}{4ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.0486413, size = 30, normalized size = 0.59

$$-\frac{(\csc(c+dx)-1)^3(3\csc(c+dx)+5)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

```
[Out] -((-1 + Csc[c + d*x])^3*(5 + 3*Csc[c + d*x]))/(12*a*d)
```

Maple [A] time = 0.085, size = 49, normalized size = 1.

$$\frac{1}{da}\left(-(\sin(dx+c))^{-1}-\frac{1}{4(\sin(dx+c))^4}+\frac{1}{3(\sin(dx+c))^3}+\frac{1}{2(\sin(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/sin(d*x+c)-1/4/sin(d*x+c)^4+1/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.10562, size = 62, normalized size = 1.22

$$-\frac{12\,\sin\left(dx+c\right)^{3}-6\,\sin\left(dx+c\right)^{2}-4\,\sin\left(dx+c\right)+3}{12\,ad\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

Fricas [A] time = 1.40033, size = 162, normalized size = 3.18

$$-\frac{6\,\cos\left(dx+c\right)^{2}-4\left(3\,\cos\left(dx+c\right)^{2}-2\right)\sin\left(dx+c\right)-3}{12\left(ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx+c\right)^{2}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(6*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^2 - 2)*sin(d*x + c) - 3)/(a*d*c
os(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cot^{5}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.32141, size = 62, normalized size = 1.22

$$-\frac{12\,\sin\left(dx+c\right)^{3}-6\,\sin\left(dx+c\right)^{2}-4\,\sin\left(dx+c\right)+3}{12\,ad\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin(
d*x + c)^4)
```

3.51 $\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=68

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

```
[Out] -Cot[c + d*x]^6/(6*a*d) + Csc[c + d*x]/(a*d) - (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.0933341, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc^5(c+dx)}{5ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^6/(6*a*d) + Csc[c + d*x]/(a*d) - (2*Csc[c + d*x]^3)/(3*a*d) +
 Csc[c + d*x]^5/(5*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^5(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^5(c+dx)\csc^2(c+dx)\,dx}{a}\\
&= -\frac{\mathrm{Subst}\left(\int x^5\,dx, x, -\cot(c+dx)\right)}{ad}+\frac{\mathrm{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^6(c+dx)}{6ad}+\frac{\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^6(c+dx)}{6ad}+\frac{\csc(c+dx)}{ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^5(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [A] time = 0.149153, size = 61, normalized size = 0.9

$$
\frac{\csc^6(c+dx)(78\sin(c+dx)-5(7\sin(3(c+dx))-3\sin(5(c+dx))+5)-15\cos(4(c+dx)))}{240ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^6*(-15*Cos[4*(c + d*x)] + 78*Sin[c + d*x] - 5*(5 + 7*Sin[3*(c
 + d*x)] - 3*Sin[5*(c + d*x)])))/(240*a*d)
```

Maple [A] time = 0.095, size = 67, normalized size = 1.

$$\frac{1}{da}\left((\sin (dx+c))^{-1}+\frac{1}{5\,(\sin (dx+c))^5}+\frac{1}{2\,(\sin (dx+c))^4}-\frac{1}{6\,(\sin (dx+c))^6}-\frac{2}{3\,(\sin (dx+c))^3}-\frac{1}{2\,(\sin (dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/sin(d*x+c)+1/5/sin(d*x+c)^5+1/2/sin(d*x+c)^4-1/6/sin(d*x+c)^6-2/3/
sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.12848, size = 89, normalized size = 1.31

$$\frac{30\,\sin (dx+c)^5-15\,\sin (dx+c)^4-20\,\sin (dx+c)^3+15\,\sin (dx+c)^2+6\,\sin (dx+c)-5}{30\,ad\sin (dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*
x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

Fricas [A] time = 1.41345, size = 248, normalized size = 3.65

$$\frac{15\,\cos (dx+c)^4-15\,\cos (dx+c)^2-2\left(15\,\cos (dx+c)^4-20\,\cos (dx+c)^2+8\right)\sin (dx+c)+5}{30\left(ad\cos (dx+c)^6-3\,ad\cos (dx+c)^4+3\,ad\cos (dx+c)^2-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*cos(d*x + c)^4 - 15*cos(d*x + c)^2 - 2*(15*cos(d*x + c)^4 - 20*cos
(d*x + c)^2 + 8)*sin(d*x + c) + 5)/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)
^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4623, size = 89, normalized size = 1.31

$$\frac{30\,\sin\left(dx+c\right)^{5}-15\,\sin\left(dx+c\right)^{4}-20\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)^{2}+6\,\sin\left(dx+c\right)-5}{30\,ad\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d*
x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

3.52 $\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\cot^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}$$

```
[Out] -Cot[c + d*x]^8/(8*a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(a*d) - (3*Cs
c[c + d*x]^5)/(5*a*d) + Csc[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.0997991, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$-\frac{\cot^8(c+dx)}{8ad}+\frac{\csc^7(c+dx)}{7ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^3(c+dx)}{ad}-\frac{\csc(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] -Cot[c + d*x]^8/(8*a*d) - Csc[c + d*x]/(a*d) + Csc[c + d*x]^3/(a*d) - (3*Cs
c[c + d*x]^5)/(5*a*d) + Csc[c + d*x]^7/(7*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^7(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^7(c+dx)\csc^2(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int x^7\,dx, x, -\cot(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int \left(-1+x^2\right)^3\,dx, x, \csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^8(c+dx)}{8ad}+\frac{\text{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)\,dx, x, \csc(c+dx)\right)}{ad}\\
&= -\frac{\cot^8(c+dx)}{8ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^7(c+dx)}{7ad}\end{aligned}$$

Mathematica [A] time = 0.208641, size = 77, normalized size = 0.92

$$\frac{\csc^8(c+dx)(-513\sin(c+dx)+371\sin(3(c+dx))-105\sin(5(c+dx))+35\sin(7(c+dx))-245\cos(2(c+dx))-35\cos}{2240ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]^8*(-245*Cos[2*(c + d*x)] - 35*Cos[6*(c + d*x)] - 513*Sin[c +
d*x] + 371*Sin[3*(c + d*x)] - 105*Sin[5*(c + d*x)] + 35*Sin[7*(c + d*x)]))/
(2240*a*d)
```

Maple [A] time = 0.115, size = 87, normalized size = 1.

$$\frac{1}{da}\left(\frac{1}{7(\sin(dx+c))^7}-(\sin(dx+c))^{-1}-\frac{1}{8(\sin(dx+c))^8}-\frac{3}{5(\sin(dx+c))^5}-\frac{3}{4(\sin(dx+c))^4}+\frac{1}{2(\sin(dx+c))^6}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(1/7/sin(d*x+c)^7-1/sin(d*x+c)-1/8/sin(d*x+c)^8-3/5/sin(d*x+c)^5-3/4/
sin(d*x+c)^4+1/2/sin(d*x+c)^6+1/sin(d*x+c)^3+1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.10832, size = 116, normalized size = 1.38

$$-\frac{280\sin(dx+c)^7-140\sin(dx+c)^6-280\sin(dx+c)^5+210\sin(dx+c)^4+168\sin(dx+c)^3-140\sin(dx+c)^2-}{280\,ad\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210*
sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c)
+ 35)/(a*d*sin(d*x + c)^8)
```

Fricas [A] time = 1.51708, size = 343, normalized size = 4.08

$$-\frac{140\cos(dx+c)^6-210\cos(dx+c)^4+140\cos(dx+c)^2-8\left(35\cos(dx+c)^6-70\cos(dx+c)^4+56\cos(dx+c)^2\right.}{280\left(ad\cos(dx+c)^8-4\,ad\cos(dx+c)^6+6\,ad\cos(dx+c)^4-4\,ad\cos(dx+c)^2+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/280*(140*cos(d*x + c)^6 - 210*cos(d*x + c)^4 + 140*cos(d*x + c)^2 - 8*(3
5*cos(d*x + c)^6 - 70*cos(d*x + c)^4 + 56*cos(d*x + c)^2 - 16)*sin(d*x + c)
 - 35)/(a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^6 + 6*a*d*cos(d*x + c)^4 -
4*a*d*cos(d*x + c)^2 + a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.43815, size = 116, normalized size = 1.38

$$-\frac{280 \sin\left(dx+c\right)^{7}-140 \sin\left(dx+c\right)^{6}-280 \sin\left(dx+c\right)^{5}+210 \sin\left(dx+c\right)^{4}+168 \sin\left(dx+c\right)^{3}-140 \sin\left(dx+c\right)^{2}-4}{280\, ad \sin\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210*
sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c)
+ 35)/(a*d*sin(d*x + c)^8)
```

3.53 $\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{\tan^7(c+dx)}{7ad}-\frac{\sec^7(c+dx)}{7ad}+\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^5)/(5*a*d) - Se
c[c + d*x]^7/(7*a*d) + Tan[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.0973616, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\tan^7(c+dx)}{7ad}-\frac{\sec^7(c+dx)}{7ad}+\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^3(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(a*d) + (3*Sec[c + d*x]^5)/(5*a*d) - Se
c[c + d*x]^7/(7*a*d) + Tan[c + d*x]^7/(7*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^6(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^7(c+dx)\,dx}{a}\\
&= \frac{\operatorname{Subst}\left(\int x^6\,dx,x,\tan(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int \left(-1+x^2\right)^3\,dx,x,\sec(c+dx)\right)}{ad}\\
&= \frac{\tan^7(c+dx)}{7ad}-\frac{\operatorname{Subst}\left(\int \left(-1+3x^2-3x^4+x^6\right)dx,x,\sec(c+dx)\right)}{ad}\\
&= \frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{ad}+\frac{3\sec^5(c+dx)}{5ad}-\frac{\sec^7(c+dx)}{7ad}+\frac{\tan^7(c+dx)}{7ad}
\end{aligned}$$

Mathematica [A] time = 0.319019, size = 146, normalized size = 1.74

$$\frac{\sec^5(c+dx)(2432\sin(c+dx)-1905\sin(2(c+dx))+320\sin(3(c+dx))-1524\sin(4(c+dx))+960\sin(5(c+dx))-38}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^5*(2912 - 7620*Cos[c + d*x] + 3760*Cos[2*(c + d*x)] - 3810*Co
s[3*(c + d*x)] + 1440*Cos[4*(c + d*x)] - 762*Cos[5*(c + d*x)] + 80*Cos[6*(c
 + d*x)] + 2432*Sin[c + d*x] - 1905*Sin[2*(c + d*x)] + 320*Sin[3*(c + d*x)]
 - 1524*Sin[4*(c + d*x)] + 960*Sin[5*(c + d*x)] - 381*Sin[6*(c + d*x)]))/(1
```

```
7920*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.057, size = 175, normalized size = 2.1

$$128\,\frac{1}{da}\left(-\frac{1}{1280\,(\tan(1/2\,dx+c/2)-1)^5}-\frac{1}{512\,(\tan(1/2\,dx+c/2)-1)^4}+\frac{1}{512\,(\tan(1/2\,dx+c/2)-1)^2}-\frac{}{2048\,\tan(1/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 128/d/a*(-1/1280/(tan(1/2*d*x+1/2*c)-1)^5-1/512/(tan(1/2*d*x+1/2*c)-1)^4+1/
512/(tan(1/2*d*x+1/2*c)-1)^2-5/2048/(tan(1/2*d*x+1/2*c)-1)-1/448/(tan(1/2*d
*x+1/2*c)+1)^7+1/128/(tan(1/2*d*x+1/2*c)+1)^6-9/1280/(tan(1/2*d*x+1/2*c)+1)
^5-1/512/(tan(1/2*d*x+1/2*c)+1)^4+1/512/(tan(1/2*d*x+1/2*c)+1)^3+3/1024/(ta
n(1/2*d*x+1/2*c)+1)^2+5/2048/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.09477, size = 456, normalized size = 5.43

$$\frac{32\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\sin(dx+c}{(\cos(dx+c)+}\right.}{35\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{20a\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{5a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{10a\sin(d}{(\cos(dx+c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 32/35*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 20*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a + 2*a*sin(d*x
 + c)/(cos(d*x + c) + 1) - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10*a*s
in(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4 + 20*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*a*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7 - 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 10*a*sin(d*x + c)^
9/(cos(d*x + c) + 1)^9 + 4*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 2*a*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11 - a*sin(d*x + c)^12/(cos(d*x + c) + 1)^
12)*d)
```

Fricas [A] time = 1.67288, size = 248, normalized size = 2.95

$$\frac{5\cos(dx+c)^6 + 15\cos(dx+c)^4 - 5\cos(dx+c)^2 + 2\left(15\cos(dx+c)^4 - 10\cos(dx+c)^2 + 3\right)\sin(dx+c) + 1}{35\left(ad\cos(dx+c)^5\sin(dx+c) + ad\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(d*x + c)^6 + 15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 2*(15*cos(d
*x + c)^4 - 10*cos(d*x + c)^2 + 3)*sin(d*x + c) + 1)/(a*d*cos(d*x + c)^5*si
n(d*x + c) + a*d*cos(d*x + c)^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^6(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

Giac [B] time = 5.31986, size = 232, normalized size = 2.76

$$-\frac{\frac{7\left(25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - 140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 33\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^5} - \frac{175\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 + 1260\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 3815\tan\left(\frac{1}{2}dx\right.}{a}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/560*(7*(25*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c)^3 + 210*tan
(1/2*d*x + 1/2*c)^2 - 140*tan(1/2*d*x + 1/2*c) + 33)/(a*(tan(1/2*d*x + 1/2*
c) - 1)^5) - (175*tan(1/2*d*x + 1/2*c)^6 + 1260*tan(1/2*d*x + 1/2*c)^5 + 38
```

```
15*tan(1/2*d*x + 1/2*c)^4 + 6020*tan(1/2*d*x + 1/2*c)^3 + 4641*tan(1/2*d*x
+ 1/2*c)^2 + 1792*tan(1/2*d*x + 1/2*c) + 281)/(a*(tan(1/2*d*x + 1/2*c) + 1)
^7))/d
```

3.54 $\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

```
[Out] -(Sec[c + d*x]/(a*d)) + (2*Sec[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^5/(5*a*d)
 + Tan[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.0918369, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2606, 194}

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]/(a*d)) + (2*Sec[c + d*x]^3)/(3*a*d) - Sec[c + d*x]^5/(5*a*d)
 + Tan[c + d*x]^5/(5*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^4(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^5(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^4\,dx,x,\tan(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\sec(c+dx)\right)}{ad}\\ &= \frac{\tan^5(c+dx)}{5ad}-\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\sec(c+dx)\right)}{ad}\\ &= -\frac{\sec(c+dx)}{ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.319816, size = 106, normalized size = 1.54

$$-\frac{\sec^3(c+dx)(-64\sin(c+dx)-178\sin(2(c+dx))+192\sin(3(c+dx))-89\sin(4(c+dx))-534\cos(c+dx)+288\,\text{co}}{960ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Sec[c + d*x]^3*(200 - 534*Cos[c + d*x] + 288*Cos[2*(c + d*x)] - 178*Cos[3
*(c + d*x)] + 24*Cos[4*(c + d*x)] - 64*Sin[c + d*x] - 178*Sin[2*(c + d*x)]
+ 192*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)]))/(960*a*d*(1 + Sin[c + d*x]))
```

Maple [B] time = 0.055, size = 130, normalized size = 1.9

$$32\frac{1}{da}\left(-\frac{1}{192\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{3}}-\frac{1}{128\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{2}}+\frac{3}{256\tan\left(1/2\,dx+c/2\right)-256}-\frac{1}{80\left(\tan\left(1/2\,dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 32/d/a*(-1/192/(tan(1/2*d*x+1/2*c)-1)^3-1/128/(tan(1/2*d*x+1/2*c)-1)^2+3/25
6/(tan(1/2*d*x+1/2*c)-1)-1/80/(tan(1/2*d*x+1/2*c)+1)^5+1/32/(tan(1/2*d*x+1/
2*c)+1)^4-1/96/(tan(1/2*d*x+1/2*c)+1)^3-1/64/(tan(1/2*d*x+1/2*c)+1)^2-3/256
/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] time = 1.08124, size = 289, normalized size = 4.19

$$-\frac{16\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+1\right)}{15\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{6a\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{6a\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{2a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{2a\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -16/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a + 2*a*sin(d*x + c)/
(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d*x
+ c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*a
*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c) + 1
)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A] time = 1.51105, size = 194, normalized size = 2.81

$$-\frac{3\cos\left(dx+c\right)^{4}+6\cos\left(dx+c\right)^{2}+4\left(3\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-1}{15\left(ad\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 4*(3*cos(d*x + c)^2 - 1)*sin(d
*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 2.56709, size = 162, normalized size = 2.35

$$\frac{\frac{5\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+11\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^3}-\frac{45\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+490\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+320\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+73}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan(
1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 240*tan(1/2*d*x + 1
/2*c)^3 + 490*tan(1/2*d*x + 1/2*c)^2 + 320*tan(1/2*d*x + 1/2*c) + 73)/(a*(t
an(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.55 $\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.0876481, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2706, 2607, 30, 2606}

$$\frac{\tan^3(c+dx)}{3ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]/(a*d) - Sec[c + d*x]^3/(3*a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a}-\frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a}\\ &= \frac{\operatorname{Subst}\left(\int x^2\,dx,x,\tan(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int\left(-1+x^2\right)dx,x,\sec(c+dx)\right)}{ad}\\ &= \frac{\sec(c+dx)}{ad}-\frac{\sec^3(c+dx)}{3ad}+\frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [B] time = 0.164187, size = 106, normalized size = 2.12

$$\frac{8\sin(c+dx)-5\sin(2(c+dx))-10\cos(c+dx)+2\cos(2(c+dx))+6}{12ad(\sin(c+dx)+1)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] (6 - 10*Cos[c + d*x] + 2*Cos[2*(c + d*x)] + 8*Sin[c + d*x] - 5*Sin[2*(c + d
*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

Maple [A] time = 0.047, size = 70, normalized size = 1.4

$$8\,\frac{1}{da}\Big(-1/16\,\left(\tan\left(1/2\,dx+c/2\right)-1\right)^{-1}-1/12\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-3}+1/8\,\left(\tan\left(1/2\,dx+c/2\right)+1\right)^{-2}+1/16\,\left(\tan\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 8/d/a*(-1/16/(tan(1/2*d*x+1/2*c)-1)-1/12/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan(
1/2*d*x+1/2*c)+1)^2+1/16/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A] time = 1.09485, size = 122, normalized size = 2.44

$$\frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{3\left(a+\frac{2a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 4/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/((a + 2*a*sin(d*x + c)/(cos(d*x
 + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4)*d)
```

Fricas [A] time = 1.37095, size = 127, normalized size = 2.54

$$\frac{\cos(dx+c)^2+2\sin(dx+c)+1}{3\left(ad\cos(dx+c)\sin(dx+c)+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^2 + 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) +
a*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [A] time = 1.65929, size = 92, normalized size = 1.84

$$-\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 12*tan
(1/2*d*x + 1/2*c) + 5)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.56 $\int \frac{1}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}$$

```
[Out] -(Cos[c + d*x]/(d*(a + a*Sin[c + d*x])))
```

Rubi [A] time = 0.0116354, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2648}

$$-\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[c + d*x])^(-1),x]
```

```
[Out] -(Cos[c + d*x]/(d*(a + a*Sin[c + d*x])))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{a+a\sin(c+dx)}\,dx = -\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

Mathematica [B] time = 0.0414675, size = 48, normalized size = 2.09

$$\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d(a\sin(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*(a + a*Sin[c
+ d*x]))
```

Maple [A] time = 0.022, size = 22, normalized size = 1.

$$-2\,\frac{1}{da\left(\tan\left(1/2\,dx+c/2\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sin(d*x+c)),x)
```

```
[Out] -2/a/d/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [A] time = 1.07772, size = 36, normalized size = 1.57

$$-\frac{2}{\left(a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/((a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Fricas [A] time = 1.46038, size = 108, normalized size = 4.7

$$-\frac{\cos\left(dx+c\right)-\sin\left(dx+c\right)+1}{ad\cos\left(dx+c\right)+ad\sin\left(dx+c\right)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c) - sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a
*d)
```

Sympy [A] time = 1.72599, size = 27, normalized size = 1.17

$$\begin{cases} -\frac{2}{ad\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+ad} & \text{for } d \neq 0 \\ \frac{x}{a\sin(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x)
```

```
[Out] Piecewise((-2/(a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x/(a*sin(c) + a), T
rue))
```

Giac [A] time = 1.27038, size = 28, normalized size = 1.22

$$-\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2/(a*d*(tan(1/2*d*x + 1/2*c) + 1))
```

3.57 $\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)
```

Rubi [A] time = 0.0512723, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2706, 3767, 8, 3770}

$$\frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \csc(c+dx)\,dx}{a}+\frac{\int \csc^2(c+dx)\,dx}{a}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\text{Subst}\left(\int 1\,dx,x,\cot(c+dx)\right)}{ad}\\ &= \frac{\tanh^{-1}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}\end{aligned}$$

Mathematica [B] time = 0.218073, size = 69, normalized size = 2.38

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)+\sin(c+dx)\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x] + (-Log[Cos[(c + d*x)/2]]
 + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]))/(2*a*d)
```

Maple [A] time = 0.068, size = 56, normalized size = 1.9

$$\frac{1}{2\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{1}{da}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a*ln(tan(1/2*d*x+
1/2*c))
```

Maxima [B] time = 1.08722, size = 95, normalized size = 3.28

$$-\frac{\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\cos(dx+c)+1}{a\sin(dx+c)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (cos(d*x + c) + 1)/(a*sin(
d*x + c)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [B] time = 1.53889, size = 173, normalized size = 5.97

$$\frac{\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-2\cos\left(dx+c\right)}{2\,ad\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cot^{2}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

Giac [B] time = 1.2855, size = 88, normalized size = 3.03

$$-\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}-\frac{2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (2*tan(
1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)))/d
```

3.58 $\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc
[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.088263, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^3(c+dx)}{3ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) - Cot[c + d*x]^3/(3*a*d) + (Cot[c + d*x]*Csc
[c + d*x])/(2*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}+\frac{\operatorname{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}\end{aligned}$$

Mathematica [B] time = 0.491465, size = 124, normalized size = 2.14

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(\cos(3(c+dx))+(3-6\sin(c+dx))\cos(c+dx)+\right.}{96ad(\sin(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2
*(Cos[3*(c + d*x)] + Cos[c + d*x]*(3 - 6*Sin[c + d*x]) + 6*(Log[Cos[(c + d*
x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3))/(96*a*d*(1 + Sin[c + d*x])
)
```

Maple [B] time = 0.075, size = 132, normalized size = 2.3

$$\frac{1}{24da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{1}{8da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{8da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}+\frac{1}{2da}\ln\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a*tan(1/2*d*x+1/2*c)^2-1/8/d/a*tan(1/2*
d*x+1/2*c)+1/8/d/a/tan(1/2*d*x+1/2*c)+1/2/d/a*ln(tan(1/2*d*x+1/2*c))-1/24/d
/a/tan(1/2*d*x+1/2*c)^3+1/8/d/a/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.02688, size = 209, normalized size = 3.6

$$-\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-1\right)(\cos(dx+c)+1)^{3}}{a\sin(dx+c)^{3}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(cos(
d*x + c) + 1))/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [B] time = 1.41589, size = 311, normalized size = 5.36

$$\frac{4\cos(dx+c)^{3}-3\left(\cos(dx+c)^{2}-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\left(\cos(dx+c)^{2}-1\right)\log\left(-\frac{1}{2}\cos(dx+c)\right.}{12\left(ad\cos(dx+c)^{2}-ad\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)
*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 6*cos(d*x + c)*sin(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x +
c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

Giac [B] time = 1.41595, size = 171, normalized size = 2.95

$$\frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{22\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d*x + 1/2*c)^3 - 3
*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1/2
*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)/(a
*tan(1/2*d*x + 1/2*c)^3))/d
```

3.59 $\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x
]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.10552, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(8*a*d) - Cot[c + d*x]^5/(5*a*d) - (3*Cot[c + d*x
]*Csc[c + d*x])/(8*a*d) + (Cot[c + d*x]^3*Csc[c + d*x])/(4*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^4(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot^3(c+dx)\csc(c+dx)}{4ad}+\frac{3\int \cot^2(c+dx)\csc(c+dx)\,dx}{4a}+\frac{\operatorname{Subst}\left(\int x^4\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}-\frac{3\int \csc(c+dx)\,dx}{8a}\\ &= \frac{3\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot^3(c+dx)\csc(c+dx)}{4ad}\end{aligned}$$

Mathematica [B] time = 0.743001, size = 189, normalized size = 2.3

$$-\frac{\csc^5(c+dx)\Big(20\sin(2(c+dx))-50\sin(4(c+dx))+80\cos(c+dx)+40\cos(3(c+dx))+8\cos(5(c+dx))+150\sin(c+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^5*(80*Cos[c + d*x] + 40*Cos[3*(c + d*x)] + 8*Cos[5*(c + d*x)
] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d*x)/2]]*Sin[
c + d*x] + 20*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)]
- 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 50*Sin[4*(c + d*x)] - 15*Log[
```

```
Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*Sin[5*(c + d*
x)]))/(640*a*d)
```

Maple [B] time = 0.088, size = 208, normalized size = 2.5

$$\frac{1}{160\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-\frac{1}{64\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}-\frac{1}{32\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}+\frac{1}{16\,da}\tan\left(\frac{dx}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/160/d/a*tan(1/2*d*x+1/2*c)^5-1/64/d/a*tan(1/2*d*x+1/2*c)^4-1/32/d/a*tan(1
/2*d*x+1/2*c)^3+1/8/d/a*tan(1/2*d*x+1/2*c)^2+1/16/d/a*tan(1/2*d*x+1/2*c)-1/
16/d/a/tan(1/2*d*x+1/2*c)-1/160/d/a/tan(1/2*d*x+1/2*c)^5+1/64/d/a/tan(1/2*d
*x+1/2*c)^4-3/8/d/a*ln(tan(1/2*d*x+1/2*c))+1/32/d/a/tan(1/2*d*x+1/2*c)^3-1/
8/d/a/tan(1/2*d*x+1/2*c)^2
```

Maxima [B] time = 1.01987, size = 316, normalized size = 3.85

$$\frac{\frac{\frac{20\sin(dx+c)}{\cos(dx+c)+1}+\frac{40\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}-\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{20\sin(}{(\cos(dx}\right.}{a\sin(dx+c)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/320*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 120*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/a + (5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2
0*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2)*(cos(d*x + c) + 1)^5/(a*sin(d*x
+ c)^5))/d
```

Fricas [B] time = 1.39609, size = 431, normalized size = 5.26

$$-\frac{16\cos\left(dx+c\right)^{5}-15\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+15\left(\cos\left(dx+c\right)^{4}-2\cos\right.}{80\left(ad\cos\left(dx+c\right)^{4}-2\,ad\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/80*(16*cos(d*x + c)^5 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2
 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(5*cos(d*x + c)^3 - 3*
cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a
*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^{6}\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

Giac [B] time = 1.57756, size = 252, normalized size = 3.07

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}-\frac{2a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-5a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-10a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+40a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+20a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{5}}-\frac{274\tan\left(\frac{1}{2}\right.}{}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/320*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (2*a^4*tan(1/2*d*x + 1/2*c)^
5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 - 10*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^4*t
```

```
an(1/2*d*x + 1/2*c)^2 + 20*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*d*x
 + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x + 1/2*c)^3 + 10*ta
n(1/2*d*x + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 2)/(a*tan(1/2*d*x + 1/2*c)^
5))/d
```

3.60 $\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+\ldots}{16ad}$$

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Co
t[c + d*x]^5*Csc[c + d*x])/(6*a*d)
```

Rubi [A] time = 0.127459, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2706, 2607, 30, 2611, 3770}

$$-\frac{\cot^7(c+dx)}{7ad}-\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{5\cot(c+dx)\csc(c+\ldots}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(16*a*d) - Cot[c + d*x]^7/(7*a*d) + (5*Cot[c + d
*x]*Csc[c + d*x])/(16*a*d) - (5*Cot[c + d*x]^3*Csc[c + d*x])/(24*a*d) + (Co
t[c + d*x]^5*Csc[c + d*x])/(6*a*d)
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx &= -\frac{\int \cot^6(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot^5(c+dx)\csc(c+dx)}{6ad}+\frac{5\int \cot^4(c+dx)\csc(c+dx)\,dx}{6a}+\frac{\mathrm{Subst}\left(\int x^6\,dx,x,-\cot(c+dx\right.}{ad}\\ &= -\frac{\cot^7(c+dx)}{7ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c+dx)\csc(c+dx)}{6ad}-\frac{5\int \cot^2(c+dx)\operatorname{csc}}{8a}\\ &= -\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+dx)\csc(c+dx)}{24ad}+\frac{\cot^5(c+dx)\csc(c}{6ad}\\ &= -\frac{5\tanh^{-1}(\cos(c+dx))}{16ad}-\frac{\cot^7(c+dx)}{7ad}+\frac{5\cot(c+dx)\csc(c+dx)}{16ad}-\frac{5\cot^3(c+dx)\csc(c+}{24ad}\end{aligned}$$

Mathematica [B] time = 0.914319, size = 284, normalized size = 2.68

$$-\frac{\csc^5(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2\left(-1190\sin(2(c+dx))+392\sin(4(c+dx))-462\sin(6(c+dx))+1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

```
[Out] -(Csc[c + d*x]^5*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(1680*Cos[c + d*x]
 + 1008*Cos[3*(c + d*x)] + 336*Cos[5*(c + d*x)] + 48*Cos[7*(c + d*x)] + 367
```

```
5*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 3675*Log[Sin[(c + d*x)/2]]*Sin[c + d
*x] - 1190*Sin[2*(c + d*x)] - 2205*Log[Cos[(c + d*x)/2]]*Sin[3*(c + d*x)] +
 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 392*Sin[4*(c + d*x)] + 735*L
og[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*Log[Sin[(c + d*x)/2]]*Sin[5*(c
+ d*x)] - 462*Sin[6*(c + d*x)] - 105*Log[Cos[(c + d*x)/2]]*Sin[7*(c + d*x)]
 + 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(86016*a*d*(1 + Sin[c + d*x
]))
```

Maple [B] time = 0.102, size = 284, normalized size = 2.7

$$\frac{1}{896\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}-\frac{1}{384\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}-\frac{1}{128\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}+\frac{3}{128\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}+\frac{3}{128\,da}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8/(a+a*sin(d*x+c)),x)
```

```
[Out] 1/896/d/a*tan(1/2*d*x+1/2*c)^7-1/384/d/a*tan(1/2*d*x+1/2*c)^6-1/128/d/a*tan
(1/2*d*x+1/2*c)^5+3/128/d/a*tan(1/2*d*x+1/2*c)^4+3/128/d/a*tan(1/2*d*x+1/2*
c)^3-15/128/d/a*tan(1/2*d*x+1/2*c)^2-5/128/d/a*tan(1/2*d*x+1/2*c)-1/896/d/a
/tan(1/2*d*x+1/2*c)^7+5/128/d/a/tan(1/2*d*x+1/2*c)+1/128/d/a/tan(1/2*d*x+1/
2*c)^5-3/128/d/a/tan(1/2*d*x+1/2*c)^4+5/16/d/a*ln(tan(1/2*d*x+1/2*c))+1/384
/d/a/tan(1/2*d*x+1/2*c)^6-3/128/d/a/tan(1/2*d*x+1/2*c)^3+15/128/d/a/tan(1/2
*d*x+1/2*c)^2
```

Maxima [B] time = 1.12476, size = 425, normalized size = 4.01

$$-\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{315\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{63\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{7\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{21\sin}{(\cos(dx}\right.}{}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2688*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 63*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a - 840*
```

```
log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (7*sin(d*x + c)/(cos(d*x + c) + 1)
 + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 - 63*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 315*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos(d*x + c
) + 1)^7/(a*sin(d*x + c)^7))/d
```

Fricas [B] time = 1.57927, size = 545, normalized size = 5.14

$$\frac{96 \cos(dx+c)^7 - 105\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)\sin(dx+c) + 105}{672\left(ad\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/672*(96*cos(d*x + c)^7 - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d
*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x + c)
^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 14*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x + c))
*sin(d*x + c))/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x
+ c)^2 - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.97865, size = 329, normalized size = 3.1

$$\frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} + \frac{3a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 7a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 21a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 315a^6\tan\left(\frac{1}{2}\right.}{a^7}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^6*tan(1/2*d*x + 1/2*c)^
7 - 7*a^6*tan(1/2*d*x + 1/2*c)^6 - 21*a^6*tan(1/2*d*x + 1/2*c)^5 + 63*a^6*t
an(1/2*d*x + 1/2*c)^4 + 63*a^6*tan(1/2*d*x + 1/2*c)^3 - 315*a^6*tan(1/2*d*x
 + 1/2*c)^2 - 105*a^6*tan(1/2*d*x + 1/2*c))/a^7 - (2178*tan(1/2*d*x + 1/2*c
)^7 - 105*tan(1/2*d*x + 1/2*c)^6 - 315*tan(1/2*d*x + 1/2*c)^5 + 63*tan(1/2*
d*x + 1/2*c)^4 + 63*tan(1/2*d*x + 1/2*c)^3 - 21*tan(1/2*d*x + 1/2*c)^2 - 7*
tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^7))/d
```

3.61 $\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{a^3}{80d(a\sin(c+dx)+a)^5}-\frac{5a^2}{64d(a\sin(c+dx)+a)^4}+\frac{21}{256d\left(a^2-a^2\sin(c+dx)\right)}+\frac{35}{256d\left(a^2\sin(c+dx)+a^2\right)}-\frac{7\tanh^{-1}}{}$$

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(128*a^2*d) + a/(192*d*(a - a*Sin[c + d*x])^3) -
 1/(32*d*(a - a*Sin[c + d*x])^2) + a^3/(80*d*(a + a*Sin[c + d*x])^5) - (5*a
^2)/(64*d*(a + a*Sin[c + d*x])^4) + (19*a)/(96*d*(a + a*Sin[c + d*x])^3) -
1/(4*d*(a + a*Sin[c + d*x])^2) + 21/(256*d*(a^2 - a^2*Sin[c + d*x])) + 35/(
256*d*(a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.148605, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{a^3}{80d(a\sin(c+dx)+a)^5}-\frac{5a^2}{64d(a\sin(c+dx)+a)^4}+\frac{21}{256d\left(a^2-a^2\sin(c+dx)\right)}+\frac{35}{256d\left(a^2\sin(c+dx)+a^2\right)}-\frac{7\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (-7*ArcTanh[Sin[c + d*x]])/(128*a^2*d) + a/(192*d*(a - a*Sin[c + d*x])^3) -
 1/(32*d*(a - a*Sin[c + d*x])^2) + a^3/(80*d*(a + a*Sin[c + d*x])^5) - (5*a
^2)/(64*d*(a + a*Sin[c + d*x])^4) + (19*a)/(96*d*(a + a*Sin[c + d*x])^3) -
1/(4*d*(a + a*Sin[c + d*x])^2) + 21/(256*d*(a^2 - a^2*Sin[c + d*x])) + 35/(
256*d*(a^2 + a^2*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^7}{(a-x)^4(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a}{64(a-x)^4} - \frac{1}{16(a-x)^3} + \frac{21}{256a(a-x)^2} - \frac{a^3}{16(a+x)^6} + \frac{5a^2}{16(a+x)^5} - \frac{19a}{32(a+x)^4} + \frac{1}{2(a+x)^3} - \frac{35}{256a(a+x)^2}\right.\right.}{d}$$

$$= \frac{a}{192d(a-a\sin(c+dx))^3} - \frac{1}{32d(a-a\sin(c+dx))^2} + \frac{a^3}{80d(a+a\sin(c+dx))^5} - \frac{5a}{64d(a+a\sin}$$

$$= -\frac{7\tanh^{-1}(\sin(c+dx))}{128a^2d} + \frac{a}{192d(a-a\sin(c+dx))^3} - \frac{1}{32d(a-a\sin(c+dx))^2} + \frac{a}{80d(a+a\sin}$$

Mathematica [A] time = 1.69255, size = 112, normalized size = 0.59

$$-\frac{210\tanh^{-1}(\sin(c+dx)) - \frac{2\left(105\sin^7(c+dx)-750\sin^6(c+dx)-815\sin^5(c+dx)+560\sin^4(c+dx)+1039\sin^3(c+dx)+78\sin^2(c+dx)-393\sin(c+dx)-144\right)}{(\sin(c+dx)-1)^3(\sin(c+dx)+1)^5}}{3840a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(210*ArcTanh[Sin[c + d*x]] - (2*(-144 - 393*Sin[c + d*x] + 78*Sin[c + d*x]
^2 + 1039*Sin[c + d*x]^3 + 560*Sin[c + d*x]^4 - 815*Sin[c + d*x]^5 - 750*Si
n[c + d*x]^6 + 105*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x
])^5))/(3840*a^2*d)
```

Maple [A] time = 0.096, size = 180, normalized size = 1.

$$-\frac{1}{192\,da^2\,(\sin(dx+c)-1)^3}-\frac{1}{32\,da^2\,(\sin(dx+c)-1)^2}-\frac{21}{256\,da^2\,(\sin(dx+c)-1)}+\frac{7\,\ln(\sin(dx+c)-1)}{256\,da^2}+\frac{}{80\,da^2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/192/d/a^2/(sin(d*x+c)-1)^3-1/32/d/a^2/(sin(d*x+c)-1)^2-21/256/d/a^2/(sin
(d*x+c)-1)+7/256/d/a^2*ln(sin(d*x+c)-1)+1/80/d/a^2/(1+sin(d*x+c))^5-5/64/d/
a^2/(1+sin(d*x+c))^4+19/96/d/a^2/(1+sin(d*x+c))^3-1/4/d/a^2/(1+sin(d*x+c))^
2+35/256/d/a^2/(1+sin(d*x+c))-7/256*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.10553, size = 273, normalized size = 1.44

$$\frac{\frac{2\left(105\,\sin(dx+c)^7-750\,\sin(dx+c)^6-815\,\sin(dx+c)^5+560\,\sin(dx+c)^4+1039\,\sin(dx+c)^3+78\,\sin(dx+c)^2-393\,\sin(dx+c)-144\right)}{a^2\sin(dx+c)^8+2\,a^2\sin(dx+c)^7-2\,a^2\sin(dx+c)^6-6\,a^2\sin(dx+c)^5+6\,a^2\sin(dx+c)^3+2\,a^2\sin(dx+c)^2-2\,a^2\sin(dx+c)-a^2}-\frac{105\,\log(\sin(dx+c)+1)}{a^2}+\frac{10}{}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(105*sin(d*x + c)^7 - 750*sin(d*x + c)^6 - 815*sin(d*x + c)^5 + 5
60*sin(d*x + c)^4 + 1039*sin(d*x + c)^3 + 78*sin(d*x + c)^2 - 393*sin(d*x +
 c) - 144)/(a^2*sin(d*x + c)^8 + 2*a^2*sin(d*x + c)^7 - 2*a^2*sin(d*x + c)^
6 - 6*a^2*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^3 + 2*a^2*sin(d*x + c)^2 - 2*
a^2*sin(d*x + c) - a^2) - 105*log(sin(d*x + c) + 1)/a^2 + 105*log(sin(d*x +
 c) - 1)/a^2)/d
```

Fricas [A] time = 1.72815, size = 599, normalized size = 3.17

$$\frac{1500\,\cos(dx+c)^6-3380\,\cos(dx+c)^4+2104\,\cos(dx+c)^2-105\left(\cos(dx+c)^8-2\,\cos(dx+c)^6\sin(dx+c)-2\,\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3840*(1500*cos(d*x + c)^6 - 3380*cos(d*x + c)^4 + 2104*cos(d*x + c)^2 - 1
05*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c) - 2*cos(d*x + c)^6)*log(
sin(d*x + c) + 1) + 105*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c) - 2
*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 500*cos(d
*x + c)^4 - 276*cos(d*x + c)^2 + 64)*sin(d*x + c) - 512)/(a^2*d*cos(d*x + c
)^8 - 2*a^2*d*cos(d*x + c)^6*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 9.84355, size = 197, normalized size = 1.04

$$-\frac{\frac{420 \log(|\sin(dx+c)+1|)}{a^2} - \frac{420 \log(|\sin(dx+c)-1|)}{a^2} + \frac{10\left(77 \sin(dx+c)^3 - 105 \sin(dx+c)^2 + 27 \sin(dx+c) + 9\right)}{a^2(\sin(dx+c)-1)^3} - \frac{959 \sin(dx+c)^5 + 6895 \sin(dx+c)^4 + 14150 \sin}{a^2(\sin}}{15360\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/15360*(420*log(abs(sin(d*x + c) + 1))/a^2 - 420*log(abs(sin(d*x + c) - 1
))/a^2 + 10*(77*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 27*sin(d*x + c) + 9)/
(a^2*(sin(d*x + c) - 1)^3) - (959*sin(d*x + c)^5 + 6895*sin(d*x + c)^4 + 14
150*sin(d*x + c)^3 + 13710*sin(d*x + c)^2 + 6555*sin(d*x + c) + 1251)/(a^2*
(sin(d*x + c) + 1)^5))/d
```

3.62 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$\frac{a^2}{32d(a\sin(c+dx)+a)^4}-\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{7}{48d(a\sin(c}$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) + a^
2/(32*d*(a + a*Sin[c + d*x])^4) - (7*a)/(48*d*(a + a*Sin[c + d*x])^3) + 1/(
4*d*(a + a*Sin[c + d*x])^2) - 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.107958, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{a^2}{32d(a\sin(c+dx)+a)^4}-\frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)}-\frac{5}{32d\left(a^2\sin(c+dx)+a^2\right)}+\frac{5\tanh^{-1}(\sin(c+dx))}{64a^2d}-\frac{7}{48d(a\sin(c}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(64*a^2*d) + 1/(64*d*(a - a*Sin[c + d*x])^2) + a^
2/(32*d*(a + a*Sin[c + d*x])^4) - (7*a)/(48*d*(a + a*Sin[c + d*x])^3) + 1/(
4*d*(a + a*Sin[c + d*x])^2) - 5/(64*d*(a^2 - a^2*Sin[c + d*x])) - 5/(32*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{32(a-x)^3} - \frac{5}{64a(a-x)^2} - \frac{a^2}{8(a+x)^5} + \frac{7a}{16(a+x)^4} - \frac{1}{2(a+x)^3} + \frac{5}{32a(a+x)^2} + \frac{5}{64a\left(a^2-x^2\right)}\right) dx, x, a\sin\right.}{d}$$

$$= \frac{1}{64d(a-a\sin(c+dx))^2} + \frac{a^2}{32d(a+a\sin(c+dx))^4} - \frac{7a}{48d(a+a\sin(c+dx))^3} + \frac{1}{4d(a+a\sin(}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{64a^2d} + \frac{1}{64d(a-a\sin(c+dx))^2} + \frac{a^2}{32d(a+a\sin(c+dx))^4} - \frac{7a}{48d(a+a\sin(}$$

Mathematica [A] time = 0.449695, size = 91, normalized size = 0.62

$$\frac{\frac{-15\sin^5(c+dx)+66\sin^4(c+dx)+74\sin^3(c+dx)-14\sin^2(c+dx)-47\sin(c+dx)-16}{(\sin(c+dx)-1)^2(\sin(c+dx)+1)^4}+15\tanh^{-1}(\sin(c+dx))}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] + (-16 - 47*Sin[c + d*x] - 14*Sin[c + d*x]^2 + 74
*Sin[c + d*x]^3 + 66*Sin[c + d*x]^4 - 15*Sin[c + d*x]^5)/((-1 + Sin[c + d*x
])^2*(1 + Sin[c + d*x])^4))/(192*a^2*d)
```

Maple [A] time = 0.084, size = 144, normalized size = 1.

$$\frac{1}{64\,da^2(\sin(dx+c)-1)^2} + \frac{5}{64\,da^2(\sin(dx+c)-1)} - \frac{5\ln(\sin(dx+c)-1)}{128\,da^2} + \frac{1}{32\,da^2(1+\sin(dx+c))^4} - \frac{7}{48\,da^2(1+\text{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/64/d/a^2/(sin(d*x+c)-1)^2+5/64/d/a^2/(sin(d*x+c)-1)-5/128/d/a^2*ln(sin(d*
x+c)-1)+1/32/d/a^2/(1+sin(d*x+c))^4-7/48/d/a^2/(1+sin(d*x+c))^3+1/4/d/a^2/(
1+sin(d*x+c))^2-5/32/d/a^2/(1+sin(d*x+c))+5/128*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.05414, size = 225, normalized size = 1.54

$$-\frac{\frac{2\left(15\sin(dx+c)^5-66\sin(dx+c)^4-74\sin(dx+c)^3+14\sin(dx+c)^2+47\sin(dx+c)+16\right)}{a^2\sin(dx+c)^6+2a^2\sin(dx+c)^5-a^2\sin(dx+c)^4-4a^2\sin(dx+c)^3-a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2}-\frac{15\log(\sin(dx+c)+1)}{a^2}+\frac{15\log(\sin(dx+c)-1)}{a^2}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(2*(15*sin(d*x + c)^5 - 66*sin(d*x + c)^4 - 74*sin(d*x + c)^3 + 14*s
in(d*x + c)^2 + 47*sin(d*x + c) + 16)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x +
 c)^5 - a^2*sin(d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 + 2*
a^2*sin(d*x + c) + a^2) - 15*log(sin(d*x + c) + 1)/a^2 + 15*log(sin(d*x + c
) - 1)/a^2)/d
```

Fricas [A] time = 1.52801, size = 532, normalized size = 3.64

$$-\frac{132\cos(dx+c)^4-236\cos(dx+c)^2-15\left(\cos(dx+c)^6-2\cos(dx+c)^4\sin(dx+c)-2\cos(dx+c)^4\right)\log(\sin(dx}{384\left(a^2d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(132*cos(d*x + c)^4 - 236*cos(d*x + c)^2 - 15*(cos(d*x + c)^6 - 2*co
s(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 15*(c
os(d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(-sin(
d*x + c) + 1) - 2*(15*cos(d*x + c)^4 + 44*cos(d*x + c)^2 - 12)*sin(d*x + c)
 + 72)/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*x + c)^4*sin(d*x + c) - 2*a^2*
```

```
d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 5.28893, size = 170, normalized size = 1.16

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^2} - \frac{60\log(|\sin(dx+c)-1|)}{a^2} + \frac{6\left(15\sin(dx+c)^2-10\sin(dx+c)-1\right)}{a^2(\sin(dx+c)-1)^2} - \frac{125\sin(dx+c)^4+740\sin(dx+c)^3+1086\sin(dx+c)^2+676\sin(dx+c)+}{a^2(\sin(dx+c)+1)^4}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(60*log(abs(sin(d*x + c) + 1))/a^2 - 60*log(abs(sin(d*x + c) - 1))/a
^2 + 6*(15*sin(d*x + c)^2 - 10*sin(d*x + c) - 1)/(a^2*(sin(d*x + c) - 1)^2)
 - (125*sin(d*x + c)^4 + 740*sin(d*x + c)^3 + 1086*sin(d*x + c)^2 + 676*sin
(d*x + c) + 157)/(a^2*(sin(d*x + c) + 1)^4))/d
```

3.63 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}+\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d}+\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{1}{4d(a\sin(c+a$$

```
[Out] -ArcTanh[Sin[c + d*x]]/(8*a^2*d) + a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(4*d
*(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) + 3/(16*d*(a^2
 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.085784, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)}+\frac{3}{16d\left(a^2\sin(c+dx)+a^2\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d}+\frac{a}{12d(a\sin(c+dx)+a)^3}-\frac{1}{4d(a\sin(c+a$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```


```
[Out] -ArcTanh[Sin[c + d*x]]/(8*a^2*d) + a/(12*d*(a + a*Sin[c + d*x])^3) - 1/(4*d
*(a + a*Sin[c + d*x])^2) + 1/(16*d*(a^2 - a^2*Sin[c + d*x])) + 3/(16*d*(a^2
 + a^2*Sin[c + d*x]))
```


Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{16a(a-x)^2} - \frac{a}{4(a+x)^4} + \frac{1}{2(a+x)^3} - \frac{3}{16a(a+x)^2} - \frac{1}{8a\left(a^2-x^2\right)}\right) dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{a}{12d(a+a\sin(c+dx))^3} - \frac{1}{4d(a+a\sin(c+dx))^2} + \frac{1}{16d\left(a^2-a^2\sin(c+dx)\right)} + \frac{3}{16d\left(a^2+a^2\right.}$$

$$= -\frac{\tanh^{-1}(\sin(c+dx))}{8a^2d} + \frac{a}{12d(a+a\sin(c+dx))^3} - \frac{1}{4d(a+a\sin(c+dx))^2} + \frac{1}{16d\left(a^2-a^2\sin\right.}$$

Mathematica [A] time = 0.310755, size = 70, normalized size = 0.67

$$-\frac{-\frac{3}{1-\sin(c+dx)} - \frac{9}{\sin(c+dx)+1} + \frac{12}{(\sin(c+dx)+1)^2} - \frac{4}{(\sin(c+dx)+1)^3} + 6\tanh^{-1}(\sin(c+dx))}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(6*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 4/(1 + Sin[c + d*x])^3 +
 12/(1 + Sin[c + d*x])^2 - 9/(1 + Sin[c + d*x]))/(48*a^2*d)
```

Maple [A] time = 0.083, size = 108, normalized size = 1.

$$-\frac{1}{16\,da^2(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)}{16\,da^2} + \frac{1}{12\,da^2(1+\sin(dx+c))^3} - \frac{1}{4\,da^2(1+\sin(dx+c))^2} + \frac{3}{16\,da^2(1+\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/16/d/a^2/(sin(d*x+c)-1)+1/16/d/a^2*ln(sin(d*x+c)-1)+1/12/d/a^2/(1+sin(d*
x+c))^3-1/4/d/a^2/(1+sin(d*x+c))^2+3/16/d/a^2/(1+sin(d*x+c))-1/16*ln(1+sin(
d*x+c))/a^2/d
```

Maxima [A] time = 1.05494, size = 149, normalized size = 1.43

$$\frac{\frac{2\left(3\,\sin(dx+c)^3-6\,\sin(dx+c)^2-7\,\sin(dx+c)-2\right)}{a^2\sin(dx+c)^4+2\,a^2\sin(dx+c)^3-2\,a^2\sin(dx+c)-a^2}-\frac{3\,\log(\sin(dx+c)+1)}{a^2}+\frac{3\,\log(\sin(dx+c)-1)}{a^2}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(2*(3*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 7*sin(d*x + c) - 2)/(a^2*sin
(d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2*a^2*sin(d*x + c) - a^2) - 3*log(sin(
d*x + c) + 1)/a^2 + 3*log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A] time = 1.53143, size = 467, normalized size = 4.49

$$\frac{12\,\cos\left(dx+c\right)^2-3\left(\cos\left(dx+c\right)^4-2\,\cos\left(dx+c\right)^2\sin\left(dx+c\right)-2\,\cos\left(dx+c\right)^2\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(\cos\left(dx+\right.\right.}{48\left(a^2d\cos\left(dx+c\right)^4-2\,a^2d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(12*cos(d*x + c)^2 - 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c)
 - 2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^4 - 2*cos(d*x
+ c)^2*sin(d*x + c) - 2*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(3*cos(d
*x + c)^2 + 4)*sin(d*x + c) - 16)/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x +
 c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 2.99776, size = 138, normalized size = 1.33

$$-\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^2}-\frac{6\log(|\sin(dx+c)-1|)}{a^2}+\frac{6\sin(dx+c)}{a^2(\sin(dx+c)-1)}-\frac{11\sin(dx+c)^3+51\sin(dx+c)^2+45\sin(dx+c)+13}{a^2(\sin(dx+c)+1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/96*(6*log(abs(sin(d*x + c) + 1))/a^2 - 6*log(abs(sin(d*x + c) - 1))/a^2
+ 6*sin(d*x + c)/(a^2*(sin(d*x + c) - 1)) - (11*sin(d*x + c)^3 + 51*sin(d*x
 + c)^2 + 45*sin(d*x + c) + 13)/(a^2*(sin(d*x + c) + 1)^3))/d
```

3.64 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}+\frac{1}{4d(a\sin(c+dx)+a)^2}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) + 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0483769, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2707, 77, 206}

$$-\frac{1}{4d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}+\frac{1}{4d(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(4*a^2*d) + 1/(4*d*(a + a*Sin[c + d*x])^2) - 1/(4*d*(
a^2 + a^2*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^3}+\frac{1}{4a(a+x)^2}+\frac{1}{4a\left(a^2-x^2\right)}\right)dx, x, a\sin(c+dx)\right)}{d} \\
&= \frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\sin(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, a\sin(c+dx)\right)}{4ad} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{4a^2d}+\frac{1}{4d(a+a\sin(c+dx))^2}-\frac{1}{4d\left(a^2+a^2\sin(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 0.0807824, size = 36, normalized size = 0.6

$$\frac{\tanh^{-1}(\sin(c+dx))-\frac{\sin(c+dx)}{(\sin(c+dx)+1)^2}}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (ArcTanh[Sin[c + d*x]] - Sin[c + d*x]/(1 + Sin[c + d*x])^2)/(4*a^2*d)
```

Maple [A] time = 0.082, size = 72, normalized size = 1.2

$$-\frac{\ln(\sin(dx+c)-1)}{8\,da^2}+\frac{1}{4\,da^2(1+\sin(dx+c))^2}-\frac{1}{4\,da^2(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8\,da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -1/8/d/a^2*ln(sin(d*x+c)-1)+1/4/d/a^2/(1+sin(d*x+c))^2-1/4/d/a^2/(1+sin(d*x
+c))+1/8*ln(1+sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.02483, size = 95, normalized size = 1.58

$$-\frac{\frac{2\sin(dx+c)}{a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c)-1)}{a^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*sin(d*x + c)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c) + a^2) - log(
sin(d*x + c) + 1)/a^2 + log(sin(d*x + c) - 1)/a^2)/d
```

Fricas [A] time = 1.45633, size = 274, normalized size = 4.57

$$\frac{\left(\cos\left(dx+c\right)^2-2\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)^2-2\sin\left(dx+c\right)-2\right)\log\left(-\sin\left(dx+c\right)+1\right)+}{8\left(a^2d\cos\left(dx+c\right)^2-2a^2d\sin\left(dx+c\right)-2a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*((cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - (cos(d*x
 + c)^2 - 2*sin(d*x + c) - 2)*log(-sin(d*x + c) + 1) + 2*sin(d*x + c))/(a^2
*d*cos(d*x + c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.84808, size = 122, normalized size = 2.03

$$\frac{\frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right|\right)}{a^2}-\frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)-2\right|\right)}{a^2}-\frac{\frac{1}{\sin(dx+c)}+\sin(dx+c)+6}{a^2\left(\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right)}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2))/a^2 - log(abs(1/sin(d*x +
 c) + sin(d*x + c) - 2))/a^2 - (1/sin(d*x + c) + sin(d*x + c) + 6)/(a^2*(1/
sin(d*x + c) + sin(d*x + c) + 2)))/d
```

3.65 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rubi [A] time = 0.0504909, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)}+\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[1 + Sin[c + d*x]]/(a^2*d) + 1/(d*(a^2 + a^2
*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^2d} - \frac{\log(1+\sin(c+dx))}{a^2d} + \frac{1}{d\left(a^2+a^2\sin(c+dx)\right)}$$

Mathematica [A] time = 0.0548377, size = 36, normalized size = 0.69

$$\frac{\frac{1}{\sin(c+dx)+1} + \log(\sin(c+dx)) - \log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Log[Sin[c + d*x]] - Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2*
d)
```

Maple [A] time = 0.034, size = 50, normalized size = 1.

$$\frac{1}{da^2\,(1+\sin(dx+c))} - \frac{\ln(1+\sin(dx+c))}{da^2} + \frac{\ln(\sin(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^2/d+ln(sin(d*x+c))/a^2/d
```

Maxima [A] time = 1.11919, size = 62, normalized size = 1.19

$$\frac{\frac{1}{a^2\sin(dx+c)+a^2} - \frac{\log(\sin(dx+c)+1)}{a^2} + \frac{\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*sin(d*x + c) + a^2) - log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c))
/a^2)/d
```

Fricas [A] time = 1.50947, size = 162, normalized size = 3.12

$$\frac{(\sin(dx+c)+1)\log\left(\frac{1}{2}\sin(dx+c)\right)-(\sin(dx+c)+1)\log(\sin(dx+c)+1)+1}{a^2 d\sin(dx+c)+a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) - (sin(d*x + c) + 1)*log(sin(d*x
+ c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.7951, size = 61, normalized size = 1.17

$$\frac{a\left(\frac{\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^3}+\frac{1}{(a\sin(dx+c)+a)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] a*(log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^3 + 1/((a*sin(d*x + c) + a)*a^2)
)/d
```

3.66 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{2\log(\sin(c+dx))}{a^2d}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (2*Log[Sin[c + d*x]])
/(a^2*d) - (2*Log[1 + Sin[c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.0593806, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc(c+dx)}{a^2d}+\frac{2\log(\sin(c+dx))}{a^2d}-\frac{2\log(\sin(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (2*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a^2*d) + (2*Log[Sin[c + d*x]])
/(a^2*d) - (2*Log[1 + Sin[c + d*x]])/(a^2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{a-x}{x^3(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{x^3}-\frac{2}{ax^2}+\frac{2}{a^2x}-\frac{2}{a^2(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{2\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\log(\sin(c+dx))}{a^2d}-\frac{2\log(1+\sin(c+dx))}{a^2d}$$

Mathematica [A] time = 0.0701507, size = 49, normalized size = 0.75

$$\frac{-\csc^2(c+dx)+4\csc(c+dx)+4\log(\sin(c+dx))-4\log(\sin(c+dx)+1)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (4*Csc[c + d*x] - Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - 4*Log[1 + Sin[c +
d*x]])/(2*a^2*d)
```

Maple [A] time = 0.106, size = 66, normalized size = 1.

$$-2\,\frac{\ln(1+\sin(dx+c))}{a^2d}-\frac{1}{2\,a^2d\,(\sin(dx+c))^2}+2\,\frac{1}{a^2d\sin(dx+c)}+2\,\frac{\ln(\sin(dx+c))}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x)
```

```
[Out] -2*ln(1+sin(d*x+c))/a^2/d-1/2/d/a^2/sin(d*x+c)^2+2/d/a^2/sin(d*x+c)+2*ln(si
n(d*x+c))/a^2/d
```

Maxima [A] time = 1.38376, size = 74, normalized size = 1.14

$$-\frac{\frac{4\log(\sin(dx+c)+1)}{a^2}-\frac{4\log(\sin(dx+c))}{a^2}-\frac{4\sin(dx+c)-1}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*log(sin(d*x + c) + 1)/a^2 - 4*log(sin(d*x + c))/a^2 - (4*sin(d*x +
c) - 1)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.54804, size = 204, normalized size = 3.14

$$\frac{4\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-4\left(\cos\left(dx+c\right)^2-1\right)\log\left(\sin\left(dx+c\right)+1\right)-4\sin\left(dx+c\right)+1}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 4*(cos(d*x + c)^2 - 1)*
log(sin(d*x + c) + 1) - 4*sin(d*x + c) + 1)/(a^2*d*cos(d*x + c)^2 - a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^3\left(c+dx\right)}{\sin^2\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 2.10221, size = 155, normalized size = 2.38

$$-\frac{\frac{32\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{16\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^2}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}+\frac{24\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(32*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 16*log(abs(tan(1/2*d*x +
1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/a^
4 + (24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/2*d
*x + 1/2*c)^2))/d
```

3.67 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(
4*a^2*d)
```

Rubi [A] time = 0.0511339, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$-\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(
4*a^2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^5}-\frac{2a}{x^4}+\frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4a^2d}$$

Mathematica [A] time = 0.074456, size = 38, normalized size = 0.69

$$\frac{\csc^4(c+dx)(8\sin(c+dx)+3\cos(2(c+dx))-6)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^4*(-6 + 3*Cos[2*(c + d*x)] + 8*Sin[c + d*x]))/(12*a^2*d)
```

Maple [A] time = 0.092, size = 39, normalized size = 0.7

$$\frac{1}{da^2}\left(-\frac{1}{4\left(\sin(dx+c)\right)^4}+\frac{2}{3\left(\sin(dx+c)\right)^3}-\frac{1}{2\left(\sin(dx+c)\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/4/sin(d*x+c)^4+2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A] time = 0.955802, size = 49, normalized size = 0.89

$$-\frac{6\sin(dx+c)^2-8\sin(dx+c)+3}{12a^2d\sin(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

Fricas [A] time = 1.4656, size = 138, normalized size = 2.51

$$\frac{6\,\cos\left(dx+c\right)^{2}+8\,\sin\left(dx+c\right)-9}{12\,\left(a^{2}d\cos\left(dx+c\right)^{4}-2\,a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*cos(d*x + c)^2 + 8*sin(d*x + c) - 9)/(a^2*d*cos(d*x + c)^4 - 2*a^2*
d*cos(d*x + c)^2 + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\cot^{5}\left(c+dx\right)}{\sin^{2}\left(c+dx\right)+2\sin\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.9624, size = 49, normalized size = 0.89

$$-\frac{6\,\sin\left(dx+c\right)^{2}-8\,\sin\left(dx+c\right)+3}{12\,a^{2}d\sin\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

3.68 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{2a^2d}$$

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5
)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)
```

Rubi [A] time = 0.0571898, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 75}

$$-\frac{\csc^6(c+dx)}{6a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5
)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3(a+x)}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4}{x^7} - \frac{2a^3}{x^6} + \frac{2a}{x^4} - \frac{1}{x^3}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^2(c+dx)}{2a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{6a^2d}$$

Mathematica [A] time = 0.07344, size = 73, normalized size = 1.

$$-\frac{\csc^6(c+dx)}{6a^2d} + \frac{2\csc^5(c+dx)}{5a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) + (2*Csc[c + d*x]^5
)/(5*a^2*d) - Csc[c + d*x]^6/(6*a^2*d)
```

Maple [A] time = 0.108, size = 49, normalized size = 0.7

$$\frac{1}{da^2}\left(\frac{2}{5\,(\sin(dx+c))^5} - \frac{1}{6\,(\sin(dx+c))^6} - \frac{2}{3\,(\sin(dx+c))^3} + \frac{1}{2\,(\sin(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(2/5/sin(d*x+c)^5-1/6/sin(d*x+c)^6-2/3/sin(d*x+c)^3+1/2/sin(d*x+c)^
2)
```

Maxima [A] time = 1.07194, size = 62, normalized size = 0.85

$$\frac{15\,\sin(dx+c)^4 - 20\,\sin(dx+c)^3 + 12\,\sin(dx+c) - 5}{30\,a^2d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*s
in(d*x + c)^6)
```

Fricas [A] time = 1.40071, size = 234, normalized size = 3.21

$$-\frac{15\,\cos\left(dx+c\right)^{4}-30\,\cos\left(dx+c\right)^{2}+4\left(5\,\cos\left(dx+c\right)^{2}-2\right)\sin\left(dx+c\right)+10}{30\left(a^{2}d\cos\left(dx+c\right)^{6}-3\,a^{2}d\cos\left(dx+c\right)^{4}+3\,a^{2}d\cos\left(dx+c\right)^{2}-a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(15*cos(d*x + c)^4 - 30*cos(d*x + c)^2 + 4*(5*cos(d*x + c)^2 - 2)*sin
(d*x + c) + 10)/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*co
s(d*x + c)^2 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.36406, size = 62, normalized size = 0.85

$$\frac{15\,\sin\left(dx+c\right)^{4}-20\,\sin\left(dx+c\right)^{3}+12\,\sin\left(dx+c\right)-5}{30\,a^{2}d\sin\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*s
in(d*x + c)^6)
```

3.69 $\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=127

$$-\frac{\csc^8(c+dx)}{8a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}+\frac{\csc^6(c+dx)}{6a^2d}-\frac{4\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) + Csc[c + d*x]^4/(
4*a^2*d) - (4*Csc[c + d*x]^5)/(5*a^2*d) + Csc[c + d*x]^6/(6*a^2*d) + (2*Csc
[c + d*x]^7)/(7*a^2*d) - Csc[c + d*x]^8/(8*a^2*d)
```

Rubi [A] time = 0.0738176, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^8(c+dx)}{8a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}+\frac{\csc^6(c+dx)}{6a^2d}-\frac{4\csc^5(c+dx)}{5a^2d}+\frac{\csc^4(c+dx)}{4a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) + Csc[c + d*x]^4/(
4*a^2*d) - (4*Csc[c + d*x]^5)/(5*a^2*d) + Csc[c + d*x]^6/(6*a^2*d) + (2*Csc
[c + d*x]^7)/(7*a^2*d) - Csc[c + d*x]^8/(8*a^2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^4(a+x)^2}{x^9}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^6}{x^9} - \frac{2a^5}{x^8} - \frac{a^4}{x^7} + \frac{4a^3}{x^6} - \frac{a^2}{x^5} - \frac{2a}{x^4} + \frac{1}{x^3}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2\csc^3(c+dx)}{3a^2d} + \frac{\csc^4(c+dx)}{4a^2d} - \frac{4\csc^5(c+dx)}{5a^2d} + \frac{\csc^6(c+dx)}{6a^2d} + \frac{2\csc^7(c}{7a^2}$$

Mathematica [A] time = 0.142755, size = 78, normalized size = 0.61

$$\frac{\csc^2(c+dx)\left(-105\csc^6(c+dx) + 240\csc^5(c+dx) + 140\csc^4(c+dx) - 672\csc^3(c+dx) + 210\csc^2(c+dx) + 560\csc(\right.}{840a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^2*(-420 + 560*Csc[c + d*x] + 210*Csc[c + d*x]^2 - 672*Csc[c +
 d*x]^3 + 140*Csc[c + d*x]^4 + 240*Csc[c + d*x]^5 - 105*Csc[c + d*x]^6))/(8
40*a^2*d)
```

Maple [A] time = 0.125, size = 79, normalized size = 0.6

$$\frac{1}{da^2}\left(\frac{2}{7\left(\sin(dx+c)\right)^7} - \frac{1}{8\left(\sin(dx+c)\right)^8} - \frac{4}{5\left(\sin(dx+c)\right)^5} + \frac{1}{4\left(\sin(dx+c)\right)^4} + \frac{1}{6\left(\sin(dx+c)\right)^6} + \frac{2}{3\left(\sin(dx+c)\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(2/7/sin(d*x+c)^7-1/8/sin(d*x+c)^8-4/5/sin(d*x+c)^5+1/4/sin(d*x+c)^
4+1/6/sin(d*x+c)^6+2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.76225, size = 103, normalized size = 0.81

$$-\frac{420\sin(dx+c)^6 - 560\sin(dx+c)^5 - 210\sin(dx+c)^4 + 672\sin(dx+c)^3 - 140\sin(dx+c)^2 - 240\sin(dx+c) + }{840\,a^2d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672*
sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(d*
x + c)^8)
```

Fricas [A] time = 1.44614, size = 331, normalized size = 2.61

$$\frac{420 \cos(dx+c)^6 - 1050 \cos(dx+c)^4 + 700 \cos(dx+c)^2 + 16\left(35 \cos(dx+c)^4 - 28 \cos(dx+c)^2 + 8\right)\sin(dx+c) - 1}{840\left(a^2 d \cos(dx+c)^8 - 4\,a^2 d \cos(dx+c)^6 + 6\,a^2 d \cos(dx+c)^4 - 4\,a^2 d \cos(dx+c)^2 + a^2 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(420*cos(d*x + c)^6 - 1050*cos(d*x + c)^4 + 700*cos(d*x + c)^2 + 16*(
35*cos(d*x + c)^4 - 28*cos(d*x + c)^2 + 8)*sin(d*x + c) - 175)/(a^2*d*cos(d
*x + c)^8 - 4*a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4 - 4*a^2*d*cos(d
*x + c)^2 + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.22548, size = 103, normalized size = 0.81

$$-\frac{420 \sin(dx+c)^6 - 560 \sin(dx+c)^5 - 210 \sin(dx+c)^4 + 672 \sin(dx+c)^3 - 140 \sin(dx+c)^2 - 240 \sin(dx+c) + 1}{840\,a^2 d \sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672*
sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(d*
x + c)^8)
```

3.70 $\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2} dx$

Optimal. Leaf size=145

$$-\frac{\csc^{10}(c+dx)}{10a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}+\frac{\csc^8(c+dx)}{4a^2d}-\frac{6\csc^7(c+dx)}{7a^2d}+\frac{6\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{2a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+$$

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(2
*a^2*d) + (6*Csc[c + d*x]^5)/(5*a^2*d) - (6*Csc[c + d*x]^7)/(7*a^2*d) + Csc
[c + d*x]^8/(4*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d) - Csc[c + d*x]^10/(10*
a^2*d)
```

Rubi [A] time = 0.0811955, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^{10}(c+dx)}{10a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}+\frac{\csc^8(c+dx)}{4a^2d}-\frac{6\csc^7(c+dx)}{7a^2d}+\frac{6\csc^5(c+dx)}{5a^2d}-\frac{\csc^4(c+dx)}{2a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] Csc[c + d*x]^2/(2*a^2*d) - (2*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(2
*a^2*d) + (6*Csc[c + d*x]^5)/(5*a^2*d) - (6*Csc[c + d*x]^7)/(7*a^2*d) + Csc
[c + d*x]^8/(4*a^2*d) + (2*Csc[c + d*x]^9)/(9*a^2*d) - Csc[c + d*x]^10/(10*
a^2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^5(a+x)^3}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^8}{x^{11}} - \frac{2a^7}{x^{10}} - \frac{2a^6}{x^9} + \frac{6a^5}{x^8} - \frac{6a^3}{x^6} + \frac{2a^2}{x^5} + \frac{2a}{x^4} - \frac{1}{x^3}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^2(c+dx)}{2a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx)}{2a^2d} + \frac{6\csc^5(c+dx)}{5a^2d} - \frac{6\csc^7(c+dx)}{7a^2d} + \frac{\csc^8(c+}{4a^2}$$

Mathematica [A] time = 0.205023, size = 88, normalized size = 0.61

$$\frac{\csc^2(c+dx)\left(-126\csc^8(c+dx) + 280\csc^7(c+dx) + 315\csc^6(c+dx) - 1080\csc^5(c+dx) + 1512\csc^3(c+dx) - 630\,\text{cs}\right.}{1260a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] (Csc[c + d*x]^2*(630 - 840*Csc[c + d*x] - 630*Csc[c + d*x]^2 + 1512*Csc[c +
 d*x]^3 - 1080*Csc[c + d*x]^5 + 315*Csc[c + d*x]^6 + 280*Csc[c + d*x]^7 - 1
26*Csc[c + d*x]^8))/(1260*a^2*d)
```

Maple [A] time = 0.139, size = 89, normalized size = 0.6

$$\frac{1}{da^2}\left(-\frac{1}{10\,(\sin(dx+c))^{10}} - \frac{6}{7\,(\sin(dx+c))^7} + \frac{1}{4\,(\sin(dx+c))^8} + \frac{6}{5\,(\sin(dx+c))^5} - \frac{1}{2\,(\sin(dx+c))^4} + \frac{2}{9\,(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/10/sin(d*x+c)^10-6/7/sin(d*x+c)^7+1/4/sin(d*x+c)^8+6/5/sin(d*x+
c)^5-1/2/sin(d*x+c)^4+2/9/sin(d*x+c)^9-2/3/sin(d*x+c)^3+1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.83669, size = 116, normalized size = 0.8

$$\frac{630 \sin(dx+c)^8 - 840 \sin(dx+c)^7 - 630 \sin(dx+c)^6 + 1512 \sin(dx+c)^5 - 1080 \sin(dx+c)^3 + 315 \sin(dx+c)^2 +}{1260\, a^2 d \sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 1512
*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x +
c) - 126)/(a^2*d*sin(d*x + c)^10)
```

Fricas [A] time = 1.67582, size = 431, normalized size = 2.97

$$-\frac{630 \cos(dx+c)^8 - 1890 \cos(dx+c)^6 + 1890 \cos(dx+c)^4 - 945 \cos(dx+c)^2 + 8\left(105 \cos(dx+c)^6 - 126 \cos(dx+\right.}{1260\left(a^2 d \cos(dx+c)^{10} - 5\, a^2 d \cos(dx+c)^8 + 10\, a^2 d \cos(dx+c)^6 - 10\, a^2 d \cos(dx+c)^4 + 5\, a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/1260*(630*cos(d*x + c)^8 - 1890*cos(d*x + c)^6 + 1890*cos(d*x + c)^4 - 9
45*cos(d*x + c)^2 + 8*(105*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d*x
 + c)^2 - 16)*sin(d*x + c) + 189)/(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d*x
+ c)^8 + 10*a^2*d*cos(d*x + c)^6 - 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*cos(d*
x + c)^2 - a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**11/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.93118, size = 116, normalized size = 0.8

$$\frac{630 \sin(dx+c)^8 - 840 \sin(dx+c)^7 - 630 \sin(dx+c)^6 + 1512 \sin(dx+c)^5 - 1080 \sin(dx+c)^3 + 315 \sin(dx+c)^2}{1260 a^2 d \sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 1512
*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x +
c) - 126)/(a^2*d*sin(d*x + c)^10)
```

3.71 $\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

Optimal. Leaf size=199

$$-\frac{\csc^{12}(c+dx)}{12a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}+\frac{3\csc^{10}(c+dx)}{10a^2d}-\frac{8\csc^9(c+dx)}{9a^2d}-\frac{\csc^8(c+dx)}{4a^2d}+\frac{12\csc^7(c+dx)}{7a^2d}-\frac{\csc^6(c+dx)}{3a^2d}-$$

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) + (3*Csc[c + d*x]^
4)/(4*a^2*d) - (8*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(3*a^2*d) + (1
2*Csc[c + d*x]^7)/(7*a^2*d) - Csc[c + d*x]^8/(4*a^2*d) - (8*Csc[c + d*x]^9)
/(9*a^2*d) + (3*Csc[c + d*x]^10)/(10*a^2*d) + (2*Csc[c + d*x]^11)/(11*a^2*d
) - Csc[c + d*x]^12/(12*a^2*d)
```

Rubi [A] time = 0.102394, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^{12}(c+dx)}{12a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}+\frac{3\csc^{10}(c+dx)}{10a^2d}-\frac{8\csc^9(c+dx)}{9a^2d}-\frac{\csc^8(c+dx)}{4a^2d}+\frac{12\csc^7(c+dx)}{7a^2d}-\frac{\csc^6(c+dx)}{3a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -Csc[c + d*x]^2/(2*a^2*d) + (2*Csc[c + d*x]^3)/(3*a^2*d) + (3*Csc[c + d*x]^
4)/(4*a^2*d) - (8*Csc[c + d*x]^5)/(5*a^2*d) - Csc[c + d*x]^6/(3*a^2*d) + (1
2*Csc[c + d*x]^7)/(7*a^2*d) - Csc[c + d*x]^8/(4*a^2*d) - (8*Csc[c + d*x]^9)
/(9*a^2*d) + (3*Csc[c + d*x]^10)/(10*a^2*d) + (2*Csc[c + d*x]^11)/(11*a^2*d
) - Csc[c + d*x]^12/(12*a^2*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^6(a+x)^4}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^{10}}{x^{13}} - \frac{2a^9}{x^{12}} - \frac{3a^8}{x^{11}} + \frac{8a^7}{x^{10}} + \frac{2a^6}{x^9} - \frac{12a^5}{x^8} + \frac{2a^4}{x^7} + \frac{8a^3}{x^6} - \frac{3a^2}{x^5} - \frac{2a}{x^4} + \frac{1}{x^3}\right) dx, x, a\sin(c+d$$

$$= -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2\csc^3(c+dx)}{3a^2d} + \frac{3\csc^4(c+dx)}{4a^2d} - \frac{8\csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{3a^2d} + \frac{12\csc$$

Mathematica [A] time = 0.329218, size = 118, normalized size = 0.59

$$-\frac{\csc^2(c+dx)\left(1155\csc^{10}(c+dx) - 2520\csc^9(c+dx) - 4158\csc^8(c+dx) + 12320\csc^7(c+dx) + 3465\csc^6(c+dx) - \right.}{13860a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^2*(6930 - 9240*Csc[c + d*x] - 10395*Csc[c + d*x]^2 + 22176*C
sc[c + d*x]^3 + 4620*Csc[c + d*x]^4 - 23760*Csc[c + d*x]^5 + 3465*Csc[c + d
*x]^6 + 12320*Csc[c + d*x]^7 - 4158*Csc[c + d*x]^8 - 2520*Csc[c + d*x]^9 +
1155*Csc[c + d*x]^10))/(13860*a^2*d)
```

Maple [A] time = 0.165, size = 119, normalized size = 0.6

$$\frac{1}{da^2}\left(\frac{3}{10\,(\sin(dx+c))^{10}} + \frac{12}{7\,(\sin(dx+c))^7} + \frac{2}{11\,(\sin(dx+c))^{11}} - \frac{1}{4\,(\sin(dx+c))^8} - \frac{8}{5\,(\sin(dx+c))^5} + \frac{3}{4\,(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(3/10/sin(d*x+c)^10+12/7/sin(d*x+c)^7+2/11/sin(d*x+c)^11-1/4/sin(d*
x+c)^8-8/5/sin(d*x+c)^5+3/4/sin(d*x+c)^4-8/9/sin(d*x+c)^9-1/12/sin(d*x+c)^1
```

```
2-1/3/sin(d*x+c)^6+2/3/sin(d*x+c)^3-1/2/sin(d*x+c)^2)
```

Maxima [A] time = 1.83541, size = 157, normalized size = 0.79

$$-\frac{6930 \sin(dx+c)^{10} - 9240 \sin(dx+c)^{9} - 10395 \sin(dx+c)^{8} + 22176 \sin(dx+c)^{7} + 4620 \sin(dx+c)^{6} - 23760 \sin}{13860\, a^2 d \sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^8
 + 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 3465
*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(d*x
 + c) + 1155)/(a^2*d*sin(d*x + c)^12)
```

Fricas [A] time = 1.65369, size = 541, normalized size = 2.72

$$\frac{6930 \cos(dx+c)^{10} - 24255 \cos(dx+c)^{8} + 32340 \cos(dx+c)^{6} - 24255 \cos(dx+c)^{4} + 9702 \cos(dx+c)^{2} + 8\left(1155 \cos\right.}{13860\left(a^2 d \cos(dx+c)^{12} - 6\, a^2 d \cos(dx+c)^{10} + 15\, a^2 d \cos(dx+c)^{8} - 20\, a^2 d \cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/13860*(6930*cos(d*x + c)^10 - 24255*cos(d*x + c)^8 + 32340*cos(d*x + c)^6
 - 24255*cos(d*x + c)^4 + 9702*cos(d*x + c)^2 + 8*(1155*cos(d*x + c)^8 - 18
48*cos(d*x + c)^6 + 1584*cos(d*x + c)^4 - 704*cos(d*x + c)^2 + 128)*sin(d*x
 + c) - 1617)/(a^2*d*cos(d*x + c)^12 - 6*a^2*d*cos(d*x + c)^10 + 15*a^2*d*c
os(d*x + c)^8 - 20*a^2*d*cos(d*x + c)^6 + 15*a^2*d*cos(d*x + c)^4 - 6*a^2*d
*cos(d*x + c)^2 + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.1608, size = 157, normalized size = 0.79

$$-\frac{6930 \sin(dx+c)^{10} - 9240 \sin(dx+c)^{9} - 10395 \sin(dx+c)^{8} + 22176 \sin(dx+c)^{7} + 4620 \sin(dx+c)^{6} - 23760 \sin}{13860\, a^{2} d \sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^8
 + 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 3465
*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(d*x
 + c) + 1155)/(a^2*d*sin(d*x + c)^12)
```

3.72 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3} dx$

Optimal. Leaf size=171

$$\frac{a^2}{40d(a\sin(c+dx)+a)^5} - \frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)} - \frac{5}{128d\left(a^3\sin(c+dx)+a^3\right)} + \frac{\tanh^{-1}(\sin(c+dx))}{128a^3d} - \frac{7a}{64d(a\sin(c+}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2) + a^
2/(40*d*(a + a*Sin[c + d*x])^5) - (7*a)/(64*d*(a + a*Sin[c + d*x])^4) + 1/(
6*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) - 1/(32*d*(
a^3 - a^3*Sin[c + d*x])) - 5/(128*d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.123351, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{a^2}{40d(a\sin(c+dx)+a)^5} - \frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)} - \frac{5}{128d\left(a^3\sin(c+dx)+a^3\right)} + \frac{\tanh^{-1}(\sin(c+dx))}{128a^3d} - \frac{7a}{64d(a\sin(c+}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(128*a^3*d) + 1/(128*a*d*(a - a*Sin[c + d*x])^2) + a^
2/(40*d*(a + a*Sin[c + d*x])^5) - (7*a)/(64*d*(a + a*Sin[c + d*x])^4) + 1/(
6*d*(a + a*Sin[c + d*x])^3) - 5/(64*a*d*(a + a*Sin[c + d*x])^2) - 1/(32*d*(
a^3 - a^3*Sin[c + d*x])) - 5/(128*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{64a(a-x)^3} - \frac{1}{32a^2(a-x)^2} - \frac{a^2}{8(a+x)^6} + \frac{7a}{16(a+x)^5} - \frac{1}{2(a+x)^4} + \frac{5}{32a(a+x)^3} + \frac{5}{128a^2(a+x)^2} + \frac{}{128a^2}\right.\right.}{d}$$

$$= \frac{1}{128ad(a-a\sin(c+dx))^2} + \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{7a}{64d(a+a\sin(c+dx))^4} + \frac{}{6d(a+a}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{128a^3d} + \frac{1}{128ad(a-a\sin(c+dx))^2} + \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{7}{64d(a+a\sin}$$

Mathematica [A] time = 0.925661, size = 102, normalized size = 0.6

$$\frac{15\tanh^{-1}(\sin(c+dx)) - \frac{15\sin^6(c+dx)+45\sin^5(c+dx)-620\sin^4(c+dx)-540\sin^3(c+dx)+157\sin^2(c+dx)+351\sin(c+dx)+112}{(\sin(c+dx)-1)^2(\sin(c+dx)+1)^5}}{1920a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (15*ArcTanh[Sin[c + d*x]] - (112 + 351*Sin[c + d*x] + 157*Sin[c + d*x]^2 -
540*Sin[c + d*x]^3 - 620*Sin[c + d*x]^4 + 45*Sin[c + d*x]^5 + 15*Sin[c + d*
x]^6)/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^5))/(1920*a^3*d)
```

Maple [A] time = 0.106, size = 162, normalized size = 1.

$$\frac{1}{128\,da^3\,(\sin(dx+c)-1)^2} + \frac{1}{32\,da^3\,(\sin(dx+c)-1)} - \frac{\ln(\sin(dx+c)-1)}{256\,da^3} + \frac{1}{40\,da^3\,(1+\sin(dx+c))^5} - \frac{}{64\,da^3\,(1+s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/128/d/a^3/(sin(d*x+c)-1)^2+1/32/d/a^3/(sin(d*x+c)-1)-1/256/d/a^3*ln(sin(d
*x+c)-1)+1/40/d/a^3/(1+sin(d*x+c))^5-7/64/d/a^3/(1+sin(d*x+c))^4+1/6/d/a^3/
(1+sin(d*x+c))^3-5/64/d/a^3/(1+sin(d*x+c))^2-5/128/d/a^3/(1+sin(d*x+c))+1/2
56*ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.96124, size = 254, normalized size = 1.49

$$-\frac{\frac{2\left(15\sin(dx+c)^6+45\sin(dx+c)^5-620\sin(dx+c)^4-540\sin(dx+c)^3+157\sin(dx+c)^2+351\sin(dx+c)+112\right)}{a^3\sin(dx+c)^7+3a^3\sin(dx+c)^6+a^3\sin(dx+c)^5-5a^3\sin(dx+c)^4-5a^3\sin(dx+c)^3+a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3}-\frac{15\log(\sin(dx+c)+1)}{a^3}+\frac{15\log(\sin(dx\cdots}{a^3}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3840*(2*(15*sin(d*x + c)^6 + 45*sin(d*x + c)^5 - 620*sin(d*x + c)^4 - 54
0*sin(d*x + c)^3 + 157*sin(d*x + c)^2 + 351*sin(d*x + c) + 112)/(a^3*sin(d*
x + c)^7 + 3*a^3*sin(d*x + c)^6 + a^3*sin(d*x + c)^5 - 5*a^3*sin(d*x + c)^4
 - 5*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) -
15*log(sin(d*x + c) + 1)/a^3 + 15*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [A] time = 1.67077, size = 657, normalized size = 3.84

$$-\frac{30\cos(dx+c)^6+1150\cos(dx+c)^4-2076\cos(dx+c)^2-15\left(3\cos(dx+c)^6-4\cos(dx+c)^4+\left(\cos(dx+c)^6-4\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3840*(30*cos(d*x + c)^6 + 1150*cos(d*x + c)^4 - 2076*cos(d*x + c)^2 - 15
*(3*cos(d*x + c)^6 - 4*cos(d*x + c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)
*sin(d*x + c))*log(sin(d*x + c) + 1) + 15*(3*cos(d*x + c)^6 - 4*cos(d*x + c
)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x + c) +
 1) - 18*(5*cos(d*x + c)^4 + 50*cos(d*x + c)^2 - 16)*sin(d*x + c) + 672)/(3
```

```
*a^3*d*cos(d*x + c)^6 - 4*a^3*d*cos(d*x + c)^4 + (a^3*d*cos(d*x + c)^6 - 4*
a^3*d*cos(d*x + c)^4)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.82376, size = 184, normalized size = 1.08

$$\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^3} - \frac{60\log(|\sin(dx+c)-1|)}{a^3} + \frac{30\left(3\sin(dx+c)^2+10\sin(dx+c)-9\right)}{a^3(\sin(dx+c)-1)^2} - \frac{137\sin(dx+c)^5+1285\sin(dx+c)^4+4970\sin(dx+c)^3+6010\sin(dx}{a^3(\sin(dx+c)+1)^5}}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15360*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c) - 1))/
a^3 + 30*(3*sin(d*x + c)^2 + 10*sin(d*x + c) - 9)/(a^3*(sin(d*x + c) - 1)^2
) - (137*sin(d*x + c)^5 + 1285*sin(d*x + c)^4 + 4970*sin(d*x + c)^3 + 6010*
sin(d*x + c)^2 + 3245*sin(d*x + c) + 673)/(a^3*(sin(d*x + c) + 1)^5))/d
```

3.73 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}+\frac{1}{16d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d}+\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{1}{6d(a\sin(c+dx}$$

```
[Out] -ArcTanh[Sin[c + d*x]]/(32*a^3*d) + a/(16*d*(a + a*Sin[c + d*x])^4) - 1/(6*
d*(a + a*Sin[c + d*x])^3) + 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d*(a^
3 - a^3*Sin[c + d*x])) + 1/(16*d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.090512, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)}+\frac{1}{16d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d}+\frac{a}{16d(a\sin(c+dx)+a)^4}-\frac{1}{6d(a\sin(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -ArcTanh[Sin[c + d*x]]/(32*a^3*d) + a/(16*d*(a + a*Sin[c + d*x])^4) - 1/(6*
d*(a + a*Sin[c + d*x])^3) + 3/(32*a*d*(a + a*Sin[c + d*x])^2) + 1/(32*d*(a^
3 - a^3*Sin[c + d*x])) + 1/(16*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{32a^2(a-x)^2} - \frac{a}{4(a+x)^5} + \frac{1}{2(a+x)^4} - \frac{3}{16a(a+x)^3} - \frac{1}{16a^2(a+x)^2} - \frac{1}{32a^2\left(a^2-x^2\right)}\right) dx, x, a\sin(c+\right.}{d}$$

$$= \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{6d(a+a\sin(c+dx))^3} + \frac{3}{32ad(a+a\sin(c+dx))^2} + \frac{}{32d\left(a^3-a\right.}$$

$$= -\frac{\tanh^{-1}(\sin(c+dx))}{32a^3d} + \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{6d(a+a\sin(c+dx))^3} + \frac{3}{32ad(a+a\sin}$$

Mathematica [A] time = 0.363215, size = 82, normalized size = 0.65

$$-\frac{-\frac{3}{1-\sin(c+dx)} - \frac{6}{\sin(c+dx)+1} - \frac{9}{(\sin(c+dx)+1)^2} + \frac{16}{(\sin(c+dx)+1)^3} - \frac{6}{(\sin(c+dx)+1)^4} + 3\tanh^{-1}(\sin(c+dx))}{96a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(3*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 6/(1 + Sin[c + d*x])^4 +
 16/(1 + Sin[c + d*x])^3 - 9/(1 + Sin[c + d*x])^2 - 6/(1 + Sin[c + d*x]))/(
96*a^3*d)
```

Maple [A] time = 0.092, size = 126, normalized size = 1.

$$-\frac{1}{32\,da^3(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)}{64\,da^3} + \frac{1}{16\,da^3(1+\sin(dx+c))^4} - \frac{1}{6\,da^3(1+\sin(dx+c))^3} + \frac{3}{32\,da^3(1+\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/32/d/a^3/(sin(d*x+c)-1)+1/64/d/a^3*ln(sin(d*x+c)-1)+1/16/d/a^3/(1+sin(d*
x+c))^4-1/6/d/a^3/(1+sin(d*x+c))^3+3/32/d/a^3/(1+sin(d*x+c))^2+1/16/d/a^3/(
1+sin(d*x+c))-1/64*ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.93841, size = 197, normalized size = 1.56

$$\frac{\frac{2\left(3\sin(dx+c)^4+9\sin(dx+c)^3-25\sin(dx+c)^2-27\sin(dx+c)-8\right)}{a^3\sin(dx+c)^5+3a^3\sin(dx+c)^4+2a^3\sin(dx+c)^3-2a^3\sin(dx+c)^2-3a^3\sin(dx+c)-a^3}-\frac{3\log(\sin(dx+c)+1)}{a^3}+\frac{3\log(\sin(dx+c)-1)}{a^3}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(2*(3*sin(d*x + c)^4 + 9*sin(d*x + c)^3 - 25*sin(d*x + c)^2 - 27*sin(
d*x + c) - 8)/(a^3*sin(d*x + c)^5 + 3*a^3*sin(d*x + c)^4 + 2*a^3*sin(d*x +
c)^3 - 2*a^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - a^3) - 3*log(sin(d*x + c
) + 1)/a^3 + 3*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [A] time = 1.58082, size = 586, normalized size = 4.65

$$\frac{6\cos(dx+c)^4+38\cos(dx+c)^2-3\left(3\cos(dx+c)^4-4\cos(dx+c)^2+\left(\cos(dx+c)^4-4\cos(dx+c)^2\right)\sin(dx+c)\right)\cdots}{192\left(3a^3d\cos(dx+c)^4-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(6*cos(d*x + c)^4 + 38*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^4 - 4*cos(d
*x + c)^2 + (cos(d*x + c)^4 - 4*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x +
 c) + 1) + 3*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4 - 4*cos
(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 18*(cos(d*x + c)^2 + 2)
*sin(d*x + c) - 60)/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + (a^3
*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 2.55233, size = 154, normalized size = 1.22

$$-\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^3}-\frac{12\log(|\sin(dx+c)-1|)}{a^3}+\frac{12(\sin(dx+c)+1)}{a^3(\sin(dx+c)-1)}-\frac{25\sin(dx+c)^4+148\sin(dx+c)^3+366\sin(dx+c)^2+260\sin(dx+c)+65}{a^3(\sin(dx+c)+1)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/768*(12*log(abs(sin(d*x + c) + 1))/a^3 - 12*log(abs(sin(d*x + c) - 1))/a
^3 + 12*(sin(d*x + c) + 1)/(a^3*(sin(d*x + c) - 1)) - (25*sin(d*x + c)^4 +
148*sin(d*x + c)^3 + 366*sin(d*x + c)^2 + 260*sin(d*x + c) + 65)/(a^3*(sin(
d*x + c) + 1)^4))/d
```

3.74 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}+\frac{1}{6d(a\sin(c+dx)+a)^3}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) + 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d
*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.0571133, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2707, 77, 206}

$$-\frac{1}{8d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{1}{8ad(a\sin(c+dx)+a)^2}+\frac{1}{6d(a\sin(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(8*a^3*d) + 1/(6*d*(a + a*Sin[c + d*x])^3) - 1/(8*a*d
*(a + a*Sin[c + d*x])^2) - 1/(8*d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^4}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^4}+\frac{1}{4a(a+x)^3}+\frac{1}{8a^2(a+x)^2}+\frac{1}{8a^2(a^2-x^2)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\sin(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{}{a^2}\right.}{}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{8a^3d}+\frac{1}{6d(a+a\sin(c+dx))^3}-\frac{1}{8ad(a+a\sin(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\sin(\right.}$$

Mathematica [A] time = 0.145046, size = 52, normalized size = 0.63

$$\frac{3\tanh^{-1}(\sin(c+dx))-\frac{3\sin^2(c+dx)+9\sin(c+dx)+2}{(\sin(c+dx)+1)^3}}{24a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]] - (2 + 9*Sin[c + d*x] + 3*Sin[c + d*x]^2)/(1 + Sin
[c + d*x])^3)/(24*a^3*d)
```

Maple [A] time = 0.095, size = 90, normalized size = 1.1

$$-\frac{\ln(\sin(dx+c)-1)}{16\,da^3}+\frac{1}{6\,da^3(1+\sin(dx+c))^3}-\frac{1}{8\,da^3(1+\sin(dx+c))^2}-\frac{1}{8\,da^3(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx}{16\,da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -1/16/d/a^3*ln(sin(d*x+c)-1)+1/6/d/a^3/(1+sin(d*x+c))^3-1/8/d/a^3/(1+sin(d*
x+c))^2-1/8/d/a^3/(1+sin(d*x+c))+1/16*ln(1+sin(d*x+c))/a^3/d
```

Maxima [A] time = 2.14301, size = 132, normalized size = 1.61

$$-\frac{\frac{2\left(3 \sin(dx+c)^2+9 \sin(dx+c)+2\right)}{a^3 \sin(dx+c)^3+3 a^3 \sin(dx+c)^2+3 a^3 \sin(dx+c)+a^3}-\frac{3 \log(\sin(dx+c)+1)}{a^3}+\frac{3 \log(\sin(dx+c)-1)}{a^3}}{48 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(3*sin(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3 + 3*a^
3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) - 3*log(sin(d*x + c) + 1)/a^3
+ 3*log(sin(d*x + c) - 1)/a^3)/d
```

Fricas [B] time = 1.55306, size = 409, normalized size = 4.99

$$-\frac{6 \cos(dx+c)^2-3\left(3 \cos(dx+c)^2+\left(\cos(dx+c)^2-4\right)\sin(dx+c)-4\right)\log(\sin(dx+c)+1)+3\left(3 \cos(dx+c)^2+\left(\right.\right.}{48\left(3 a^3 d \cos(dx+c)^2-4 a^3 d+\left(a^3 d \cos(dx+c)^2-4 a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(6*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d*
x + c) - 4)*log(sin(d*x + c) + 1) + 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 -
 4)*sin(d*x + c) - 4)*log(-sin(d*x + c) + 1) - 18*sin(d*x + c) - 10)/(3*a^3
*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 4*a^3*d)*sin(d*x + c)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x)
 + 1), x)/a**3
```

Giac [A] time = 2.18984, size = 109, normalized size = 1.33

$$\frac{\frac{6 \log\left(\left|\sin\left(dx+c\right)+1\right|\right)}{a^3} - \frac{6 \log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{a^3} - \frac{11 \sin\left(dx+c\right)^3+45 \sin\left(dx+c\right)^2+69 \sin\left(dx+c\right)+19}{a^3\left(\sin\left(dx+c\right)+1\right)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c) - 1))/a^3 -
 (11*sin(d*x + c)^3 + 45*sin(d*x + c)^2 + 69*sin(d*x + c) + 19)/(a^3*(sin(d
*x + c) + 1)^3))/d
```

3.75 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=74

$$\frac{1}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.0582604, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 44}

$$\frac{1}{d\left(a^3\sin(c+dx)+a^3\right)}+\frac{\log(\sin(c+dx))}{a^3d}-\frac{\log(\sin(c+dx)+1)}{a^3d}+\frac{1}{2ad(a\sin(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[1 + Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a + a
*Sin[c + d*x])^2) + 1/(d*(a^3 + a^3*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(1+\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{1}{d\left(a^3+a^3\sin(c+dx\right.}$$

Mathematica [A] time = 0.175268, size = 52, normalized size = 0.7

$$\frac{\frac{2\sin(c+dx)+3}{(\sin(c+dx)+1)^2} + 2\log(\sin(c+dx)) - 2\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (3 + 2*Sin[c + d*x])/(1 +
Sin[c + d*x])^2)/(2*a^3*d)
```

Maple [A] time = 0.037, size = 68, normalized size = 0.9

$$\frac{1}{2\,da^3\,(1+\sin(dx+c))^2} + \frac{1}{da^3\,(1+\sin(dx+c))} - \frac{\ln(1+\sin(dx+c))}{da^3} + \frac{\ln(\sin(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3/(1+sin(d*x+c))^2+1/d/a^3/(1+sin(d*x+c))-ln(1+sin(d*x+c))/a^3/d+ln
(sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.76862, size = 97, normalized size = 1.31

$$\frac{\frac{2\sin(dx+c)+3}{a^3\sin(dx+c)^2+2\,a^3\sin(dx+c)+a^3} - \frac{2\log(\sin(dx+c)+1)}{a^3} + \frac{2\log(\sin(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((2*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3) -
 2*log(sin(d*x + c) + 1)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

Fricas [A] time = 1.6075, size = 284, normalized size = 3.84

$$\frac{2\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-2\left(\cos\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)-2}{2\left(a^{3}d\cos\left(dx+c\right)^{2}-2\,a^{3}d\sin\left(dx+c\right)-2\,a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(1/2*sin(d*x + c)) - 2*(cos
(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*sin(d*x + c) -
3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x)
 + 1), x)/a**3
```

Giac [A] time = 1.9801, size = 80, normalized size = 1.08

$$-\frac{\frac{2\,\log\left(|\sin\left(dx+c\right)+1|\right)}{a^{3}}-\frac{2\,\log\left(|\sin\left(dx+c\right)|\right)}{a^{3}}-\frac{2\,\sin\left(dx+c\right)+3}{a^{3}\left(\sin\left(dx+c\right)+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (2*
sin(d*x + c) + 3)/(a^3*(sin(d*x + c) + 1)^2))/d
```

3.76 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{5\log(\sin(c+dx))}{a^3d}-\frac{5\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (5*Log[Sin[c + d*x]])
/(a^3*d) - (5*Log[1 + Sin[c + d*x]])/(a^3*d) + 2/(d*(a^3 + a^3*Sin[c + d*x]
))
```

Rubi [A] time = 0.0697399, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$\frac{2}{d\left(a^3\sin(c+dx)+a^3\right)}-\frac{\csc^2(c+dx)}{2a^3d}+\frac{3\csc(c+dx)}{a^3d}+\frac{5\log(\sin(c+dx))}{a^3d}-\frac{5\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] (3*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^3*d) + (5*Log[Sin[c + d*x]])
/(a^3*d) - (5*Log[1 + Sin[c + d*x]])/(a^3*d) + 2/(d*(a^3 + a^3*Sin[c + d*x]
))
```


Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)^2}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3} - \frac{3}{a^2x^2} + \frac{5}{a^3x} - \frac{2}{a^2(a+x)^2} - \frac{5}{a^3(a+x)}\right) dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{3\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^3d} + \frac{5\log(\sin(c+dx))}{a^3d} - \frac{5\log(1+\sin(c+dx))}{a^3d} + \frac{2}{d\left(a^3+a^3\sin\right.}$$

Mathematica [A] time = 0.19094, size = 61, normalized size = 0.71

$$\frac{\frac{4}{\sin(c+dx)+1} - \csc^2(c+dx) + 6\csc(c+dx) + 10\log(\sin(c+dx)) - 10\log(\sin(c+dx)+1)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (6*Csc[c + d*x] - Csc[c + d*x]^2 + 10*Log[Sin[c + d*x]] - 10*Log[1 + Sin[c
+ d*x]] + 4/(1 + Sin[c + d*x]))/(2*a^3*d)
```

Maple [A] time = 0.129, size = 84, normalized size = 1.

$$2\,\frac{1}{da^3(1+\sin(dx+c))} - 5\,\frac{\ln(1+\sin(dx+c))}{da^3} - \frac{1}{2\,da^3(\sin(dx+c))^2} + 3\,\frac{1}{da^3\sin(dx+c)} + 5\,\frac{\ln(\sin(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3/(1+sin(d*x+c))-5*ln(1+sin(d*x+c))/a^3/d-1/2/d/a^3/sin(d*x+c)^2+3/d/
a^3/sin(d*x+c)+5*ln(sin(d*x+c))/a^3/d
```

Maxima [A] time = 1.95102, size = 108, normalized size = 1.26

$$\frac{\frac{10 \sin(dx+c)^2+5 \sin(dx+c)-1}{a^3 \sin(dx+c)^3+a^3 \sin(dx+c)^2} - \frac{10 \log(\sin(dx+c)+1)}{a^3} + \frac{10 \log(\sin(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((10*sin(d*x + c)^2 + 5*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^3 + a^3*sin
(d*x + c)^2) - 10*log(sin(d*x + c) + 1)/a^3 + 10*log(sin(d*x + c))/a^3)/d
```

Fricas [A] time = 1.54676, size = 393, normalized size = 4.57

$$\frac{10\,\cos\left(dx+c\right)^2 + 10\left(\cos\left(dx+c\right)^2 + \left(\cos\left(dx+c\right)^2 - 1\right)\sin\left(dx+c\right) - 1\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right) - 10\left(\cos\left(dx+c\right)^2 + \left(\cos\right.\right.}{2\left(a^3d\cos\left(dx+c\right)^2 - a^3d + \left(a^3d\cos\left(dx+c\right)^2 - a^3d\right)\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(10*cos(d*x + c)^2 + 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x
+ c) - 1)*log(1/2*sin(d*x + c)) - 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)
*sin(d*x + c) - 1)*log(sin(d*x + c) + 1) - 5*sin(d*x + c) - 9)/(a^3*d*cos(d
*x + c)^2 - a^3*d + (a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.73369, size = 208, normalized size = 2.42

$$-\frac{\frac{80\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{40\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^3}-\frac{30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+53\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^2a^3}+\frac{a^3\tan\cdots}{\cdots}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(80*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 40*log(abs(tan(1/2*d*x +
1/2*c)))/a^3 - (30*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 + 53*
tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/2*c
)^2 + tan(1/2*d*x + 1/2*c))^2*a^3) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a^3*t
an(1/2*d*x + 1/2*c))/a^6)/d
```

3.77 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=96

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

```
[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3
*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 +
 Sin[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.0712619, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{a^3d}-\frac{2\csc^2(c+dx)}{a^3d}+\frac{4\csc(c+dx)}{a^3d}+\frac{4\log(\sin(c+dx))}{a^3d}-\frac{4\log(\sin(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (4*Csc[c + d*x])/(a^3*d) - (2*Csc[c + d*x]^2)/(a^3*d) + Csc[c + d*x]^3/(a^3
*d) - Csc[c + d*x]^4/(4*a^3*d) + (4*Log[Sin[c + d*x]])/(a^3*d) - (4*Log[1 +
 Sin[c + d*x]])/(a^3*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2}{x^5(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{x^5} - \frac{3}{x^4} + \frac{4}{ax^3} - \frac{4}{a^2x^2} + \frac{4}{a^3x} - \frac{4}{a^3(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{4\csc(c+dx)}{a^3d} - \frac{2\csc^2(c+dx)}{a^3d} + \frac{\csc^3(c+dx)}{a^3d} - \frac{\csc^4(c+dx)}{4a^3d} + \frac{4\log(\sin(c+dx))}{a^3d} - \frac{4\log}{}$$

Mathematica [A] time = 0.327739, size = 69, normalized size = 0.72

$$\frac{-\csc^4(c+dx) + 4\csc^3(c+dx) - 8\csc^2(c+dx) + 16\csc(c+dx) + 16\log(\sin(c+dx)) - 16\log(\sin(c+dx)+1)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (16*Csc[c + d*x] - 8*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 - Csc[c + d*x]^4 + 1
6*Log[Sin[c + d*x]] - 16*Log[1 + Sin[c + d*x]])/(4*a^3*d)
```

Maple [A] time = 0.126, size = 97, normalized size = 1.

$$-4\,\frac{\ln(1+\sin(dx+c))}{a^3d} - \frac{1}{4\,a^3d(\sin(dx+c))^4} + \frac{1}{a^3d(\sin(dx+c))^3} - 2\,\frac{1}{a^3d(\sin(dx+c))^2} + 4\,\frac{1}{a^3d\sin(dx+c)} + 4\,\frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x)
```

```
[Out] -4*ln(1+sin(d*x+c))/a^3/d-1/4/d/a^3/sin(d*x+c)^4+1/d/a^3/sin(d*x+c)^3-2/d/a
^3/sin(d*x+c)^2+4/d/a^3/sin(d*x+c)+4*ln(sin(d*x+c))/a^3/d
```

Maxima [A] time = 2.05557, size = 101, normalized size = 1.05

$$-\frac{\frac{16\log(\sin(dx+c)+1)}{a^3} - \frac{16\log(\sin(dx+c))}{a^3} - \frac{16\sin(dx+c)^3 - 8\sin(dx+c)^2 + 4\sin(dx+c) - 1}{a^3\sin(dx+c)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(16*log(sin(d*x + c) + 1)/a^3 - 16*log(sin(d*x + c))/a^3 - (16*sin(d*x
 + c)^3 - 8*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^4))/d
```

Fricas [A] time = 1.65155, size = 348, normalized size = 3.62

$$\frac{8\cos\left(dx+c\right)^{2}+16\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-16\left(\cos\left(dx+c\right)^{4}-2\cos\left(dx+c\right)^{2}+1\right)\mathrm{lo}}{4\left(a^{3}d\cos\left(dx+c\right)^{4}-2\,a^{3}d\cos\left(dx+c\right)^{2}+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*cos(d*x + c)^2 + 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2*
sin(d*x + c)) - 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(sin(d*x + c)
 + 1) - 4*(4*cos(d*x + c)^2 - 5)*sin(d*x + c) - 9)/(a^3*d*cos(d*x + c)^4 -
2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^{5}\left(c+dx\right)}{\sin^{3}\left(c+dx\right)+3\sin^{2}\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 2.57501, size = 235, normalized size = 2.45

$$-\frac{\frac{1536 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{768 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{1600 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-456 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+108 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3}{a^3 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/192*(1536*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 768*log(abs(tan(1/2*d
*x + 1/2*c)))/a^3 + (1600*tan(1/2*d*x + 1/2*c)^4 - 456*tan(1/2*d*x + 1/2*c)
^3 + 108*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(1/2
*d*x + 1/2*c)^4) + 3*(a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x + 1/2*
c)^3 + 36*a^9*tan(1/2*d*x + 1/2*c)^2 - 152*a^9*tan(1/2*d*x + 1/2*c))/a^12)/
d
```

3.78 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=73

$$-\frac{\csc^6(c+dx)}{6a^3d}+\frac{3\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{3a^3d}$$

```
[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + (3*Csc[c + d*x]^5
)/(5*a^3*d) - Csc[c + d*x]^6/(6*a^3*d)
```

Rubi [A] time = 0.0574368, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 43}

$$-\frac{\csc^6(c+dx)}{6a^3d}+\frac{3\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + (3*Csc[c + d*x]^5
)/(5*a^3*d) - Csc[c + d*x]^6/(6*a^3*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3}{x^7}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^7} - \frac{3a^2}{x^6} + \frac{3a}{x^5} - \frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^3(c+dx)}{3a^3d} - \frac{3\csc^4(c+dx)}{4a^3d} + \frac{3\csc^5(c+dx)}{5a^3d} - \frac{\csc^6(c+dx)}{6a^3d}$$

Mathematica [A] time = 0.098613, size = 48, normalized size = 0.66

$$\frac{\csc^3(c+dx)\left(-10\csc^3(c+dx)+36\csc^2(c+dx)-45\csc(c+dx)+20\right)}{60a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(20 - 45*Csc[c + d*x] + 36*Csc[c + d*x]^2 - 10*Csc[c + d*x]
^3))/(60*a^3*d)
```

Maple [A] time = 0.119, size = 49, normalized size = 0.7

$$\frac{1}{da^3}\left(\frac{3}{5\,(\sin(dx+c))^5} - \frac{3}{4\,(\sin(dx+c))^4} - \frac{1}{6\,(\sin(dx+c))^6} + \frac{1}{3\,(\sin(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(3/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4-1/6/sin(d*x+c)^6+1/3/sin(d*x+c)^
3)
```

Maxima [A] time = 1.53887, size = 62, normalized size = 0.85

$$\frac{20\,\sin(dx+c)^3 - 45\,\sin(dx+c)^2 + 36\,\sin(dx+c) - 10}{60\,a^3d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d*
sin(d*x + c)^6)
```

Fricas [A] time = 1.46143, size = 208, normalized size = 2.85

$$-\frac{45 \cos(dx+c)^2 - 4\left(5 \cos(dx+c)^2 - 14\right)\sin(dx+c) - 55}{60\left(a^3 d\cos(dx+c)^6 - 3a^3 d\cos(dx+c)^4 + 3a^3 d\cos(dx+c)^2 - a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(45*cos(d*x + c)^2 - 4*(5*cos(d*x + c)^2 - 14)*sin(d*x + c) - 55)/(a^
3*d*cos(d*x + c)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^2 - a^3*
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.64026, size = 62, normalized size = 0.85

$$\frac{20 \sin(dx+c)^3 - 45 \sin(dx+c)^2 + 36 \sin(dx+c) - 10}{60 a^3 d\sin(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d*
sin(d*x + c)^6)
```

3.79 $\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\csc^8(c+dx)}{8a^3d}+\frac{3\csc^7(c+dx)}{7a^3d}-\frac{\csc^6(c+dx)}{3a^3d}-\frac{2\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{\csc^3(c+dx)}{3a^3d}$$

```
[Out] -Csc[c + d*x]^3/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - (2*Csc[c + d*x]^
5)/(5*a^3*d) - Csc[c + d*x]^6/(3*a^3*d) + (3*Csc[c + d*x]^7)/(7*a^3*d) - Cs
c[c + d*x]^8/(8*a^3*d)
```

Rubi [A] time = 0.0689768, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 75}

$$-\frac{\csc^8(c+dx)}{8a^3d}+\frac{3\csc^7(c+dx)}{7a^3d}-\frac{\csc^6(c+dx)}{3a^3d}-\frac{2\csc^5(c+dx)}{5a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{\csc^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -Csc[c + d*x]^3/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - (2*Csc[c + d*x]^
5)/(5*a^3*d) - Csc[c + d*x]^6/(3*a^3*d) + (3*Csc[c + d*x]^7)/(7*a^3*d) - Cs
c[c + d*x]^8/(8*a^3*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^4(a+x)}{x^9}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^5}{x^9}-\frac{3a^4}{x^8}+\frac{2a^3}{x^7}+\frac{2a^2}{x^6}-\frac{3a}{x^5}+\frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{2\csc^5(c+dx)}{5a^3d}-\frac{\csc^6(c+dx)}{3a^3d}+\frac{3\csc^7(c+dx)}{7a^3d}-\frac{\csc^8(c}{8a}$$

Mathematica [A] time = 0.0721869, size = 68, normalized size = 0.62

$$-\frac{\csc^3(c+dx)\left(105\csc^5(c+dx)-360\csc^4(c+dx)+280\csc^3(c+dx)+336\csc^2(c+dx)-630\csc(c+dx)+280\right)}{840a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] -(Csc[c + d*x]^3*(280 - 630*Csc[c + d*x] + 336*Csc[c + d*x]^2 + 280*Csc[c +
 d*x]^3 - 360*Csc[c + d*x]^4 + 105*Csc[c + d*x]^5))/(840*a^3*d)
```

Maple [A] time = 0.136, size = 69, normalized size = 0.6

$$\frac{1}{da^3}\left(\frac{3}{7\left(\sin(dx+c)\right)^7}-\frac{1}{8\left(\sin(dx+c)\right)^8}-\frac{2}{5\left(\sin(dx+c)\right)^5}+\frac{3}{4\left(\sin(dx+c)\right)^4}-\frac{1}{3\left(\sin(dx+c)\right)^6}-\frac{1}{3\left(\sin(dx+c)\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(3/7/sin(d*x+c)^7-1/8/sin(d*x+c)^8-2/5/sin(d*x+c)^5+3/4/sin(d*x+c)^
4-1/3/sin(d*x+c)^6-1/3/sin(d*x+c)^3)
```

Maxima [A] time = 1.21198, size = 89, normalized size = 0.82

$$-\frac{280\sin(dx+c)^5-630\sin(dx+c)^4+336\sin(dx+c)^3+280\sin(dx+c)^2-360\sin(dx+c)+105}{840\,a^3d\sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/840*(280*sin(d*x + c)^5 - 630*sin(d*x + c)^4 + 336*sin(d*x + c)^3 + 280*
sin(d*x + c)^2 - 360*sin(d*x + c) + 105)/(a^3*d*sin(d*x + c)^8)
```

Fricas [A] time = 1.47629, size = 302, normalized size = 2.77

$$\frac{630 \cos(dx+c)^4 - 980 \cos(dx+c)^2 - 8\left(35 \cos(dx+c)^4 - 112 \cos(dx+c)^2 + 32\right)\sin(dx+c) + 245}{840\left(a^3 d \cos(dx+c)^8 - 4 a^3 d \cos(dx+c)^6 + 6 a^3 d \cos(dx+c)^4 - 4 a^3 d \cos(dx+c)^2 + a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(630*cos(d*x + c)^4 - 980*cos(d*x + c)^2 - 8*(35*cos(d*x + c)^4 - 112
*cos(d*x + c)^2 + 32)*sin(d*x + c) + 245)/(a^3*d*cos(d*x + c)^8 - 4*a^3*d*c
os(d*x + c)^6 + 6*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.61935, size = 89, normalized size = 0.82

$$-\frac{280 \sin(dx+c)^5 - 630 \sin(dx+c)^4 + 336 \sin(dx+c)^3 + 280 \sin(dx+c)^2 - 360 \sin(dx+c) + 105}{840 a^3 d \sin(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/840*(280*sin(d*x + c)^5 - 630*sin(d*x + c)^4 + 336*sin(d*x + c)^3 + 280*
sin(d*x + c)^2 - 360*sin(d*x + c) + 105)/(a^3*d*sin(d*x + c)^8)
```

3.80 $\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{\csc^{10}(c+dx)}{10a^3d}+\frac{\csc^9(c+dx)}{3a^3d}-\frac{\csc^8(c+dx)}{8a^3d}-\frac{5\csc^7(c+dx)}{7a^3d}+\frac{5\csc^6(c+dx)}{6a^3d}+\frac{\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{\csc^3}{3}$$

```
[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + Csc[c + d*x]^5/(5
*a^3*d) + (5*Csc[c + d*x]^6)/(6*a^3*d) - (5*Csc[c + d*x]^7)/(7*a^3*d) - Csc
[c + d*x]^8/(8*a^3*d) + Csc[c + d*x]^9/(3*a^3*d) - Csc[c + d*x]^10/(10*a^3*
d)
```

Rubi [A] time = 0.0805763, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^{10}(c+dx)}{10a^3d}+\frac{\csc^9(c+dx)}{3a^3d}-\frac{\csc^8(c+dx)}{8a^3d}-\frac{5\csc^7(c+dx)}{7a^3d}+\frac{5\csc^6(c+dx)}{6a^3d}+\frac{\csc^5(c+dx)}{5a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{\csc^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] Csc[c + d*x]^3/(3*a^3*d) - (3*Csc[c + d*x]^4)/(4*a^3*d) + Csc[c + d*x]^5/(5
*a^3*d) + (5*Csc[c + d*x]^6)/(6*a^3*d) - (5*Csc[c + d*x]^7)/(7*a^3*d) - Csc
[c + d*x]^8/(8*a^3*d) + Csc[c + d*x]^9/(3*a^3*d) - Csc[c + d*x]^10/(10*a^3*
d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^5(a+x)^2}{x^{11}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^7}{x^{11}} - \frac{3a^6}{x^{10}} + \frac{a^5}{x^9} + \frac{5a^4}{x^8} - \frac{5a^3}{x^7} - \frac{a^2}{x^6} + \frac{3a}{x^5} - \frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\csc^3(c+dx)}{3a^3d} - \frac{3\csc^4(c+dx)}{4a^3d} + \frac{\csc^5(c+dx)}{5a^3d} + \frac{5\csc^6(c+dx)}{6a^3d} - \frac{5\csc^7(c+dx)}{7a^3d} - \frac{\csc^8(c+}{8a^3}$$

Mathematica [A] time = 0.11307, size = 88, normalized size = 0.61

$$\frac{\csc^3(c+dx)\left(-84\csc^7(c+dx)+280\csc^6(c+dx)-105\csc^5(c+dx)-600\csc^4(c+dx)+700\csc^3(c+dx)+168\csc^2(\right.}{840a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(280 - 630*Csc[c + d*x] + 168*Csc[c + d*x]^2 + 700*Csc[c +
d*x]^3 - 600*Csc[c + d*x]^4 - 105*Csc[c + d*x]^5 + 280*Csc[c + d*x]^6 - 84*
Csc[c + d*x]^7))/(840*a^3*d)
```

Maple [A] time = 0.153, size = 89, normalized size = 0.6

$$\frac{1}{da^3}\left(-\frac{1}{10\,(\sin(dx+c))^{10}} - \frac{5}{7\,(\sin(dx+c))^7} - \frac{1}{8\,(\sin(dx+c))^8} + \frac{1}{5\,(\sin(dx+c))^5} - \frac{3}{4\,(\sin(dx+c))^4} + \frac{1}{3\,(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(-1/10/sin(d*x+c)^10-5/7/sin(d*x+c)^7-1/8/sin(d*x+c)^8+1/5/sin(d*x+
c)^5-3/4/sin(d*x+c)^4+1/3/sin(d*x+c)^9+5/6/sin(d*x+c)^6+1/3/sin(d*x+c)^3)
```

Maxima [A] time = 1.96466, size = 116, normalized size = 0.8

$$\frac{280 \sin(dx+c)^7 - 630 \sin(dx+c)^6 + 168 \sin(dx+c)^5 + 700 \sin(dx+c)^4 - 600 \sin(dx+c)^3 - 105 \sin(dx+c)^2 + 28}{840\, a^3 d \sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(280*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 168*sin(d*x + c)^5 + 700*s
in(d*x + c)^4 - 600*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 280*sin(d*x + c)
- 84)/(a^3*d*sin(d*x + c)^10)
```

Fricas [A] time = 1.59062, size = 398, normalized size = 2.74

$$-\frac{630 \cos(dx+c)^6 - 1190 \cos(dx+c)^4 + 595 \cos(dx+c)^2 - 8\left(35 \cos(dx+c)^6 - 126 \cos(dx+c)^4 + 72 \cos(dx+c)^2\right.}{840\left(a^3 d \cos(dx+c)^{10} - 5\, a^3 d \cos(dx+c)^8 + 10\, a^3 d \cos(dx+c)^6 - 10\, a^3 d \cos(dx+c)^4 + 5\, a^3 d \cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/840*(630*cos(d*x + c)^6 - 1190*cos(d*x + c)^4 + 595*cos(d*x + c)^2 - 8*(
35*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d*x + c)^2 - 16)*sin(d*x +
c) - 119)/(a^3*d*cos(d*x + c)^10 - 5*a^3*d*cos(d*x + c)^8 + 10*a^3*d*cos(d*
x + c)^6 - 10*a^3*d*cos(d*x + c)^4 + 5*a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**11/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.01292, size = 116, normalized size = 0.8

$$\frac{280 \sin(dx+c)^7 - 630 \sin(dx+c)^6 + 168 \sin(dx+c)^5 + 700 \sin(dx+c)^4 - 600 \sin(dx+c)^3 - 105 \sin(dx+c)^2 +}{840\, a^3 d \sin(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(280*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 168*sin(d*x + c)^5 + 700*s
in(d*x + c)^4 - 600*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 280*sin(d*x + c)
- 84)/(a^3*d*sin(d*x + c)^10)
```

3.81 $\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{\csc^{12}(c+dx)}{12a^3d}+\frac{3\csc^{11}(c+dx)}{11a^3d}-\frac{8\csc^9(c+dx)}{9a^3d}+\frac{3\csc^8(c+dx)}{4a^3d}+\frac{6\csc^7(c+dx)}{7a^3d}-\frac{4\csc^6(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-$$

```
[Out] -Csc[c + d*x]^3/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - (4*Csc[c + d*x]^
6)/(3*a^3*d) + (6*Csc[c + d*x]^7)/(7*a^3*d) + (3*Csc[c + d*x]^8)/(4*a^3*d)
- (8*Csc[c + d*x]^9)/(9*a^3*d) + (3*Csc[c + d*x]^11)/(11*a^3*d) - Csc[c + d
*x]^12/(12*a^3*d)
```

Rubi [A] time = 0.0783676, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^{12}(c+dx)}{12a^3d}+\frac{3\csc^{11}(c+dx)}{11a^3d}-\frac{8\csc^9(c+dx)}{9a^3d}+\frac{3\csc^8(c+dx)}{4a^3d}+\frac{6\csc^7(c+dx)}{7a^3d}-\frac{4\csc^6(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^3,x]
```


```
[Out] -Csc[c + d*x]^3/(3*a^3*d) + (3*Csc[c + d*x]^4)/(4*a^3*d) - (4*Csc[c + d*x]^
6)/(3*a^3*d) + (6*Csc[c + d*x]^7)/(7*a^3*d) + (3*Csc[c + d*x]^8)/(4*a^3*d)
- (8*Csc[c + d*x]^9)/(9*a^3*d) + (3*Csc[c + d*x]^11)/(11*a^3*d) - Csc[c + d
*x]^12/(12*a^3*d)
```


Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^6(a+x)^3}{x^{13}}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^9}{x^{13}} - \frac{3a^8}{x^{12}} + \frac{8a^6}{x^{10}} - \frac{6a^5}{x^9} - \frac{6a^4}{x^8} + \frac{8a^3}{x^7} - \frac{3a}{x^5} + \frac{1}{x^4}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= -\frac{\csc^3(c+dx)}{3a^3d} + \frac{3\csc^4(c+dx)}{4a^3d} - \frac{4\csc^6(c+dx)}{3a^3d} + \frac{6\csc^7(c+dx)}{7a^3d} + \frac{3\csc^8(c+dx)}{4a^3d} - \frac{8\,\text{cs}}{}$$

Mathematica [A] time = 0.117899, size = 88, normalized size = 0.61

$$\frac{\csc^3(c+dx)\left(-231\csc^9(c+dx)+756\csc^8(c+dx)-2464\csc^6(c+dx)+2079\csc^5(c+dx)+2376\csc^4(c+dx)-3696\right.}{2772a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (Csc[c + d*x]^3*(-924 + 2079*Csc[c + d*x] - 3696*Csc[c + d*x]^3 + 2376*Csc[
c + d*x]^4 + 2079*Csc[c + d*x]^5 - 2464*Csc[c + d*x]^6 + 756*Csc[c + d*x]^8
 - 231*Csc[c + d*x]^9))/(2772*a^3*d)
```

Maple [A] time = 0.174, size = 89, normalized size = 0.6

$$\frac{1}{da^3}\left(\frac{6}{7(\sin(dx+c))^7} + \frac{3}{11(\sin(dx+c))^{11}} + \frac{3}{4(\sin(dx+c))^8} + \frac{3}{4(\sin(dx+c))^4} - \frac{8}{9(\sin(dx+c))^9} - \frac{1}{12(\sin(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(6/7/sin(d*x+c)^7+3/11/sin(d*x+c)^11+3/4/sin(d*x+c)^8+3/4/sin(d*x+c
)^4-8/9/sin(d*x+c)^9-1/12/sin(d*x+c)^12-4/3/sin(d*x+c)^6-1/3/sin(d*x+c)^3)
```

Maxima [A] time = 2.73298, size = 116, normalized size = 0.8

$$-\frac{924 \sin(dx+c)^9 - 2079 \sin(dx+c)^8 + 3696 \sin(dx+c)^6 - 2376 \sin(dx+c)^5 - 2079 \sin(dx+c)^4 + 2464 \sin(dx+}{2772\, a^3 d \sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 - 2
376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(d*
x + c) + 231)/(a^3*d*sin(d*x + c)^12)
```

Fricas [A] time = 1.66886, size = 498, normalized size = 3.43

$$\frac{2079 \cos(dx+c)^8 - 4620 \cos(dx+c)^6 + 3465 \cos(dx+c)^4 - 1386 \cos(dx+c)^2 - 4\left(231 \cos(dx+c)^8 - 924 \cos(dx\right.}{2772\left(a^3 d \cos(dx+c)^{12} - 6\, a^3 d \cos(dx+c)^{10} + 15\, a^3 d \cos(dx+c)^8 - 20\, a^3 d \cos(dx+c)^6 + 15\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2772*(2079*cos(d*x + c)^8 - 4620*cos(d*x + c)^6 + 3465*cos(d*x + c)^4 - 1
386*cos(d*x + c)^2 - 4*(231*cos(d*x + c)^8 - 924*cos(d*x + c)^6 + 792*cos(d
*x + c)^4 - 352*cos(d*x + c)^2 + 64)*sin(d*x + c) + 231)/(a^3*d*cos(d*x + c
)^12 - 6*a^3*d*cos(d*x + c)^10 + 15*a^3*d*cos(d*x + c)^8 - 20*a^3*d*cos(d*x
 + c)^6 + 15*a^3*d*cos(d*x + c)^4 - 6*a^3*d*cos(d*x + c)^2 + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.99282, size = 116, normalized size = 0.8

$$-\frac{924 \sin(dx+c)^9 - 2079 \sin(dx+c)^8 + 3696 \sin(dx+c)^6 - 2376 \sin(dx+c)^5 - 2079 \sin(dx+c)^4 + 2464 \sin(dx}{2772\, a^3 d \sin(dx+c)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 - 2
376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(d*
x + c) + 231)/(a^3*d*sin(d*x + c)^12)
```

3.82 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=195

$$\frac{a^2}{48d(a\sin(c+dx)+a)^6}-\frac{3}{256d\left(a^4-a^4\sin(c+dx)\right)}-\frac{1}{256d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{256d\left(a^2-a^2\sin(c+dx)\right)^2}-\frac{}{256d}$$

```
[Out] -ArcTanh[Sin[c + d*x]]/(128*a^4*d) + a^2/(48*d*(a + a*Sin[c + d*x])^6) - (7
*a)/(80*d*(a + a*Sin[c + d*x])^5) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 5/(96*
a*d*(a + a*Sin[c + d*x])^3) + 1/(256*d*(a^2 - a^2*Sin[c + d*x])^2) - 5/(256
*d*(a^2 + a^2*Sin[c + d*x])^2) - 3/(256*d*(a^4 - a^4*Sin[c + d*x])) - 1/(25
6*d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.134569, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 88, 206}

$$\frac{a^2}{48d(a\sin(c+dx)+a)^6}-\frac{3}{256d\left(a^4-a^4\sin(c+dx)\right)}-\frac{1}{256d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{256d\left(a^2-a^2\sin(c+dx)\right)^2}-\frac{}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -ArcTanh[Sin[c + d*x]]/(128*a^4*d) + a^2/(48*d*(a + a*Sin[c + d*x])^6) - (7
*a)/(80*d*(a + a*Sin[c + d*x])^5) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 5/(96*
a*d*(a + a*Sin[c + d*x])^3) + 1/(256*d*(a^2 - a^2*Sin[c + d*x])^2) - 5/(256
*d*(a^2 + a^2*Sin[c + d*x])^2) - 3/(256*d*(a^4 - a^4*Sin[c + d*x])) - 1/(25
6*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a-x)^3(a+x)^7}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{128a^2(a-x)^3} - \frac{3}{256a^3(a-x)^2} - \frac{a^2}{8(a+x)^7} + \frac{7a}{16(a+x)^6} - \frac{1}{2(a+x)^5} + \frac{5}{32a(a+x)^4} + \frac{5}{128a^2(a+x)^3} + \ldots\right.\right.}{d}$$

$$= \frac{a^2}{48d(a+a\sin(c+dx))^6} - \frac{7a}{80d(a+a\sin(c+dx))^5} + \frac{1}{8d(a+a\sin(c+dx))^4} - \frac{5}{96ad(a+a\,\mathrm{s}\ldots}$$

$$= -\frac{\tanh^{-1}(\sin(c+dx))}{128a^4d} + \frac{a^2}{48d(a+a\sin(c+dx))^6} - \frac{7a}{80d(a+a\sin(c+dx))^5} + \frac{1}{8d(a+a\sin\ldots}$$

Mathematica [A] time = 1.48884, size = 112, normalized size = 0.57

$$-\frac{30\tanh^{-1}(\sin(c+dx)) - \frac{2\left(15\sin^7(c+dx)+60\sin^6(c+dx)+65\sin^5(c+dx)+440\sin^4(c+dx)+257\sin^3(c+dx)-132\sin^2(c+dx)-177\sin(c+dx)-48\right)}{(\sin(c+dx)-1)^2(\sin(c+dx)+1)^6}}{3840a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(30*ArcTanh[Sin[c + d*x]] - (2*(-48 - 177*Sin[c + d*x] - 132*Sin[c + d*x]^
2 + 257*Sin[c + d*x]^3 + 440*Sin[c + d*x]^4 + 65*Sin[c + d*x]^5 + 60*Sin[c
+ d*x]^6 + 15*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^6)
)/(3840*a^4*d)
```

Maple [A] time = 0.088, size = 180, normalized size = 0.9

$$\frac{1}{256\,d a^4 (\sin(dx+c)-1)^2}+\frac{3}{256\,d a^4 (\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{256\,d a^4}+\frac{1}{48\,d a^4 (1+\sin(dx+c))^6}-\frac{7}{80\,d a^4 (1+s$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/256/d/a^4/(sin(d*x+c)-1)^2+3/256/d/a^4/(sin(d*x+c)-1)+1/256/d/a^4*ln(sin(
d*x+c)-1)+1/48/d/a^4/(1+sin(d*x+c))^6-7/80/d/a^4/(1+sin(d*x+c))^5+1/8/d/a^4
/(1+sin(d*x+c))^4-5/96/d/a^4/(1+sin(d*x+c))^3-5/256/d/a^4/(1+sin(d*x+c))^2-
1/256/d/a^4/(1+sin(d*x+c))-1/256*ln(1+sin(d*x+c))/a^4/d
```

Maxima [A] time = 3.47638, size = 288, normalized size = 1.48

$$\frac{\frac{2\left(15\sin(dx+c)^7+60\sin(dx+c)^6+65\sin(dx+c)^5+440\sin(dx+c)^4+257\sin(dx+c)^3-132\sin(dx+c)^2-177\sin(dx+c)-48\right)}{a^4\sin(dx+c)^8+4a^4\sin(dx+c)^7+4a^4\sin(dx+c)^6-4a^4\sin(dx+c)^5-10a^4\sin(dx+c)^4-4a^4\sin(dx+c)^3+4a^4\sin(dx+c)^2+4a^4\sin(dx+c)+a^4}-\frac{15\log(\sin(dx+c)+1)}{a^4}}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3840*(2*(15*sin(d*x + c)^7 + 60*sin(d*x + c)^6 + 65*sin(d*x + c)^5 + 440*
sin(d*x + c)^4 + 257*sin(d*x + c)^3 - 132*sin(d*x + c)^2 - 177*sin(d*x + c)
 - 48)/(a^4*sin(d*x + c)^8 + 4*a^4*sin(d*x + c)^7 + 4*a^4*sin(d*x + c)^6 -
4*a^4*sin(d*x + c)^5 - 10*a^4*sin(d*x + c)^4 - 4*a^4*sin(d*x + c)^3 + 4*a^4
*sin(d*x + c)^2 + 4*a^4*sin(d*x + c) + a^4) - 15*log(sin(d*x + c) + 1)/a^4
+ 15*log(sin(d*x + c) - 1)/a^4)/d
```

Fricas [A] time = 1.73448, size = 775, normalized size = 3.97

$$-\frac{120\cos(dx+c)^6-1240\cos(dx+c)^4+1856\cos(dx+c)^2+15\left(\cos(dx+c)^8-8\cos(dx+c)^6+8\cos(dx+c)^4-4\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3840*(120*cos(d*x + c)^6 - 1240*cos(d*x + c)^4 + 1856*cos(d*x + c)^2 + 1
5*(cos(d*x + c)^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)^6
 - 2*cos(d*x + c)^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)
^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)^6 - 2*cos(d*x +
c)^4)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(15*cos(d*x + c)^6 - 110*cos
(d*x + c)^4 + 432*cos(d*x + c)^2 - 160)*sin(d*x + c) - 640)/(a^4*d*cos(d*x
+ c)^8 - 8*a^4*d*cos(d*x + c)^6 + 8*a^4*d*cos(d*x + c)^4 - 4*(a^4*d*cos(d*x
 + c)^6 - 2*a^4*d*cos(d*x + c)^4)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 5.53021, size = 197, normalized size = 1.01

$$-\frac{\frac{60\,\log(|\sin(dx+c)+1|)}{a^4}-\frac{60\,\log(|\sin(dx+c)-1|)}{a^4}+\frac{30\left(3\,\sin(dx+c)^2-12\,\sin(dx+c)+7\right)}{a^4(\sin(dx+c)-1)^2}-\frac{147\,\sin(dx+c)^6+822\,\sin(dx+c)^5+1605\,\sin(dx+c)^4+340\,\sin(dx-}{a^4(\sin(dx+c)+1)^6}}{15360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/15360*(60*log(abs(sin(d*x + c) + 1))/a^4 - 60*log(abs(sin(d*x + c) - 1))
/a^4 + 30*(3*sin(d*x + c)^2 - 12*sin(d*x + c) + 7)/(a^4*(sin(d*x + c) - 1)^
2) - (147*sin(d*x + c)^6 + 822*sin(d*x + c)^5 + 1605*sin(d*x + c)^4 + 340*s
in(d*x + c)^3 - 675*sin(d*x + c)^2 - 522*sin(d*x + c) - 117)/(a^4*(sin(d*x
+ c) + 1)^6))/d
```

3.83 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=132

$$\frac{1}{64d\left(a^4-a^4\sin(c+dx)\right)}+\frac{1}{64d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{32d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{a}{20d(a\sin(c+dx)+a)^5}-\frac{}{8d(a\sin(}$$

```
[Out] a/(20*d*(a + a*Sin[c + d*x])^5) - 1/(8*d*(a + a*Sin[c + d*x])^4) + 1/(16*a*
d*(a + a*Sin[c + d*x])^3) + 1/(32*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(64*d*(
a^4 - a^4*Sin[c + d*x])) + 1/(64*d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.0887882, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$\frac{1}{64d\left(a^4-a^4\sin(c+dx)\right)}+\frac{1}{64d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{32d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{a}{20d(a\sin(c+dx)+a)^5}-\frac{}{8d(a\sin(}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] a/(20*d*(a + a*Sin[c + d*x])^5) - 1/(8*d*(a + a*Sin[c + d*x])^4) + 1/(16*a*
d*(a + a*Sin[c + d*x])^3) + 1/(32*d*(a^2 + a^2*Sin[c + d*x])^2) + 1/(64*d*(
a^4 - a^4*Sin[c + d*x])) + 1/(64*d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a-x)^2(a+x)^6}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{64a^3(a-x)^2} - \frac{a}{4(a+x)^6} + \frac{1}{2(a+x)^5} - \frac{3}{16a(a+x)^4} - \frac{1}{16a^2(a+x)^3} - \frac{1}{64a^3(a+x)^2}\right) dx, x, a\sin(c+ \ldots}{d}$$

$$= \frac{a}{20d(a+a\sin(c+dx))^5} - \frac{1}{8d(a+a\sin(c+dx))^4} + \frac{1}{16ad(a+a\sin(c+dx))^3} + \frac{\ldots}{32d\left(a^2+a \ldots\right.}$$

Mathematica [A] time = 0.115064, size = 50, normalized size = 0.38

$$-\frac{5\sin^2(c+dx)+4\sin(c+dx)+1}{20a^4d(\sin(c+dx)-1)(\sin(c+dx)+1)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] -(1 + 4*Sin[c + d*x] + 5*Sin[c + d*x]^2)/(20*a^4*d*(-1 + Sin[c + d*x])*(1 +
 Sin[c + d*x])^5)
```

Maple [A] time = 0.088, size = 81, normalized size = 0.6

$$\frac{1}{da^4}\left(-\frac{1}{64\sin(dx+c)-64} + \frac{1}{20(1+\sin(dx+c))^5} - \frac{1}{8(1+\sin(dx+c))^4} + \frac{1}{16(1+\sin(dx+c))^3} + \frac{1}{32(1+\sin(dx \ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d/a^4*(-1/64/(sin(d*x+c)-1)+1/20/(1+sin(d*x+c))^5-1/8/(1+sin(d*x+c))^4+1/
16/(1+sin(d*x+c))^3+1/32/(1+sin(d*x+c))^2+1/64/(1+sin(d*x+c)))
```

Maxima [A] time = 2.93891, size = 128, normalized size = 0.97

$$-\frac{5\sin(dx+c)^2+4\sin(dx+c)+1}{20\left(a^4\sin(dx+c)^6+4a^4\sin(dx+c)^5+5a^4\sin(dx+c)^4-5a^4\sin(dx+c)^2-4a^4\sin(dx+c)-a^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/20*(5*sin(d*x + c)^2 + 4*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^6 + 4*a^4*
sin(d*x + c)^5 + 5*a^4*sin(d*x + c)^4 - 5*a^4*sin(d*x + c)^2 - 4*a^4*sin(d*
x + c) - a^4)*d)
```

Fricas [A] time = 1.35404, size = 250, normalized size = 1.89

$$-\frac{5\cos(dx+c)^2 - 4\sin(dx+c) - 6}{20\left(a^4d\cos(dx+c)^6 - 8a^4d\cos(dx+c)^4 + 8a^4d\cos(dx+c)^2 - 4\left(a^4d\cos(dx+c)^4 - 2a^4d\cos(dx+c)^2\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/20*(5*cos(d*x + c)^2 - 4*sin(d*x + c) - 6)/(a^4*d*cos(d*x + c)^6 - 8*a^4
*d*cos(d*x + c)^4 + 8*a^4*d*cos(d*x + c)^2 - 4*(a^4*d*cos(d*x + c)^4 - 2*a^
4*d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 2.77507, size = 103, normalized size = 0.78

$$-\frac{\frac{5}{a^4(\sin(dx+c)-1)} - \frac{5\sin(dx+c)^4+30\sin(dx+c)^3+80\sin(dx+c)^2+50\sin(dx+c)+11}{a^4(\sin(dx+c)+1)^5}}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/320*(5/(a^4*(sin(d*x + c) - 1)) - (5*sin(d*x + c)^4 + 30*sin(d*x + c)^3
+ 80*sin(d*x + c)^2 + 50*sin(d*x + c) + 11)/(a^4*(sin(d*x + c) + 1)^5))/d
```

3.84 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=105

$$-\frac{1}{16d\left(a^4\sin(c+dx)+a^4\right)}-\frac{1}{16d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\tanh^{-1}(\sin(c+dx))}{16a^4d}-\frac{1}{12ad(a\sin(c+dx)+a)^3}+\frac{1}{8d(a\sin(c-}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(16*a^4*d) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 1/(12*a
*d*(a + a*Sin[c + d*x])^3) - 1/(16*d*(a^2 + a^2*Sin[c + d*x])^2) - 1/(16*d*
(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.0653541, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2707, 77, 206}

$$-\frac{1}{16d\left(a^4\sin(c+dx)+a^4\right)}-\frac{1}{16d\left(a^2\sin(c+dx)+a^2\right)^2}+\frac{\tanh^{-1}(\sin(c+dx))}{16a^4d}-\frac{1}{12ad(a\sin(c+dx)+a)^3}+\frac{1}{8d(a\sin(c-}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```


```
[Out] ArcTanh[Sin[c + d*x]]/(16*a^4*d) + 1/(8*d*(a + a*Sin[c + d*x])^4) - 1/(12*a
*d*(a + a*Sin[c + d*x])^3) - 1/(16*d*(a^2 + a^2*Sin[c + d*x])^2) - 1/(16*d*
(a^4 + a^4*Sin[c + d*x]))
```


Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a-x)(a+x)^5}\,dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+x)^5}+\frac{1}{4a(a+x)^4}+\frac{1}{8a^2(a+x)^3}+\frac{1}{16a^3(a+x)^2}+\frac{1}{16a^3\left(a^2-x^2\right)}\right)dx, x, a\sin(c+dx)\right)}{d}$$

$$= \frac{1}{8d(a+a\sin(c+dx))^4}-\frac{1}{12ad(a+a\sin(c+dx))^3}-\frac{1}{16d\left(a^2+a^2\sin(c+dx)\right)^2}-\frac{}{16d\left(a^4+\right.}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{16a^4d}+\frac{1}{8d(a+a\sin(c+dx))^4}-\frac{1}{12ad(a+a\sin(c+dx))^3}-\frac{1}{16d\left(a^2+a^2\text{si}\right.}$$

Mathematica [A] time = 0.25388, size = 62, normalized size = 0.59

$$\frac{3\tanh^{-1}(\sin(c+dx))-\frac{3\sin^3(c+dx)+12\sin^2(c+dx)+19\sin(c+dx)+4}{(\sin(c+dx)+1)^4}}{48a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]] - (4 + 19*Sin[c + d*x] + 12*Sin[c + d*x]^2 + 3*Sin
[c + d*x]^3)/(1 + Sin[c + d*x])^4)/(48*a^4*d)
```

Maple [A] time = 0.092, size = 108, normalized size = 1.

$$-\frac{\ln(\sin(dx+c)-1)}{32\,da^4}+\frac{1}{8\,da^4(1+\sin(dx+c))^4}-\frac{1}{12\,da^4(1+\sin(dx+c))^3}-\frac{1}{16\,da^4(1+\sin(dx+c))^2}-\frac{1}{16\,da^4(1+\text{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -1/32/d/a^4*ln(sin(d*x+c)-1)+1/8/d/a^4/(1+sin(d*x+c))^4-1/12/d/a^4/(1+sin(d
*x+c))^3-1/16/d/a^4/(1+sin(d*x+c))^2-1/16/d/a^4/(1+sin(d*x+c))+1/32*ln(1+si
n(d*x+c))/a^4/d
```

Maxima [A] time = 3.1573, size = 163, normalized size = 1.55

$$-\frac{\frac{2\left(3 \sin(dx+c)^{3}+12 \sin(dx+c)^{2}+19 \sin(dx+c)+4\right)}{a^{4} \sin(dx+c)^{4}+4 a^{4} \sin(dx+c)^{3}+6 a^{4} \sin(dx+c)^{2}+4 a^{4} \sin(dx+c)+a^{4}}-\frac{3 \log(\sin(dx+c)+1)}{a^{4}}+\frac{3 \log(\sin(dx+c)-1)}{a^{4}}}{96 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(3*sin(d*x + c)^3 + 12*sin(d*x + c)^2 + 19*sin(d*x + c) + 4)/(a^4*
sin(d*x + c)^4 + 4*a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 4*a^4*sin(d*
x + c) + a^4) - 3*log(sin(d*x + c) + 1)/a^4 + 3*log(sin(d*x + c) - 1)/a^4)/
d
```

Fricas [B] time = 1.5909, size = 525, normalized size = 5.

$$\frac{24 \cos(dx+c)^{2}+3\left(\cos(dx+c)^{4}-8 \cos(dx+c)^{2}-4\left(\cos(dx+c)^{2}-2\right)\sin(dx+c)+8\right)\log(\sin(dx+c)+1)-3\left(\mathrm{co}\right.}{96\left(a^{4} d \cos(dx+c)^{4}-8 a^{4} d \cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/96*(24*cos(d*x + c)^2 + 3*(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*x
 + c)^2 - 2)*sin(d*x + c) + 8)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^4 -
8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)*log(-sin(d*x +
c) + 1) + 2*(3*cos(d*x + c)^2 - 22)*sin(d*x + c) - 32)/(a^4*d*cos(d*x + c)^
4 - 8*a^4*d*cos(d*x + c)^2 + 8*a^4*d - 4*(a^4*d*cos(d*x + c)^2 - 2*a^4*d)*s
in(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d*x)
**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 2.14057, size = 123, normalized size = 1.17

$$\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^4}-\frac{12\log(|\sin(dx+c)-1|)}{a^4}-\frac{25\sin(dx+c)^4+124\sin(dx+c)^3+246\sin(dx+c)^2+252\sin(dx+c)+57}{a^4(\sin(dx+c)+1)^4}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/384*(12*log(abs(sin(d*x + c) + 1))/a^4 - 12*log(abs(sin(d*x + c) - 1))/a^
4 - (25*sin(d*x + c)^4 + 124*sin(d*x + c)^3 + 246*sin(d*x + c)^2 + 252*sin(
d*x + c) + 57)/(a^4*(sin(d*x + c) + 1)^4))/d
```

3.85 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=106

$$\frac{5}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{9\log(\sin(c+dx))}{a^4d}-\frac{9\log(\sin(c+dx}{a^4d}$$

```
[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (9*Log[Sin[c + d*x]])
/(a^4*d) - (9*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]
)^2) + 5/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rubi [A] time = 0.082628, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 77}

$$\frac{5}{d\left(a^4\sin(c+dx)+a^4\right)}+\frac{1}{d\left(a^2\sin(c+dx)+a^2\right)^2}-\frac{\csc^2(c+dx)}{2a^4d}+\frac{4\csc(c+dx)}{a^4d}+\frac{9\log(\sin(c+dx))}{a^4d}-\frac{9\log(\sin(c+dx}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^4*d) + (9*Log[Sin[c + d*x]])
/(a^4*d) - (9*Log[1 + Sin[c + d*x]])/(a^4*d) + 1/(d*(a^2 + a^2*Sin[c + d*x]
)^2) + 5/(d*(a^4 + a^4*Sin[c + d*x]))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{a-x}{x^3(a+x)^3}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x^3} - \frac{4}{a^3x^2} + \frac{9}{a^4x} - \frac{2}{a^2(a+x)^3} - \frac{5}{a^3(a+x)^2} - \frac{9}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{4\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{9\log(\sin(c+dx))}{a^4d} - \frac{9\log(1+\sin(c+dx))}{a^4d} + \frac{1}{d\left(a^2+a^2\sin\right.}$$

Mathematica [A] time = 0.838887, size = 73, normalized size = 0.69

$$\frac{\frac{10}{\sin(c+dx)+1} + \frac{2}{(\sin(c+dx)+1)^2} - \csc^2(c+dx) + 8\csc(c+dx) + 18\log(\sin(c+dx)) - 18\log(\sin(c+dx)+1)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (8*Csc[c + d*x] - Csc[c + d*x]^2 + 18*Log[Sin[c + d*x]] - 18*Log[1 + Sin[c
+ d*x]] + 2/(1 + Sin[c + d*x])^2 + 10/(1 + Sin[c + d*x]))/(2*a^4*d)
```

Maple [A] time = 0.139, size = 101, normalized size = 1.

$$\frac{1}{da^4(1+\sin(dx+c))^2} + 5\frac{1}{da^4(1+\sin(dx+c))} - 9\frac{\ln(1+\sin(dx+c))}{da^4} - \frac{1}{2da^4(\sin(dx+c))^2} + 4\frac{1}{da^4\sin(dx+c)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/d/a^4/(1+sin(d*x+c))^2+5/d/a^4/(1+sin(d*x+c))-9*ln(1+sin(d*x+c))/a^4/d-1/
2/d/a^4/sin(d*x+c)^2+4/d/a^4/sin(d*x+c)+9*ln(sin(d*x+c))/a^4/d
```

Maxima [A] time = 2.61648, size = 139, normalized size = 1.31

$$\frac{\frac{18 \sin(dx+c)^3+27 \sin(dx+c)^2+6 \sin(dx+c)-1}{a^4 \sin(dx+c)^4+2 a^4 \sin(dx+c)^3+a^4 \sin(dx+c)^2} - \frac{18 \log(\sin(dx+c)+1)}{a^4} + \frac{18 \log(\sin(dx+c))}{a^4}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*((18*sin(d*x + c)^3 + 27*sin(d*x + c)^2 + 6*sin(d*x + c) - 1)/(a^4*sin(
d*x + c)^4 + 2*a^4*sin(d*x + c)^3 + a^4*sin(d*x + c)^2) - 18*log(sin(d*x +
c) + 1)/a^4 + 18*log(sin(d*x + c))/a^4)/d
```

Fricas [A] time = 1.58272, size = 522, normalized size = 4.92

$$-\frac{27 \cos(dx+c)^2 - 18 \left(\cos(dx+c)^4 - 3 \cos(dx+c)^2 - 2 \left(\cos(dx+c)^2 - 1\right) \sin(dx+c) + 2\right) \log\left(\frac{1}{2} \sin(dx+c)\right) + 18 (}{2 \left(a^4 d \cos(dx+c)^4 - 3 a^4 d \cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(27*cos(d*x + c)^2 - 18*(cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 2*(cos(d*
x + c)^2 - 1)*sin(d*x + c) + 2)*log(1/2*sin(d*x + c)) + 18*(cos(d*x + c)^4
- 3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 2)*log(sin(d*x +
 c) + 1) + 6*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) - 26)/(a^4*d*cos(d*x + c)^
4 - 3*a^4*d*cos(d*x + c)^2 + 2*a^4*d - 2*(a^4*d*cos(d*x + c)^2 - a^4*d)*sin
(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\, dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.81917, size = 250, normalized size = 2.36

$$-\frac{\frac{144 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a^4} - \frac{72 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)\right|\right)}{a^4} + \frac{108 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 - 16 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1}{a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2} + \frac{a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2 - 16 a^4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)}{a^8} - }{8 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*(144*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 72*log(abs(tan(1/2*d*x +
 1/2*c)))/a^4 + (108*tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x + 1/2*c) + 1)/
(a^4*tan(1/2*d*x + 1/2*c)^2) + (a^4*tan(1/2*d*x + 1/2*c)^2 - 16*a^4*tan(1/2
*d*x + 1/2*c))/a^8 - 4*(75*tan(1/2*d*x + 1/2*c)^4 + 272*tan(1/2*d*x + 1/2*c
)^3 + 402*tan(1/2*d*x + 1/2*c)^2 + 272*tan(1/2*d*x + 1/2*c) + 75)/(a^4*(tan
(1/2*d*x + 1/2*c) + 1)^4))/d
```

3.86 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=135

$$-\frac{\csc^6(c+dx)}{6a^4d}+\frac{4\csc^5(c+dx)}{5a^4d}-\frac{7\csc^4(c+dx)}{4a^4d}+\frac{8\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{8\csc(c+dx)}{a^4d}+\frac{8\log(\sin(c+dx))}{a^4d}$$

```
[Out] (8*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (8*Csc[c + d*x]^3)/
(3*a^4*d) - (7*Csc[c + d*x]^4)/(4*a^4*d) + (4*Csc[c + d*x]^5)/(5*a^4*d) - C
sc[c + d*x]^6/(6*a^4*d) + (8*Log[Sin[c + d*x]])/(a^4*d) - (8*Log[1 + Sin[c
+ d*x]])/(a^4*d)
```

Rubi [A] time = 0.0856235, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2707, 88}

$$-\frac{\csc^6(c+dx)}{6a^4d}+\frac{4\csc^5(c+dx)}{5a^4d}-\frac{7\csc^4(c+dx)}{4a^4d}+\frac{8\csc^3(c+dx)}{3a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{8\csc(c+dx)}{a^4d}+\frac{8\log(\sin(c+dx))}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (8*Csc[c + d*x])/(a^4*d) - (4*Csc[c + d*x]^2)/(a^4*d) + (8*Csc[c + d*x]^3)/
(3*a^4*d) - (7*Csc[c + d*x]^4)/(4*a^4*d) + (4*Csc[c + d*x]^5)/(5*a^4*d) - C
sc[c + d*x]^6/(6*a^4*d) + (8*Log[Sin[c + d*x]])/(a^4*d) - (8*Log[1 + Sin[c
+ d*x]])/(a^4*d)
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^3}{x^7(a+x)}\,dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^7}-\frac{4a}{x^6}+\frac{7}{x^5}-\frac{8}{ax^4}+\frac{8}{a^2x^3}-\frac{8}{a^3x^2}+\frac{8}{a^4x}-\frac{8}{a^4(a+x)}\right)dx, x, a\sin(c+dx)\right)}{d}$$
$$= \frac{8\csc(c+dx)}{a^4d}-\frac{4\csc^2(c+dx)}{a^4d}+\frac{8\csc^3(c+dx)}{3a^4d}-\frac{7\csc^4(c+dx)}{4a^4d}+\frac{4\csc^5(c+dx)}{5a^4d}-\frac{\csc^6(}{6}$$

Mathematica [A] time = 0.170585, size = 89, normalized size = 0.66

$$\frac{-10\csc^6(c+dx)+48\csc^5(c+dx)-105\csc^4(c+dx)+160\csc^3(c+dx)-240\csc^2(c+dx)+480\csc(c+dx)+480\,\text{lo}}{60a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (480*Csc[c + d*x] - 240*Csc[c + d*x]^2 + 160*Csc[c + d*x]^3 - 105*Csc[c + d
*x]^4 + 48*Csc[c + d*x]^5 - 10*Csc[c + d*x]^6 + 480*Log[Sin[c + d*x]] - 480
*Log[1 + Sin[c + d*x]])/(60*a^4*d)
```

Maple [A] time = 0.148, size = 130, normalized size = 1.

$$-8\frac{\ln(1+\sin(dx+c))}{a^4d}-\frac{1}{6a^4d(\sin(dx+c))^6}+\frac{4}{5a^4d(\sin(dx+c))^5}-\frac{7}{4a^4d(\sin(dx+c))^4}+\frac{8}{3a^4d(\sin(dx+c))^3}-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x)
```

```
[Out] -8*ln(1+sin(d*x+c))/a^4/d-1/6/d/a^4/sin(d*x+c)^6+4/5/d/a^4/sin(d*x+c)^5-7/4
/d/a^4/sin(d*x+c)^4+8/3/d/a^4/sin(d*x+c)^3-4/d/a^4/sin(d*x+c)^2+8/d/a^4/sin
(d*x+c)+8*ln(sin(d*x+c))/a^4/d
```

Maxima [A] time = 2.68077, size = 128, normalized size = 0.95

$$-\frac{\frac{480\,\log(\sin(dx+c)+1)}{a^4}-\frac{480\,\log(\sin(dx+c))}{a^4}-\frac{480\,\sin(dx+c)^5-240\,\sin(dx+c)^4+160\,\sin(dx+c)^3-105\,\sin(dx+c)^2+48\,\sin(dx+c)-10}{a^4\sin(dx+c)^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/60*(480*log(sin(d*x + c) + 1)/a^4 - 480*log(sin(d*x + c))/a^4 - (480*sin
(d*x + c)^5 - 240*sin(d*x + c)^4 + 160*sin(d*x + c)^3 - 105*sin(d*x + c)^2
+ 48*sin(d*x + c) - 10)/(a^4*sin(d*x + c)^6))/d
```

Fricas [A] time = 1.59832, size = 502, normalized size = 3.72

$$\frac{240\,\cos(dx+c)^4-585\,\cos(dx+c)^2+480\left(\cos(dx+c)^6-3\,\cos(dx+c)^4+3\,\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\,\sin(dx+c)\right)-}{60\left(a^4d\cos(dx+c)^6-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/60*(240*cos(d*x + c)^4 - 585*cos(d*x + c)^2 + 480*(cos(d*x + c)^6 - 3*cos
(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 480*(cos(d*x +
c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(sin(d*x + c) + 1) - 16*
(30*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 43)*sin(d*x + c) + 355)/(a^4*d*cos
(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.93045, size = 313, normalized size = 2.32

$$-\frac{\frac{30720 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{15360 \log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}+\frac{37632 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-10080 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+2835 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-880 \tan\left(\frac{1}{2}dx\right.}{a^4 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/1920*(30720*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 15360*log(abs(tan(1
/2*d*x + 1/2*c)))/a^4 + (37632*tan(1/2*d*x + 1/2*c)^6 - 10080*tan(1/2*d*x +
 1/2*c)^5 + 2835*tan(1/2*d*x + 1/2*c)^4 - 880*tan(1/2*d*x + 1/2*c)^3 + 240*
tan(1/2*d*x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 5)/(a^4*tan(1/2*d*x + 1/
2*c)^6) + (5*a^20*tan(1/2*d*x + 1/2*c)^6 - 48*a^20*tan(1/2*d*x + 1/2*c)^5 +
 240*a^20*tan(1/2*d*x + 1/2*c)^4 - 880*a^20*tan(1/2*d*x + 1/2*c)^3 + 2835*a
^20*tan(1/2*d*x + 1/2*c)^2 - 10080*a^20*tan(1/2*d*x + 1/2*c))/a^24)/d
```

3.87 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=127

$$\frac{8\tan^9(c+dx)}{9a^4d}+\frac{16\tan^7(c+dx)}{7a^4d}+\frac{9\tan^5(c+dx)}{5a^4d}+\frac{\tan^3(c+dx)}{3a^4d}-\frac{8\sec^9(c+dx)}{9a^4d}+\frac{12\sec^7(c+dx)}{7a^4d}-\frac{4\sec^5(c+dx)}{5a^4d}$$

```
[Out] (-4*Sec[c + d*x]^5)/(5*a^4*d) + (12*Sec[c + d*x]^7)/(7*a^4*d) - (8*Sec[c +
d*x]^9)/(9*a^4*d) + Tan[c + d*x]^3/(3*a^4*d) + (9*Tan[c + d*x]^5)/(5*a^4*d)
 + (16*Tan[c + d*x]^7)/(7*a^4*d) + (8*Tan[c + d*x]^9)/(9*a^4*d)
```

Rubi [A] time = 0.309682, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2711, 2607, 270, 2606, 14}

$$\frac{8\tan^9(c+dx)}{9a^4d}+\frac{16\tan^7(c+dx)}{7a^4d}+\frac{9\tan^5(c+dx)}{5a^4d}+\frac{\tan^3(c+dx)}{3a^4d}-\frac{8\sec^9(c+dx)}{9a^4d}+\frac{12\sec^7(c+dx)}{7a^4d}-\frac{4\sec^5(c+dx)}{5a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (-4*Sec[c + d*x]^5)/(5*a^4*d) + (12*Sec[c + d*x]^7)/(7*a^4*d) - (8*Sec[c +
d*x]^9)/(9*a^4*d) + Tan[c + d*x]^3/(3*a^4*d) + (9*Tan[c + d*x]^5)/(5*a^4*d)
 + (16*Tan[c + d*x]^7)/(7*a^4*d) + (8*Tan[c + d*x]^9)/(9*a^4*d)
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(a^4\sec^8(c+dx)\tan^2(c+dx)-4a^4\sec^7(c+dx)\tan^3(c+dx)+6a^4\sec^6(c+dx)\tan^4(c\right.}{a^8}$$

$$= \frac{\int \sec^8(c+dx)\tan^2(c+dx)\,dx}{a^4}+\frac{\int \sec^4(c+dx)\tan^6(c+dx)\,dx}{a^4}-\frac{4\int \sec^7(c+dx)\tan^3(}{a^4}$$

$$= \frac{\text{Subst}\left(\int x^6\left(1+x^2\right)dx,x,\tan(c+dx)\right)}{a^4d}+\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^3dx,x,\tan(c+dx)\right)}{a^4d}-\frac{4\,\text{S}}{}$$

$$= \frac{\text{Subst}\left(\int \left(x^6+x^8\right)dx,x,\tan(c+dx)\right)}{a^4d}+\frac{\text{Subst}\left(\int \left(x^2+3x^4+3x^6+x^8\right)dx,x,\tan(c+d\right.}{a^4d}$$

$$= -\frac{4\sec^5(c+dx)}{5a^4d}+\frac{12\sec^7(c+dx)}{7a^4d}-\frac{8\sec^9(c+dx)}{9a^4d}+\frac{\tan^3(c+dx)}{3a^4d}+\frac{9\tan^5(c+dx)}{5a^4d}+\frac{16}{}$$

Mathematica [A] time = 0.415533, size = 124, normalized size = 0.98

$$\frac{\sec(c+dx)(34944\sin(c+dx)+1776\sin(2(c+dx))-9504\sin(3(c+dx))-296\sin(4(c+dx))+352\sin(5(c+dx))+}{80640a^4d(\sin(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (Sec[c + d*x]*(16128 + 1554*Cos[c + d*x] - 16896*Cos[2*(c + d*x)] - 999*Cos
[3*(c + d*x)] + 2816*Cos[4*(c + d*x)] + 37*Cos[5*(c + d*x)] + 34944*Sin[c +
```

```
d*x] + 1776*Sin[2*(c + d*x)] - 9504*Sin[3*(c + d*x)] - 296*Sin[4*(c + d*x)
] + 352*Sin[5*(c + d*x)]))/(80640*a^4*d*(1 + Sin[c + d*x])^4)
```

Maple [A] time = 0.076, size = 158, normalized size = 1.2

$$8\,\frac{1}{da^4}\left(-\frac{1}{128\,\tan(1/2\,dx+c/2)-128}-2/9\,(\tan(1/2\,dx+c/2)+1)^{-9}+(\tan(1/2\,dx+c/2)+1)^{-8}-\frac{29}{14\,(\tan(1/2\,dx+c/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 8/d/a^4*(-1/128/(tan(1/2*d*x+1/2*c)-1)-2/9/(tan(1/2*d*x+1/2*c)+1)^9+1/(tan(
1/2*d*x+1/2*c)+1)^8-29/14/(tan(1/2*d*x+1/2*c)+1)^7+31/12/(tan(1/2*d*x+1/2*c
)+1)^6-83/40/(tan(1/2*d*x+1/2*c)+1)^5+17/16/(tan(1/2*d*x+1/2*c)+1)^4-29/96/
(tan(1/2*d*x+1/2*c)+1)^3+1/64/(tan(1/2*d*x+1/2*c)+1)^2+1/128/(tan(1/2*d*x+1
/2*c)+1))
```

Maxima [B] time = 3.08174, size = 481, normalized size = 3.79

$$\frac{8\left(\frac{16\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{54\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{201\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{294\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{378\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{210\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{105\,\sin(dx+}{(\cos(dx+c)+}\right.}{315\left(a^4+\frac{8\,a^4\sin(dx+c)}{\cos(dx+c)+1}+\frac{27\,a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{48\,a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{42\,a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{42\,a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{48\,a^4\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{27\,a^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{8\,a^4}{(\cos}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 8/315*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 54*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 201*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 294*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 210*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 105*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 2)/((
a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 27*a^4*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 48*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 42*a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 -
 48*a^4*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 27*a^4*sin(d*x + c)^8/(cos(d*
x + c) + 1)^8 - 8*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^4*sin(d*x + c
)^10/(cos(d*x + c) + 1)^10)*d)
```

Fricas [A] time = 1.49638, size = 338, normalized size = 2.66

$$\frac{88 \cos (dx+c)^4-220 \cos (dx+c)^2+\left(22 \cos (dx+c)^4-165 \cos (dx+c)^2+175\right)\sin (dx+c)+140}{315\left(a^4 d\cos (dx+c)^5-8\,a^4 d\cos (dx+c)^3+8\,a^4 d\cos (dx+c)-4\left(a^4 d\cos (dx+c)^3-2\,a^4 d\cos (dx+c)\right)\sin (dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/315*(88*cos(d*x + c)^4 - 220*cos(d*x + c)^2 + (22*cos(d*x + c)^4 - 165*co
s(d*x + c)^2 + 175)*sin(d*x + c) + 140)/(a^4*d*cos(d*x + c)^5 - 8*a^4*d*cos
(d*x + c)^3 + 8*a^4*d*cos(d*x + c) - 4*(a^4*d*cos(d*x + c)^3 - 2*a^4*d*cos(
d*x + c))*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\tan^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(tan(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 2.53876, size = 197, normalized size = 1.55

$$-\frac{\frac{315}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}-\frac{315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+3150\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1050\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+630\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8064\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-6006\,\tan\left(\frac{1}{2}\,dx\right.}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^9}}{5040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/5040*(315/(a^4*(tan(1/2*d*x + 1/2*c) - 1)) - (315*tan(1/2*d*x + 1/2*c)^8
 + 3150*tan(1/2*d*x + 1/2*c)^7 + 1050*tan(1/2*d*x + 1/2*c)^6 + 630*tan(1/2*
d*x + 1/2*c)^5 - 8064*tan(1/2*d*x + 1/2*c)^4 - 6006*tan(1/2*d*x + 1/2*c)^3
- 5274*tan(1/2*d*x + 1/2*c)^2 - 846*tan(1/2*d*x + 1/2*c) - 59)/(a^4*(tan(1/
2*d*x + 1/2*c) + 1)^9))/d
```

3.88 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=108

$$-\frac{\cot(c+dx)}{a^4d}+\frac{4\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{104\cot(c+dx)}{15a^4d(\csc(c+dx)+1)}+\frac{31\cot(c+dx)}{15a^4d(\csc(c+dx)+1)^2}-\frac{2\cot(c+dx)}{5a^4d(\csc(c+dx)+1)^3}$$

```
[Out] (4*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^4*d) - (2*Cot[c + d*x])
/(5*a^4*d*(1 + Csc[c + d*x])^3) + (31*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c +
d*x])^2) - (104*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c + d*x]))
```

Rubi [A] time = 0.317935, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3770, 3767, 8, 3777, 3922, 3919, 3794}

$$-\frac{\cot(c+dx)}{a^4d}+\frac{4\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{104\cot(c+dx)}{15a^4d(\csc(c+dx)+1)}+\frac{31\cot(c+dx)}{15a^4d(\csc(c+dx)+1)^2}-\frac{2\cot(c+dx)}{5a^4d(\csc(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (4*ArcTanh[Cos[c + d*x]])/(a^4*d) - Cot[c + d*x]/(a^4*d) - (2*Cot[c + d*x])
/(5*a^4*d*(1 + Csc[c + d*x])^3) + (31*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c +
d*x])^2) - (104*Cot[c + d*x])/(15*a^4*d*(1 + Csc[c + d*x]))
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*
x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x
])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ
[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(\frac{9}{a^2} - \frac{4\csc(c+dx)}{a^2} + \frac{\csc^2(c+dx)}{a^2} - \frac{2}{a^2(1+\csc(c+dx))^3} + \frac{9}{a^2(1+\csc(c+dx))^2} - \frac{16}{a^2(1+\csc(c+dx))}\right)dx}{a^2}$$

$$= \frac{9x}{a^4} + \frac{\int \csc^2(c+dx)\,dx}{a^4} - \frac{2\int \frac{1}{(1+\csc(c+dx))^3}\,dx}{a^4} - \frac{4\int \csc(c+dx)\,dx}{a^4} + \frac{9\int \frac{1}{(1+\csc(c+dx))^2}\,dx}{a^4}$$

$$= \frac{9x}{a^4} + \frac{4\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3} + \frac{3\cot(c+dx)}{a^4d(1+\csc(c+dx))^2} - \frac{16\cot}{a^4d(1+}$$

$$= \frac{2x}{a^4} + \frac{4\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^4d} - \frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3} + \frac{31\cot(c+dx)}{15a^4d(1+\csc(c+}$$

$$= \frac{4\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^4d} - \frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3} + \frac{31\cot(c+dx)}{15a^4d(1+\csc(c+dx))^2}$$

$$= \frac{4\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^4d} - \frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3} + \frac{31\cot(c+dx)}{15a^4d(1+\csc(c+dx))^2}$$

Mathematica [B] time = 0.41973, size = 315, normalized size = 2.92

$$\frac{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3\left(24\sin\left(\frac{1}{2}(c+dx)\right)+316\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4-3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] ((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(24*Sin[(c + d*x)/2] - 12*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]) + 76*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^2 - 38*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 316*Sin[(c
+ d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 15*Cot[(c + d*x)/2]*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 120*Log[Cos[(c + d*x)/2]]*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^5 - 120*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^5 + 15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*Tan[(c
+ d*x)/2]))/(30*d*(a + a*Sin[c + d*x])^4)
```

Maple [A] time = 0.125, size = 161, normalized size = 1.5

$$\frac{1}{2da^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{16}{5da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5} + 8\frac{1}{da^4(\tan(1/2\,dx+c/2)+1)^4} - \frac{44}{3da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3} + 14$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/2/d/a^4*tan(1/2*d*x+1/2*c)-16/5/d/a^4/(tan(1/2*d*x+1/2*c)+1)^5+8/d/a^4/(t
an(1/2*d*x+1/2*c)+1)^4-44/3/d/a^4/(tan(1/2*d*x+1/2*c)+1)^3+14/d/a^4/(tan(1/
2*d*x+1/2*c)+1)^2-18/d/a^4/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^4/tan(1/2*d*x+1/2
*c)-4/d/a^4*ln(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 3.72867, size = 389, normalized size = 3.6

$$-\frac{\frac{\frac{491 \sin(dx+c)}{\cos(dx+c)+1}+\frac{1690 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2570 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1815 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{555 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+15}{\frac{a^4 \sin(dx+c)}{\cos(dx+c)+1}+\frac{5 a^4 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10 a^4 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10 a^4 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5 a^4 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{120 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}-\frac{15 \sin(dx+c)}{a^4(\cos(dx+c)+1)}}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/30*((491*sin(d*x + c)/(cos(d*x + c) + 1) + 1690*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 2570*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1815*sin(d*x + c)^
4/(cos(d*x + c) + 1)^4 + 555*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15)/(a^4
*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + 10*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^4*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 5*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) + 120*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^4
 - 15*sin(d*x + c)/(a^4*(cos(d*x + c) + 1)))/d
```

Fricas [B] time = 1.59742, size = 992, normalized size = 9.19

$$94 \cos(dx+c)^4 + 222 \cos(dx+c)^3 - 115 \cos(dx+c)^2 + 30 \left(\cos(dx+c)^4 - 2 \cos(dx+c)^3 - 5 \cos(dx+c)^2 - \left(\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/15*(94*cos(d*x + c)^4 + 222*cos(d*x + c)^3 - 115*cos(d*x + c)^2 + 30*(cos
(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - (cos(d*x + c)^3 + 3*cos
(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*log(1/
2*cos(d*x + c) + 1/2) - 30*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x +
 c)^2 - (cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x +
c) + 2*cos(d*x + c) + 4)*log(-1/2*cos(d*x + c) + 1/2) + (94*cos(d*x + c)^3
- 128*cos(d*x + c)^2 - 243*cos(d*x + c) - 6)*sin(d*x + c) - 237*cos(d*x + c
) + 6)/(a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 5*a^4*d*cos(d*x + c
)^2 + 2*a^4*d*cos(d*x + c) + 4*a^4*d - (a^4*d*cos(d*x + c)^3 + 3*a^4*d*cos(
d*x + c)^2 - 2*a^4*d*cos(d*x + c) - 4*a^4*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 1.94505, size = 182, normalized size = 1.69

$$-\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}-\frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}-\frac{15\left(8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\frac{4\left(135\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+605\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+385}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/30*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 15*tan(1/2*d*x + 1/2*c)/a^4
 - 15*(8*tan(1/2*d*x + 1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)) + 4*(135*tan(
1/2*d*x + 1/2*c)^4 + 435*tan(1/2*d*x + 1/2*c)^3 + 605*tan(1/2*d*x + 1/2*c)^
2 + 385*tan(1/2*d*x + 1/2*c) + 104)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.89 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=120

$$-\frac{\cot^3(c+dx)}{3a^4d}-\frac{9\cot(c+dx)}{a^4d}+\frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}-\frac{44\cot(c+dx)}{3a^4d(\csc(c+dx)+1)}+\frac{4\,\mathrm{c}}{3a^4d(\mathrm{cs}}$$

```
[Out] (14*ArcTanh[Cos[c + d*x]])/(a^4*d) - (9*Cot[c + d*x])/(a^4*d) - Cot[c + d*x
]^3/(3*a^4*d) + (2*Cot[c + d*x]*Csc[c + d*x])/(a^4*d) + (4*Cot[c + d*x])/(3
*a^4*d*(1 + Csc[c + d*x])^2) - (44*Cot[c + d*x])/(3*a^4*d*(1 + Csc[c + d*x]
))
```

Rubi [A] time = 0.248752, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2709, 3770, 3767, 8, 3768, 3777, 3919, 3794}

$$-\frac{\cot^3(c+dx)}{3a^4d}-\frac{9\cot(c+dx)}{a^4d}+\frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}-\frac{44\cot(c+dx)}{3a^4d(\csc(c+dx)+1)}+\frac{4\,\mathrm{c}}{3a^4d(\mathrm{cs}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (14*ArcTanh[Cos[c + d*x]])/(a^4*d) - (9*Cot[c + d*x])/(a^4*d) - Cot[c + d*x
]^3/(3*a^4*d) + (2*Cot[c + d*x]*Csc[c + d*x])/(a^4*d) + (4*Cot[c + d*x])/(3
*a^4*d*(1 + Csc[c + d*x])^2) - (44*Cot[c + d*x])/(3*a^4*d*(1 + Csc[c + d*x]
))
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(16-12\csc(c+dx)+8\csc^2(c+dx)-4\csc^3(c+dx)+\csc^4(c+dx)+\frac{4}{(1+\csc(c+dx))^2}-\frac{}{1+c\ldots}\right.}{a^4}$$

$$= \frac{16x}{a^4}+\frac{\int \csc^4(c+dx)\,dx}{a^4}-\frac{4\int \csc^3(c+dx)\,dx}{a^4}+\frac{4\int \frac{1}{(1+\csc(c+dx))^2}\,dx}{a^4}+\frac{8\int \csc^2(c+dx)\,dx}{a^4}$$

$$= \frac{16x}{a^4}+\frac{12\tanh^{-1}(\cos(c+dx))}{a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}+\frac{4\cot(c+dx)}{3a^4d(1+\csc(c+dx))^2}-\frac{20}{a^4d(\ldots}$$

$$= \frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{9\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}+\frac{4}{3a^4d(\ldots}$$

$$= \frac{14\tanh^{-1}(\cos(c+dx))}{a^4d}-\frac{9\cot(c+dx)}{a^4d}-\frac{\cot^3(c+dx)}{3a^4d}+\frac{2\cot(c+dx)\csc(c+dx)}{a^4d}+\frac{4}{3a^4d(\ldots}$$

Mathematica [B] time = 6.0893, size = 589, normalized size = 4.91

$$\frac{80\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^7}{3d(a\sin(c+dx)+a)^4}-\frac{4\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^6}{3d(a\sin(c+dx)+a)^4}+\frac{8\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\ldots\right.}{3d(a\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (8*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(3*d*(a + a*Si
n[c + d*x])^4) - (4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)/(3*d*(a + a*Si
n[c + d*x])^4) + (80*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^7)/(3*d*(a + a*Sin[c + d*x])^4) - (13*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^8)/(3*d*(a + a*Sin[c + d*x])^4) + (Csc[(c + d*x)/2]^2*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2*d*(a + a*Sin[c + d*x])^4) - (Cot[
(c + d*x)/2]*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2
4*d*(a + a*Sin[c + d*x])^4) + (14*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^8)/(d*(a + a*Sin[c + d*x])^4) - (14*Log[Sin[(c + d*x)/2]
]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(d*(a + a*Sin[c + d*x])^4) - (Se
c[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2*d*(a + a*Sin[c
 + d*x])^4) + (13*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])
/(3*d*(a + a*Sin[c + d*x])^4) + (Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(24*d*(a + a*Sin[c + d*x])^4)
```

Maple [A] time = 0.138, size = 195, normalized size = 1.6

$$\frac{1}{24\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{2\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{35}{8\,d a^4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{16}{3\,d a^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+8\,\frac{}{d a^4\,(\tan(1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/24/d/a^4*tan(1/2*d*x+1/2*c)^3-1/2/d/a^4*tan(1/2*d*x+1/2*c)^2+35/8/d/a^4*t
an(1/2*d*x+1/2*c)-16/3/d/a^4/(tan(1/2*d*x+1/2*c)+1)^3+8/d/a^4/(tan(1/2*d*x+
1/2*c)+1)^2-32/d/a^4/(tan(1/2*d*x+1/2*c)+1)-1/24/d/a^4/tan(1/2*d*x+1/2*c)^3
+1/2/d/a^4/tan(1/2*d*x+1/2*c)^2-35/8/d/a^4/tan(1/2*d*x+1/2*c)-14/d/a^4*ln(t
an(1/2*d*x+1/2*c))
```

Maxima [B] time = 3.06899, size = 385, normalized size = 3.21

$$\frac{\frac{\frac{9\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{72\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{984\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{1647\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{873\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3\,a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{12\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^4}-\frac{336\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 72*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 984*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1647*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 873*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*sin(d*
x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 3*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6) + (105*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^4 - 336*log(sin(d*
x + c)/(cos(d*x + c) + 1))/a^4)/d
```

Fricas [B] time = 1.58112, size = 1187, normalized size = 9.89

$$-\frac{66\,\cos(dx+c)^5-24\,\cos(dx+c)^4-147\,\cos(dx+c)^3+29\,\cos(dx+c)^2-21\left(\cos(dx+c)^5+2\,\cos(dx+c)^4-2\,\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(66*cos(d*x + c)^5 - 24*cos(d*x + c)^4 - 147*cos(d*x + c)^3 + 29*cos(d
*x + c)^2 - 21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*co
s(d*x + c)^2 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos(d*
x + c) + 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(1/2*cos(d*x + c) + 1/2) +
21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*cos(d*x + c)^2
 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos(d*x + c) + 2)*
sin(d*x + c) + cos(d*x + c) + 2)*log(-1/2*cos(d*x + c) + 1/2) - (66*cos(d*x
 + c)^4 + 90*cos(d*x + c)^3 - 57*cos(d*x + c)^2 - 86*cos(d*x + c) - 4)*sin(
d*x + c) + 82*cos(d*x + c) - 4)/(a^4*d*cos(d*x + c)^5 + 2*a^4*d*cos(d*x + c
)^4 - 2*a^4*d*cos(d*x + c)^3 - 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c)
+ 2*a^4*d + (a^4*d*cos(d*x + c)^4 - a^4*d*cos(d*x + c)^3 - 3*a^4*d*cos(d*x
+ c)^2 + a^4*d*cos(d*x + c) + 2*a^4*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\cot^4(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Integral(cot(c + d*x)**4/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d
*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

Giac [A] time = 2.40079, size = 242, normalized size = 2.02

$$-\frac{\frac{336\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}-\frac{308\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+51\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-723\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-676\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-72\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^3a^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/24*(336*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - (308*tan(1/2*d*x + 1/2*c)^6
 + 51*tan(1/2*d*x + 1/2*c)^5 - 723*tan(1/2*d*x + 1/2*c)^4 - 676*tan(1/2*d*x
 + 1/2*c)^3 - 72*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 1)/((tan
(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c))^3*a^4) - (a^8*tan(1/2*d*x + 1/2
*c)^3 - 12*a^8*tan(1/2*d*x + 1/2*c)^2 + 105*a^8*tan(1/2*d*x + 1/2*c))/a^12)
/d
```

3.90 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

Optimal. Leaf size=133

$$-\frac{\cot^5(c+dx)}{5a^4d}-\frac{3\cot^3(c+dx)}{a^4d}-\frac{16\cot(c+dx)}{a^4d}+\frac{27\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\cot(c+dx)\csc^3(c+dx)}{a^4d}+\frac{11\cot(c+dx)}{2a^4d}$$

```
[Out] (27*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - (16*Cot[c + d*x])/(a^4*d) - (3*Cot[c
 + d*x]^3)/(a^4*d) - Cot[c + d*x]^5/(5*a^4*d) + (11*Cot[c + d*x]*Csc[c + d*
x])/(2*a^4*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(a^4*d) - (8*Cot[c + d*x])/(a
^4*d*(1 + Csc[c + d*x]))
```

Rubi [A] time = 0.249307, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2709, 3770, 3767, 8, 3768, 3777}

$$-\frac{\cot^5(c+dx)}{5a^4d}-\frac{3\cot^3(c+dx)}{a^4d}-\frac{16\cot(c+dx)}{a^4d}+\frac{27\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{\cot(c+dx)\csc^3(c+dx)}{a^4d}+\frac{11\cot(c+dx)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]
```


```
[Out] (27*ArcTanh[Cos[c + d*x]])/(2*a^4*d) - (16*Cot[c + d*x])/(a^4*d) - (3*Cot[c
 + d*x]^3)/(a^4*d) - Cot[c + d*x]^5/(5*a^4*d) + (11*Cot[c + d*x]*Csc[c + d*
x])/(2*a^4*d) + (Cot[c + d*x]*Csc[c + d*x]^3)/(a^4*d) - (8*Cot[c + d*x])/(a
^4*d*(1 + Csc[c + d*x]))
```


Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Intege
rQ[2*n]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \left(8a^2 - 8a^2\csc(c+dx) + 8a^2\csc^2(c+dx) - 8a^2\csc^3(c+dx) + 7a^2\csc^4(c+dx) - 4a^2\operatorname{cs}\right.}{a^6}$$

$$= \frac{8x}{a^4} + \frac{\int \csc^6(c+dx)\,dx}{a^4} - \frac{4\int \csc^5(c+dx)\,dx}{a^4} + \frac{7\int \csc^4(c+dx)\,dx}{a^4} - \frac{8\int \csc(c+dx)\,dx}{a^4}$$

$$= \frac{8x}{a^4} + \frac{8\tanh^{-1}(\cos(c+dx))}{a^4d} + \frac{4\cot(c+dx)\csc(c+dx)}{a^4d} + \frac{\cot(c+dx)\csc^3(c+dx)}{a^4d} - \frac{}{a^4d}$$

$$= \frac{12\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{16\cot(c+dx)}{a^4d} - \frac{3\cot^3(c+dx)}{a^4d} - \frac{\cot^5(c+dx)}{5a^4d} + \frac{11\cot(c+dx}{2a^4}$$

$$= \frac{27\tanh^{-1}(\cos(c+dx))}{2a^4d} - \frac{16\cot(c+dx)}{a^4d} - \frac{3\cot^3(c+dx)}{a^4d} - \frac{\cot^5(c+dx)}{5a^4d} + \frac{11\cot(c+dx}{2a^4}$$

Mathematica [B] time = 6.14507, size = 733, normalized size = 5.51

$$\frac{16\sin\left(\frac{1}{2}(c+dx)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^7}{d(a\sin(c+dx)+a)^4}+\frac{27\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^8}{2d(a\sin(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]
```

```
[Out] (16*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)/(d*(a + a*Sin
[c + d*x])^4) - (33*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
8)/(5*d*(a + a*Sin[c + d*x])^4) + (11*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) - (53*Cot[(c + d*x)/2]*
Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a + a*S
in[c + d*x])^4) + (Csc[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
^8)/(16*d*(a + a*Sin[c + d*x])^4) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4*(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a + a*Sin[c + d*x])^4) + (27
*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2*d*(a + a
*Sin[c + d*x])^4) - (27*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^8)/(2*d*(a + a*Sin[c + d*x])^4) - (11*Sec[(c + d*x)/2]^2*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) - (Sec[(c + d*
x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(16*d*(a + a*Sin[c + d*x])
^4) + (33*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(5*d*(a
 + a*Sin[c + d*x])^4) + (53*Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^8*Tan[(c + d*x)/2])/(160*d*(a + a*Sin[c + d*x])^4) + (Sec[(c + d*
x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(160*d*(a
 + a*Sin[c + d*x])^4)
```

Maple [A] time = 0.149, size = 229, normalized size = 1.7

$$\frac{1}{160\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{1}{16\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4+\frac{11}{32\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{3}{2\,da^4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+\frac{111}{16\,da^4}\tan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x)
```

```
[Out] 1/160/d/a^4*tan(1/2*d*x+1/2*c)^5-1/16/d/a^4*tan(1/2*d*x+1/2*c)^4+11/32/d/a^
4*tan(1/2*d*x+1/2*c)^3-3/2/d/a^4*tan(1/2*d*x+1/2*c)^2+111/16/d/a^4*tan(1/2*
d*x+1/2*c)-16/d/a^4/(tan(1/2*d*x+1/2*c)+1)-1/160/d/a^4/tan(1/2*d*x+1/2*c)^5
+1/16/d/a^4/tan(1/2*d*x+1/2*c)^4-11/32/d/a^4/tan(1/2*d*x+1/2*c)^3+3/2/d/a^4
```

```
/tan(1/2*d*x+1/2*c)^2-111/16/d/a^4/tan(1/2*d*x+1/2*c)-27/2/d/a^4*ln(tan(1/2
*d*x+1/2*c))
```

Maxima [B] time = 3.61884, size = 377, normalized size = 2.83

$$\frac{\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1}-\frac{45 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{185 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{870 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3670 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^4 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{1110 \sin(dx+c)}{\cos(dx+c)+1}-\frac{240 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{55 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{10 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^4}}{160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/160*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 185*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 870*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 3670*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) +
 (1110*sin(d*x + c)/(cos(d*x + c) + 1) - 240*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 55*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 10*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^4 - 2160*log(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4)/d
```

Fricas [B] time = 1.78661, size = 1191, normalized size = 8.95

$$424 \cos(dx+c)^6 + 154 \cos(dx+c)^5 - 1060 \cos(dx+c)^4 - 340 \cos(dx+c)^3 + 800 \cos(dx+c)^2 + 135 \left(\cos(dx+c)^6 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/20*(424*cos(d*x + c)^6 + 154*cos(d*x + c)^5 - 1060*cos(d*x + c)^4 - 340*c
os(d*x + c)^3 + 800*cos(d*x + c)^2 + 135*(cos(d*x + c)^6 - 3*cos(d*x + c)^4
 + 3*cos(d*x + c)^2 - (cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 -
 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sin(d*x + c) - 1)*log(1/2*cos(d*x + c
) + 1/2) - 135*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - (cos
(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*
x + c) + 1)*sin(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(212*cos(d*x
```

```
 + c)^5 + 135*cos(d*x + c)^4 - 395*cos(d*x + c)^3 - 225*cos(d*x + c)^2 + 17
5*cos(d*x + c) + 80)*sin(d*x + c) + 190*cos(d*x + c) - 160)/(a^4*d*cos(d*x
+ c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d - (a^4*d*c
os(d*x + c)^5 + a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 2*a^4*d*cos
(d*x + c)^2 + a^4*d*cos(d*x + c) + a^4*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+a*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.73458, size = 275, normalized size = 2.07

$$-\frac{\frac{2160\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}+\frac{2560}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}-\frac{4932\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-1110\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+240\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-55\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}\right.}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}}{160d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/160*(2160*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 2560/(a^4*(tan(1/2*d*x +
1/2*c) + 1)) - (4932*tan(1/2*d*x + 1/2*c)^5 - 1110*tan(1/2*d*x + 1/2*c)^4 +
 240*tan(1/2*d*x + 1/2*c)^3 - 55*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x +
1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)^5) - (a^16*tan(1/2*d*x + 1/2*c)^5 - 1
0*a^16*tan(1/2*d*x + 1/2*c)^4 + 55*a^16*tan(1/2*d*x + 1/2*c)^3 - 240*a^16*t
an(1/2*d*x + 1/2*c)^2 + 1110*a^16*tan(1/2*d*x + 1/2*c))/a^20)/d
```

3.91 $\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$

Optimal. Leaf size=162

$$\frac{5\tan^3(e+fx)\sqrt{a(\sin(e+fx)+1)}}{12f}+\frac{29\tan(e+fx)\sqrt{a\sin(e+fx)+a}}{12f}-\frac{\sec^3(e+fx)\sqrt{a(\sin(e+fx)+1)}}{12f}-\frac{27\sec(e$$

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(8*Sqrt[2]*f) - (27*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])])/(8*f) - (
Sec[e + f*x]^3*Sqrt[a*(1 + Sin[e + f*x])])/(12*f) + (29*Sqrt[a + a*Sin[e +
f*x]]*Tan[e + f*x])/(12*f) + (5*Sqrt[a*(1 + Sin[e + f*x])]*Tan[e + f*x]^3)/
(12*f)
```

Rubi [A] time = 0.923052, antiderivative size = 195, normalized size of antiderivative = 1.2, number of steps used = 15, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {2714, 2646, 4401, 2675, 2687, 2650, 2649, 206, 2878, 2855}

$$\frac{11a^2\cos(e+fx)}{8f(a\sin(e+fx)+a)^{3/2}}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{4\sec^3(e+fx)(a\sin(e+fx)+a)^{3/2}}{3af}-\frac{7\sec^3(e+fx)\sqrt{a\sin(e+fx)+}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(8*Sqrt[2]*f) + (11*a^2*Cos[e + f*x])/(8*f*(a + a*Sin[e + f*x])^(3/2))
 - (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (11*a*Sec[e + f*x])/(6
*f*Sqrt[a + a*Sin[e + f*x]]) - (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x]])/
(3*f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*a*f)
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{a+a\sin(e+fx)}\,dx - \int \sec^4(e+fx)\sqrt{a+a\sin(e+fx)}\left(1-2\sin^2(e+fx\right.$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \int \left(\sec^4(e+fx)\sqrt{a(1+\sin(e+fx))} - 2\sec^2(e+fx)\sqrt{\right.$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + 2\int \sec^2(e+fx)\sqrt{a(1+\sin(e+fx))}\tan^2(e+fx)\,dx -$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\sec^3(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{4\sec^3(e+fx)(a+a}{3af}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{5a\sec(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} - \frac{7\sec^3(e+fx)\sqrt{a+a\sin(e+f}}{3f}$$

$$= \frac{5a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{11a\sec(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} - \frac{7\sec}{}$$

$$= \frac{11a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{11a\sec(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} - \frac{7\sec}{}$$

$$= \frac{5\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{2}f} + \frac{11a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

$$= \frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{2}f} + \frac{11a^2\cos(e+fx)}{8f(a+a\sin(e+fx))^{3/2}} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

Mathematica [C] time = 5.54044, size = 394, normalized size = 2.43

$$\frac{\sqrt{a(\sin(e+fx)+1)}\left(-48\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\cos\left(\frac{fx}{2}\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2+48\left(\sin\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}\right)\right)\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]
```

```
[Out] (((6*Sin[(f*x)/2])/(Cos[e/2] + Sin[e/2]) - (3*(Cos[e/2] - Sin[e/2])*(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]))/(Cos[e/2] + Sin[e/2]) + (33 + 33*I)*(-1)^(3
/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(f*x)/4]*(Cos[(2*e + f*x)/4] - Sin[(
2*e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 48*Cos[(f*x)/2]*(
```

```
Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 48*(Cos[e/2]
 + Sin[e/2])*Sin[(f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (4*(Cos
[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
3 - (36*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2] - Sin[(e
 + f*x)/2]))*Sqrt[a*(1 + Sin[e + f*x]])/(24*f*(Cos[(e + f*x)/2] + Sin[(e +
 f*x)/2])^3)
```

Maple [A] time = 0.464, size = 172, normalized size = 1.1

$$-\frac{1}{\left(-48+48\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(96\,a^{5/2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}+\left(33\,\left(a-a\sin\left(fx+e\right)\right)^{3/2}\sqrt{2}\mathrm{Artanh}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] -1/48/a^(3/2)*(96*a^(5/2)*sin(f*x+e)*cos(f*x+e)^2+(33*(a-a*sin(f*x+e))^(3/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+20*a^(5/2))
*sin(f*x+e)-162*a^(5/2)*cos(f*x+e)^2+33*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a-4*a^(5/2))/(-1+sin(f*x+e)
)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(fx+e\right)+a}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^4, x)
```

Fricas [A] time = 2.00592, size = 548, normalized size = 3.38

$$\frac{33\sqrt{2}\sqrt{a}\cos\left(fx+e\right)^{3}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\sqrt{a\sin\left(fx+e\right)+a}\left(\sqrt{2}\cos\left(fx+e\right)-\sqrt{2}\sin\left(fx+e\right)+\sqrt{2}\right)\sqrt{a}+3\,a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)-2\,a\right)\sin\left(fx+e\right)}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right.}{96\,f\cos\left(fx+e\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(33*sqrt(2)*sqrt(a)*cos(f*x + e)^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a*s
in(f*x + e) + a)*(sqrt(2)*cos(f*x + e) - sqrt(2)*sin(f*x + e) + sqrt(2))*sq
rt(a) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(
f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(81*c
os(f*x + e)^2 - 2*(24*cos(f*x + e)^2 + 5)*sin(f*x + e) + 2)*sqrt(a*sin(f*x
+ e) + a))/(f*cos(f*x + e)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sin \left(fx+e\right)+a} \tan \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^4, x)
```

3.92 $\int \sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$

Optimal. Leaf size=101

$$-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}+\frac{5\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}f}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[2]*f)) + (5*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f - (2*Sec[e +
 f*x]*(a + a*Sin[e + f*x])^(3/2))/(a*f)
```

Rubi [A] time = 0.184754, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2713, 2855, 2649, 206}

$$-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}+\frac{5\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(Sqrt[2]*f)) + (5*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f - (2*Sec[e +
 f*x]*(a + a*Sin[e + f*x])^(3/2))/(a*f)
```

Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
```

```
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx = -\frac{2\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{af} + \frac{2\int \sec^2(e+fx)\sqrt{a+a\sin(e+fx)}\left(\frac{3a}{2}+\right.}{a}$$

$$= \frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f} - \frac{2\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{af} + \frac{1}{2}a\int$$

$$= \frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f} - \frac{2\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{af} - \frac{a\,\mathrm{Subst}}{}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}f} + \frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f} - \frac{2\sec(e+f}{}$$

Mathematica [C] time = 0.327891, size = 114, normalized size = 1.13

$$\frac{\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(-2\sin(e+fx)+(1-i)\sqrt[4]{-1}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]
```

```
[Out] (Sec[e + f*x]*(3 + (1 - I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(f
*x)/4]*(Cos[(2*e + f*x)/4] - Sin[(2*e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]) - 2*Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])])/f
```

Maple [A] time = 0.502, size = 89, normalized size = 0.9

$$-\frac{1+\sin\left(fx+e\right)}{2f\cos\left(fx+e\right)}\left(\sqrt{a}\sqrt{2}\mathrm{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{a}}\right)\sqrt{a-a\sin\left(fx+e\right)}+4a\sin\left(fx+e\right)-6a\right)\frac{1}{\sqrt{a+a\sin}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] -1/2*(1+sin(f*x+e))*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*(a-a*sin(f*x+e))^(1/2)+4*a*sin(f*x+e)-6*a)/cos(f*x+e)/(a+a*si
n(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(fx+e\right)+a}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^2, x)
```

Fricas [A] time = 1.82154, size = 459, normalized size = 4.54

$$\frac{\sqrt{2}\sqrt{a}\cos\left(fx+e\right)\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)+3a\cos\left(fx+e\right)-\left(a\cos\left(fx+e\right)-2a\right)\sin\left(fx+e\right)+2a}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-2}\right)-4}{4f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(a)*cos(f*x + e)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a
*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x
+ e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(a*sin(f*x + e) + a)*
(2*sin(f*x + e) - 3))/(f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\tan^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\sin\left(fx+e\right)+a}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^2, x)
```

3.93 $\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=89

$$\frac{3a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f) + (
3*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/f
```

Rubi [A] time = 0.192413, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2716, 2981, 2773, 206}

$$\frac{3a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f) + (
3*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Sqrt[a + a*S
in[e + f*x]])/f
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
```

```
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f} + \frac{\int \csc(e+fx)\left(\frac{a}{2}-\frac{3}{2}a\sin(e+fx)\right)\sqrt{a+a\sin}}{a}$$

$$= \frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f} + \frac{1}{2}\int \csc(e+fx)\sqrt{a+a}$$

$$= \frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, -\right.}{f}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\sqrt{a+a\sin(e+}}{f}$$

Mathematica [B] time = 0.954969, size = 206, normalized size = 2.31

$$\frac{\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(4\sin\left(\frac{1}{2}(e+fx)\right)+2\sin\left(\frac{3}{2}(e+fx)\right)-4\cos\left(\frac{1}{2}(e+fx)\right)+2\cos\left(\frac{3}{2}(e+fx)\right)-\sin\right.}{f\left(\cot\left(\frac{1}{2}(e+fx)\right)+1\right)\left(\csc\left(\frac{1}{4}(e+fx)\right)-\sec\left(\frac{1}{4}(e\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(-4*Cos[(e + f*x)/2] + 2*Cos
[(3*(e + f*x))/2] + 4*Sin[(e + f*x)/2] - Log[1 + Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]]*Sin[e + f*x] + Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[
e + f*x] + 2*Sin[(3*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x
)/4] - Sec[(e + f*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

Maple [A] time = 0.55, size = 125, normalized size = 1.4

$$\frac{1+\sin\left(fx+e\right)}{\sin\left(fx+e\right)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\sin\left(fx+e\right)\left(2\sqrt{a-a\sin\left(fx+e\right)}a^{3/2}-\operatorname{Artanh}\left(\sqrt{a-a\sin\left(fx\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] (1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*(2*(a-a*sin(f*x+e))^(
1/2)*a^(3/2)-arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^2)-(a-a*sin(f*x+e))^
(1/2)*a^(3/2))/sin(f*x+e)/a^(3/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(fx+e\right)+a}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^2, x)
```

Fricas [B] time = 1.93572, size = 749, normalized size = 8.42

$$\frac{\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\log\left(\frac{a\cos\left(fx+e\right)^{3}-7a\cos\left(fx+e\right)^{2}-4\left(\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+3\right)\sin\left(fx+e\right)-2\right.}{\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)}\right.}{4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a*
cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3
)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a
*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)
/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos
(f*x + e) - 1)) - 4*(2*cos(f*x + e)^2 + (2*cos(f*x + e) + 3)*sin(f*x + e) -
 cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x
 + e) + f)*sin(f*x + e) - f)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*cot(e + f*x)**2, x)
```

Giac [B] time = 2.0909, size = 570, normalized size = 6.4

$$\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+1\right)}{\sqrt{-a}}-\sqrt{a}\log\left(\left|-\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)+\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}\right|\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e
)^2 + a))/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/sqrt(-a) - sqrt(a)*log(ab
```

```
s(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)))*sgn(
tan(1/2*f*x + 1/2*e) + 1) + 2*a^(3/2)*sgn(tan(1/2*f*x + 1/2*e) + 1)/((sqrt(
a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a) - (2*s
qrt(2)*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - sqrt(2)*sqrt(-a)*sq
rt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 2*a*arctan((sqrt(2)*sqrt(a) + sqrt(a
))/sqrt(-a)) - sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 5*sqrt(2)*
sqrt(-a)*sqrt(a) + 11*sqrt(-a)*sqrt(a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt
(2)*sqrt(-a) + sqrt(-a)) + (5*a*sgn(tan(1/2*f*x + 1/2*e) + 1) + (a*sgn(tan(
1/2*f*x + 1/2*e) + 1)*tan(1/2*f*x + 1/2*e) - 4*a*sgn(tan(1/2*f*x + 1/2*e) +
 1))*tan(1/2*f*x + 1/2*e))/sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))/f
```

3.94 $\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=163

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{11a\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}+\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)}}{3f}$$

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) + (11*a*Cot[e + f*x])/(8
*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cot[e + f*x]*Csc[e + f*x])/(12*f*Sqrt[a +
 a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/
(3*f)
```

Rubi [A] time = 0.37672, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2718, 2646, 3044, 2980, 2772, 2773, 206}

$$-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{11a\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}+\frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) + (11*a*Cot[e + f*x])/(8
*f*Sqrt[a + a*Sin[e + f*x]]) - (a*Cot[e + f*x]*Csc[e + f*x])/(12*f*Sqrt[a +
 a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/
(3*f)
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \int \sqrt{a+a\sin(e+fx)}\,dx + \int \csc^4(e+fx)\sqrt{a+a\sin(e+fx)}\left(1-2\sin^2(e+fx)\right.$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{\int \csc^3(e}{}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a}}{3f}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{11a\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \mathrm{co}$$

$$= -\frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{11a\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \mathrm{co}$$

$$= \frac{11\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{11a\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} - \frac{a}{}$$

Mathematica [A] time = 1.56776, size = 309, normalized size = 1.9

$$\frac{\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(-252\sin\left(\frac{1}{2}(e+fx)\right)-250\sin\left(\frac{3}{2}(e+fx)\right)+114\sin\left(\frac{5}{2}(e+fx)\right)+48\sin\left(\frac{7}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(252*Cos[(e + f*x)/2] - 250
*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/2] + 48*Cos[(7*(e + f*x))/2]
- 252*Sin[(e + f*x)/2] + 99*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Si
n[e + f*x] - 99*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] -
 250*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] - 33*Log[1 + Cos[(e +
f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 33*Log[1 - Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 48*Sin[(7*(e + f*x))/2]))/(24*f*(1 +
 Cot[(e + f*x)/2])*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

Maple [A] time = 0.701, size = 170, normalized size = 1.

$$\frac{1+\sin\left(fx+e\right)}{24\left(\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(-48\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}a^{7/2}\left(\sin\left(fx+e\right)\right)^{3}+33\left(-a\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)/a^(7/2)*(-48*(-a*(-1+sin(f*x
+e)))^(1/2)*a^(7/2)*sin(f*x+e)^3+33*(-a*(-1+sin(f*x+e)))^(5/2)*a^(3/2)+33*a
rctanh((-a*(-1+sin(f*x+e)))^(1/2)/a^(1/2))*a^4*sin(f*x+e)^3-56*(-a*(-1+sin(
f*x+e)))^(3/2)*a^(5/2)+15*(-a*(-1+sin(f*x+e)))^(1/2)*a^(7/2))/sin(f*x+e)^3/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\sin\left(fx+e\right)+a}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^4, x)
```

Fricas [B] time = 1.81584, size = 1025, normalized size = 6.29

$$33\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(33*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e
)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7
*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2
*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a
*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 +
 cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) +
4*(48*cos(f*x + e)^4 - 33*cos(f*x + e)^3 - 139*cos(f*x + e)^2 + (48*cos(f*x
 + e)^3 + 81*cos(f*x + e)^2 - 58*cos(f*x + e) - 83)*sin(f*x + e) + 25*cos(f
*x + e) + 83)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e
)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*sin(f*x +
e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.1154, size = 950, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(66*a*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/
2*e)^2 + a))/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/sqrt(-a) - 33*sqrt(a)*
log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))
)*sgn(tan(1/2*f*x + 1/2*e) + 1) - (330*sqrt(2)*a*arctan((sqrt(2)*sqrt(a) +
sqrt(a))/sqrt(-a)) - 165*sqrt(2)*sqrt(-a)*sqrt(a)*log(sqrt(2)*sqrt(a) + sqr
t(a)) + 462*a*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 231*sqrt(-a)*s
qrt(a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 992*sqrt(2)*sqrt(-a)*sqrt(a) + 1422
*sqrt(-a)*sqrt(a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(5*sqrt(2)*sqrt(-a) + 7*sq
rt(-a)) + (130*a*sgn(tan(1/2*f*x + 1/2*e) + 1) - (99*a*sgn(tan(1/2*f*x + 1/
```

```
2*e) + 1) - (32*a*sgn(tan(1/2*f*x + 1/2*e) + 1) - (2*a*sgn(tan(1/2*f*x + 1/
2*e) + 1)*tan(1/2*f*x + 1/2*e) + 3*a*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2
*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))/sqrt(a*tan(1/2*f
*x + 1/2*e)^2 + a) - 2*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*
x + 1/2*e)^2 + a))^5*a*sgn(tan(1/2*f*x + 1/2*e) + 1) - 30*(sqrt(a)*tan(1/2*
f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*sgn(tan(1/2*f*
x + 1/2*e) + 1) + 72*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1
/2*e)^2 + a))^2*a^(5/2)*sgn(tan(1/2*f*x + 1/2*e) + 1) - 3*(sqrt(a)*tan(1/2*
f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*sgn(tan(1/2*f*x + 1/
2*e) + 1) - 34*a^(7/2)*sgn(tan(1/2*f*x + 1/2*e) + 1))/((sqrt(a)*tan(1/2*f*x
 + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^3)/f
```

3.95 $\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=167

$$\frac{2a^3\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^{3/2}}-\frac{4a^2\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}f}+\frac{\sec^3(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}-\frac{7\ldots}{}$$

```
[Out] -(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])
])/(2*Sqrt[2]*f) + (2*a^3*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^(3/2))
- (4*a^2*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (7*a*Sec[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(2*f) + (Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/
(3*f)
```

Rubi [A] time = 0.975016, antiderivative size = 195, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2714, 2647, 2646, 4401, 2675, 2649, 206, 2878, 2855}

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}f}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}+\frac{4\sec^3(e+fx)(a\sin(e+f\ldots}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] -(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])
])/(2*Sqrt[2]*f) - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2
*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (a*Sec[e + f*x]*Sqrt[a +
a*Sin[e + f*x]])/(2*f) - (23*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*
f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(5/2))/(a*f)
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2675

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos
[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,
f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m
+ 1/2, 2*p]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
```

```
b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx = \int (a+a\sin(e+fx))^{3/2}\,dx - \int \sec^4(e+fx)(a+a\sin(e+fx))^{3/2}\left(1-2\sin^2(e\right.$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{1}{3}(4a)\int \sqrt{a+a\sin(e+fx)}\,dx - \int \left(\mathrm{sec}\right.$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + 2\int \sec^2(e+f$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\sec^3(e+fx)(a}{}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{a\sec(e+fx)\sqrt{}}{2}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} + \frac{a\sec(e+fx)\sqrt{}}{2}$$

$$= \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a}}{3f}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+}}{3f}$$

Mathematica [C] time = 5.56233, size = 141, normalized size = 0.84

$$\frac{a\sec^3(e+fx)\sqrt{a(\sin(e+fx)+1)}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2\left(54\sin(e+fx)+\sin(3(e+fx))+6\cos(2(e+f\right.}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a*Sec[e + f*x]^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[a*(1 + Sin[e
 + f*x])]*(-45 + 6*Cos[2*(e + f*x)] + (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 + I
/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2
])^3 + 54*Sin[e + f*x] + Sin[3*(e + f*x)]))/(6*f)
```

Maple [A] time = 0.574, size = 139, normalized size = 0.8

$$\frac{1+\sin\left(fx+e\right)}{12\,a\left(-1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(3\,a^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)\left(a-a\sin\left(fx+e\right)\right)^{3/2}-8\,a^{3}\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x)
```

```
[Out] 1/12*(1+sin(f*x+e))/a/(-1+sin(f*x+e))*(3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(3/2)-8*a^3*sin(f*x+e)*c
os(f*x+e)^2-24*a^3*cos(f*x+e)^2-106*a^3*sin(f*x+e)+102*a^3)/cos(f*x+e)/(a+a
*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.90228, size = 648, normalized size = 3.88

$$\frac{3\left(\sqrt{2}a\cos\left(fx+e\right)\sin\left(fx+e\right)-\sqrt{2}a\cos\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{a\sin\left(fx+e\right)+a}\left(\sqrt{2}\cos\left(fx+e\right)-\sqrt{2}\sin\left(fx+e\right)+\sqrt{2}\right)\sqrt{a}+3a}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-c}\right.}{24\left(f\cos\left(fx+e\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(sqrt(2)*a*cos(f*x + e)*sin(f*x + e) - sqrt(2)*a*cos(f*x + e))*sqrt
(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a*sin(f*x + e) + a)*(sqrt(2)*cos(f*x +
e) - sqrt(2)*sin(f*x + e) + sqrt(2))*sqrt(a) + 3*a*cos(f*x + e) - (a*cos(f*
x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2)) - 4*(12*a*cos(f*x + e)^2 + (4*a*cos(f*x + e)^
2 + 53*a)*sin(f*x + e) - 51*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)*si
n(f*x + e) - f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*tan(f*x + e)^4, x)
```

3.96 $\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}+\frac{7\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

```
[Out] (11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]*(a +
 a*Sin[e + f*x])^(3/2))/(3*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(5/2))
/(3*a*f)
```

Rubi [A] time = 0.195731, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2713, 2855, 2646}

$$\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}+\frac{7\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]*(a +
 a*Sin[e + f*x])^(3/2))/(3*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(5/2))
/(3*a*f)
```

Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx &= -\frac{2\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{3af}+\frac{2\int \sec^2(e+fx)(a+a\sin(e+fx))^{3/2}\big(}{3a}\\ &= \frac{7\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{3f}-\frac{2\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{3af}-\frac{1}{6}(1\\ &= \frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}+\frac{7\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{3f}-\frac{2\sec(e+fx)(a}{3}\end{aligned}$$

Mathematica [A] time = 3.99156, size = 46, normalized size = 0.52

$$\frac{a\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}(-8\sin(e+fx)+\cos(2(e+fx))+15)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (a*Sec[e + f*x]*(15 + Cos[2*(e + f*x)] - 8*Sin[e + f*x])*Sqrt[a*(1 + Sin[e
+ f*x])])/(3*f)
```

Maple [A] time = 0.402, size = 55, normalized size = 0.6

$$-\frac{2a^2\left(1+\sin\left(fx+e\right)\right)\left(\left(\sin\left(fx+e\right)\right)^2+4\sin\left(fx+e\right)-8\right)}{3f\cos\left(fx+e\right)}\frac{1}{\sqrt{a+a\sin\left(fx+e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x)
```

```
[Out] -2/3*a^2*(1+sin(f*x+e))*(sin(f*x+e)^2+4*sin(f*x+e)-8)/cos(f*x+e)/(a+a*sin(f
*x+e))^(1/2)/f
```

Maxima [A] time = 1.54691, size = 196, normalized size = 2.23

$$-\frac{8\left(2\,a^{\frac{3}{2}}-\frac{2\,a^{\frac{3}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{3\,a^{\frac{3}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{2\,a^{\frac{3}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{2\,a^{\frac{3}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}\right)}{3\,f\left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-1\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -8/3*(2*a^(3/2) - 2*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^(3/2)*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e) +
 1)^3 + 2*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(f*(sin(f*x + e)/(co
s(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

Fricas [A] time = 1.7535, size = 123, normalized size = 1.4

$$\frac{2\left(a\cos\left(fx+e\right)^{2}-4\,a\sin\left(fx+e\right)+7\,a\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 2/3*(a*cos(f*x + e)^2 - 4*a*sin(f*x + e) + 7*a)*sqrt(a*sin(f*x + e) + a)/(f
*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*tan(f*x + e)^2, x)
```

3.97 $\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=121

$$\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{5a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)(a\sin(e+fx)+}{f}$$

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (
11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (5*a*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(3*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/f
```

Rubi [A] time = 0.320568, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2716, 2976, 2981, 2773, 206}

$$\frac{11a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{5a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)(a\sin(e+fx)+}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (
11*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (5*a*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(3*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/f
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
```

```
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = -\frac{\cot(e+fx)(a+a\sin(e+fx))^{3/2}}{f} + \frac{\int \csc(e+fx)\left(\frac{3a}{2}-\frac{5}{2}a\sin(e+fx)\right)(a+$$

$$= \frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+fx)(a+a\sin(e+fx))^{3/2}}{f} + \frac{2\int c$$

$$= \frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+fx)(a+$$

$$= \frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+fx)(a+$$

$$= -\frac{3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{5a\cos(e+fx)\sqrt{a+}}{3f}$$

Mathematica [A] time = 0.759996, size = 233, normalized size = 1.93

$$-\frac{a\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(-14\sin\left(\frac{1}{2}(e+fx)\right)-9\sin\left(\frac{3}{2}(e+fx)\right)-\sin\left(\frac{5}{2}(e+fx)\right)+14\cos\left(\frac{1}{2}(e+fx)\right)\right.}{3f\left(\cot\left(\frac{1}{2}(e+fx)\right)+1\right)\left(c\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a*Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(14*Cos[(e + f*x)/2] - 9*
Cos[(3*(e + f*x))/2] + Cos[(5*(e + f*x))/2] - 14*Sin[(e + f*x)/2] + 9*Log[1
 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 9*Log[1 - Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 9*Sin[(3*(e + f*x))/2] - Sin[(5*(
e + f*x))/2]))/(3*f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f*x
)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

Maple [A] time = 0.642, size = 144, normalized size = 1.2

$$-\frac{1+\sin(fx+e)}{3\sin(fx+e)\cos(fx+e)f}\sqrt{-a(-1+\sin(fx+e))}\left(\sin(fx+e)\left(-12\sqrt{a-a\sin(fx+e)}a^{3/2}+2\sqrt{a}\,(a-a\sin(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/3*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*(-12*(a-a*sin(f*
x+e))^(1/2)*a^(3/2)+2*a^(1/2)*(a-a*sin(f*x+e))^(3/2)+9*arctanh((a-a*sin(f*x
+e))^(1/2)/a^(1/2))*a^2)+3*(a-a*sin(f*x+e))^(1/2)*a^(3/2))/sin(f*x+e)/a^(1/
2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^2, x)
```

Fricas [B] time = 1.84636, size = 833, normalized size = 6.88

$$\frac{9\left(a\cos\left(fx+e\right)^{2}-\left(a\cos\left(fx+e\right)+a\right)\sin\left(fx+e\right)-a\right)\sqrt{a}\log\left(\frac{a\cos\left(fx+e\right)^{3}-7\,a\cos\left(fx+e\right)^{2}-4\left(\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+3\right)\sin\left(fx+e\right.\right.}{\cos\left(fx+e\right)^{3}+\cos\left(f\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(9*(a*cos(f*x + e)^2 - (a*cos(f*x + e) + a)*sin(f*x + e) - a)*sqrt(a)*
log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x +
 e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a
) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x +
e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e
) - cos(f*x + e) - 1)) + 4*(2*a*cos(f*x + e)^3 - 8*a*cos(f*x + e)^2 + a*cos
(f*x + e) - (2*a*cos(f*x + e)^2 + 10*a*cos(f*x + e) + 11*a)*sin(f*x + e) +
11*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e) + f)*si
```

```
n(f*x + e) - f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.63422, size = 671, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/6*(18*a^2*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/
2*e)^2 + a))/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/sqrt(-a) - 9*a^(3/2)*l
og(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)))
*sgn(tan(1/2*f*x + 1/2*e) + 1) + 6*a^(5/2)*sgn(tan(1/2*f*x + 1/2*e) + 1)/((
sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a) -
 (18*sqrt(2)*a^2*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 9*sqrt(2)*s
qrt(-a)*a^(3/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 18*a^2*arctan((sqrt(2)*sqr
t(a) + sqrt(a))/sqrt(-a)) - 9*sqrt(-a)*a^(3/2)*log(sqrt(2)*sqrt(a) + sqrt(a
)) + 19*sqrt(2)*sqrt(-a)*a^(3/2) + 41*sqrt(-a)*a^(3/2))*sgn(tan(1/2*f*x + 1
/2*e) + 1)/(sqrt(2)*sqrt(-a) + sqrt(-a)) + (23*a^3*sgn(tan(1/2*f*x + 1/2*e)
 + 1) - (12*a^3*sgn(tan(1/2*f*x + 1/2*e) + 1) - (18*a^3*sgn(tan(1/2*f*x + 1
/2*e) + 1) + (3*a^3*sgn(tan(1/2*f*x + 1/2*e) + 1)*tan(1/2*f*x + 1/2*e) - 20
*a^3*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2
*e))*tan(1/2*f*x + 1/2*e))/(a*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2))/f
```

3.98 $\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

Optimal. Leaf size=197

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{29a^2\cot(e+fx)}{24f\sqrt{a\sin(e+fx)+a}}+\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-$$

```
[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (29*a^2*Cot[e + f*
x])/(24*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(3*f) - (a*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f)
 - (Cot[e + f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.496362, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2718, 2647, 2646, 3044, 2975, 2980, 2773, 206}

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{29a^2\cot(e+fx)}{24f\sqrt{a\sin(e+fx)+a}}+\frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8f}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2),x]
```


```
[Out] (37*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) + (29*a^2*Cot[e + f*
x])/(24*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(3*f) - (a*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*f)
 - (Cot[e + f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2))/(3*f)
```


Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
```

```
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \int (a+a\sin(e+fx))^{3/2}\,dx + \int \csc^4(e+fx)(a+a\sin(e+fx))^{3/2}\left(1-2\sin^2(e\right.$$

$$= -\frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{\cot(e+fx)\csc^2(e+fx)(a+a\sin(e+f}{3f}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{a\cot(e+fx)\,\mathrm{cs}}{}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(}}{3f}$$

$$= -\frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(}}{3f}$$

$$= \frac{37a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+f}}$$

Mathematica [A] time = 1.35636, size = 334, normalized size = 1.7

$$-\frac{a\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(276\sin\left(\frac{1}{2}(e+fx)\right)+326\sin\left(\frac{3}{2}(e+fx)\right)-78\sin\left(\frac{5}{2}(e+fx)\right)-72\sin\left(\frac{7}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(a*Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(-276*Cos[(e + f*x)/2] +
 326*Cos[(3*(e + f*x))/2] + 78*Cos[(5*(e + f*x))/2] - 72*Cos[(7*(e + f*x))/
2] + 8*Cos[(9*(e + f*x))/2] + 276*Sin[(e + f*x)/2] - 333*Log[1 + Cos[(e + f
*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 333*Log[1 - Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]]*Sin[e + f*x] + 326*Sin[(3*(e + f*x))/2] - 78*Sin[(5*(e + f*
x))/2] + 111*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)]
- 111*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 72*Si
n[(7*(e + f*x))/2] - 8*Sin[(9*(e + f*x))/2]))/(24*f*(1 + Cot[(e + f*x)/2])*
(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

Maple [A] time = 0.651, size = 196, normalized size = 1.

$$-\frac{1+\sin\left(fx+e\right)}{24\left(\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(96\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}a^{5/2}\left(\sin\left(fx+e\right)\right)^{3}-16\left(-a\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/24*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(96*(-a*(-1+sin(f*x+e)))^(1
/2)*a^(5/2)*sin(f*x+e)^3-16*(-a*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)^3*a^(3/2)
-111*arctanh((-a*(-1+sin(f*x+e)))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+15*(-a*(-
1+sin(f*x+e)))^(1/2)*a^(5/2)+8*(-a*(-1+sin(f*x+e)))^(3/2)*a^(3/2)-15*(-a*(-
1+sin(f*x+e)))^(5/2)*a^(1/2))/a^(3/2)/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+
e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^4, x)
```

Fricas [B] time = 1.72276, size = 1123, normalized size = 5.7

$$\frac{111 \left(a \cos\left(fx+e\right)^4 - 2\,a \cos\left(fx+e\right)^2 - \left(a \cos\left(fx+e\right)^3 + a \cos\left(fx+e\right)^2 - a \cos\left(fx+e\right) - a\right) \sin\left(fx+e\right) + a\right)\sqrt{a}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(111*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 - (a*cos(f*x + e)^3 + a*co
s(f*x + e)^2 - a*cos(f*x + e) - a)*sin(f*x + e) + a)*sqrt(a)*log((a*cos(f*x
 + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f
*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*
x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f
*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x +
e) - 1)) - 4*(16*a*cos(f*x + e)^5 - 64*a*cos(f*x + e)^4 - 17*a*cos(f*x + e)
^3 + 165*a*cos(f*x + e)^2 + 9*a*cos(f*x + e) - (16*a*cos(f*x + e)^4 + 80*a*
cos(f*x + e)^3 + 63*a*cos(f*x + e)^2 - 102*a*cos(f*x + e) - 93*a)*sin(f*x +
 e) - 93*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*cos(f*x + e)^
2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*sin(f*x + e)
 + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.10235, size = 1058, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] -1/48*(222*a^2*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x +
 1/2*e)^2 + a))/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/sqrt(-a) - 111*a^(3
/2)*log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 +
 a)))*sgn(tan(1/2*f*x + 1/2*e) + 1) - (1110*sqrt(2)*a^2*arctan((sqrt(2)*sqr
t(a) + sqrt(a))/sqrt(-a)) - 555*sqrt(2)*sqrt(-a)*a^(3/2)*log(sqrt(2)*sqrt(a
) + sqrt(a)) + 1554*a^2*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 777*
sqrt(-a)*a^(3/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 1192*sqrt(2)*sqrt(-a)*a^(
3/2) + 1706*sqrt(-a)*a^(3/2))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(5*sqrt(2)*sqrt
(-a) + 7*sqrt(-a)) + (182*a^3*sgn(tan(1/2*f*x + 1/2*e) + 1) - (105*a^3*sgn(
tan(1/2*f*x + 1/2*e) + 1) - (138*a^3*sgn(tan(1/2*f*x + 1/2*e) + 1) - (178*a
^3*sgn(tan(1/2*f*x + 1/2*e) + 1) - (18*a^3*sgn(tan(1/2*f*x + 1/2*e) + 1) -
(2*a^3*sgn(tan(1/2*f*x + 1/2*e) + 1)*tan(1/2*f*x + 1/2*e) + 9*a^3*sgn(tan(1
/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))*tan(1/2*f
*x + 1/2*e))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))/(a*tan(1/2*f*x + 1
/2*e)^2 + a)^(3/2) - 2*(9*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*
x + 1/2*e)^2 + a))^5*a^2*sgn(tan(1/2*f*x + 1/2*e) + 1) - 18*(sqrt(a)*tan(1/
2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*sgn(tan(1/2*
f*x + 1/2*e) + 1) + 48*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x +
 1/2*e)^2 + a))^2*a^(7/2)*sgn(tan(1/2*f*x + 1/2*e) + 1) - 9*(sqrt(a)*tan(1/
2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^4*sgn(tan(1/2*f*x +
1/2*e) + 1) - 22*a^(9/2)*sgn(tan(1/2*f*x + 1/2*e) + 1))/((sqrt(a)*tan(1/2*f
*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^3)/f
```

3.99 $\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=151

$$-\frac{2a^5\cos^5(e+fx)}{5f(a\sin(e+fx)+a)^{5/2}}+\frac{8a^4\cos^3(e+fx)}{3f(a\sin(e+fx)+a)^{3/2}}-\frac{12a^3\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{8a^2\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}+\frac{2a\,\mathrm{se}}{}$$

```
[Out] (-2*a^5*Cos[e + f*x]^5)/(5*f*(a + a*Sin[e + f*x])^(5/2)) + (8*a^4*Cos[e + f
*x]^3)/(3*f*(a + a*Sin[e + f*x])^(3/2)) - (12*a^3*Cos[e + f*x])/(f*Sqrt[a +
 a*Sin[e + f*x]]) - (8*a^2*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f + (2*a*
Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*f)
```

Rubi [A] time = 0.979098, antiderivative size = 208, normalized size of antiderivative = 1.38, number of steps used = 10, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2714, 2647, 2646, 4401, 2673, 2878, 2855}

$$-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{64a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{46a^2\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{2a\cos(e+fx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^4,x]
```

```
[Out] (-64*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (46*a^2*Sec[e + f*x]*Sqrt[a + a*Sin[e
 + f*x]])/(3*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (2*
a*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*f) + (26*Sec[e + f*x]^3*(a
+ a*Sin[e + f*x])^(5/2))/(3*f) - (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(7/
2))/(a*f)
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
```

```
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2673

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]
```

Rule 2878

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*g*(m + p + 2)), x] + Dist[1/(b*(m
 + p + 2)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m*(b*(m + 1) - a*(p
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 -
 b^2, 0] && NeQ[m + p + 2, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx = \int (a+a\sin(e+fx))^{5/2}\,dx - \int \sec^4(e+fx)(a+a\sin(e+fx))^{5/2}\left(1-2\sin^2(e\right.$$
$$= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} + \frac{1}{5}(8a)\int (a+a\sin(e+fx))^{3/2}\,dx - \int$$
$$= -\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} +$$
$$= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+f}{}$$
$$= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+f}{}$$
$$= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{46a^2\sec(e+}{}$$

Mathematica [A] time = 5.45924, size = 112, normalized size = 0.74

$$\frac{a^2\sqrt{a(\sin(e+fx)+1)}(1488\sin(e+fx)+16\sin(3(e+fx))+204\cos(2(e+fx))-3\cos(4(e+fx))-1225)}{60f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^4,x]
```

```
[Out] (a^2*Sqrt[a*(1 + Sin[e + f*x])]*(-1225 + 204*Cos[2*(e + f*x)] - 3*Cos[4*(e
+ f*x)] + 1488*Sin[e + f*x] + 16*Sin[3*(e + f*x)]))/(60*f*(Cos[(e + f*x)/2]
 - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

Maple [A] time = 0.404, size = 87, normalized size = 0.6

$$\frac{2a^3\left(1+\sin\left(fx+e\right)\right)\left(3\left(\sin\left(fx+e\right)\right)^4+8\left(\sin\left(fx+e\right)\right)^3+48\left(\sin\left(fx+e\right)\right)^2-192\sin\left(fx+e\right)+128\right)}{\left(-15+15\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\frac{1}{\sqrt{a+a\sin(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x)
```

```
[Out] 2/15*a^3*(1+sin(f*x+e))/(-1+sin(f*x+e))*(3*sin(f*x+e)^4+8*sin(f*x+e)^3+48*s
in(f*x+e)^2-192*sin(f*x+e)+128)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [B] time = 1.76375, size = 374, normalized size = 2.48

$$\frac{32\left(8\,a^{\frac{5}{2}}-\frac{24\,a^{\frac{5}{2}}\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}+\frac{44\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}-\frac{68\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}+\frac{75\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}}-\frac{68\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}}+\frac{44\,a^{\frac{5}{2}}\sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}-\frac{24\,a^{\frac{5}{2}}\sin\left(fx\right.}{\left(\cos\left(fx+e\right)\right.}}{15\,f\left(\frac{3\,\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1}-\frac{3\,\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+\frac{\sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}}-1\right)\left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 32/15*(8*a^(5/2) - 24*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 44*a^(5/2)*
sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 68*a^(5/2)*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 75*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 68*a^(5/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 44*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - 24*a^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 8*a^(5/2)*sin(f*x
 + e)^8/(cos(f*x + e) + 1)^8)/(f*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin
(f*x + e)^2/(cos(f*x + e) + 1)^2 + sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)
*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 1.43246, size = 246, normalized size = 1.63

$$\frac{2\left(3\,a^{2}\cos\left(fx+e\right)^{4}-54\,a^{2}\cos\left(fx+e\right)^{2}+179\,a^{2}-8\left(a^{2}\cos\left(fx+e\right)^{2}+23\,a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{15\left(f\cos\left(fx+e\right)\sin\left(fx+e\right)-f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 2/15*(3*a^2*cos(f*x + e)^4 - 54*a^2*cos(f*x + e)^2 + 179*a^2 - 8*(a^2*cos(f
*x + e)^2 + 23*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e)*
sin(f*x + e) - f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*tan(f*x + e)^4, x)
```

3.100 $\int (a + a\sin(e + fx))^{5/2} \tan^2(e + fx)\, dx$

Optimal. Leaf size=118

$$\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{124a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}+\frac{9\sec(e+fx)(a\sin$$

```
[Out] (124*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (9*Sec[e + f*x]*(a + a*Sin[e + f*x])^
(5/2))/(5*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(5*a*f)
```

Rubi [A] time = 0.214287, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2713, 2855, 2647, 2646}

$$\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{124a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}+\frac{9\sec(e+fx)(a\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]
```


```
[Out] (124*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*
x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (9*Sec[e + f*x]*(a + a*Sin[e + f*x])^
(5/2))/(5*f) - (2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(5*a*f)
```


Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx = -\frac{2\sec(e+fx)(a+a\sin(e+fx))^{7/2}}{5af} + \frac{2\int \sec^2(e+fx)(a+a\sin(e+fx))^{5/2}\left(\right.}{5a}$$

$$= \frac{9\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{5f} - \frac{2\sec(e+fx)(a+a\sin(e+fx))^{7/2}}{5af} - \frac{1}{10}($$

$$= \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{9\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{5f} - \frac{2\,\mathrm{s}}{}$$

$$= \frac{124a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{9\sec(e+fx)(}{}$$

Mathematica [A] time = 5.46323, size = 60, normalized size = 0.51

$$\frac{a^2\sec(e+fx)\sqrt{a(\sin(e+fx)+1)}(-185\sin(e+fx)+3\sin(3(e+fx))+22\cos(2(e+fx))+330)}{30f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]
```

```
[Out] (a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(330 + 22*Cos[2*(e + f*x)] - 1
85*Sin[e + f*x] + 3*Sin[3*(e + f*x)]))/(30*f)
```

Maple [A] time = 0.412, size = 67, normalized size = 0.6

$$-\frac{2 a^{3} \left(1+\sin \left(f x +e \right)\right) \left(3 \left(\sin \left(f x +e \right)\right)^{3}+11 \left(\sin \left(f x +e \right)\right)^{2}+44 \sin \left(f x +e \right)-88\right)}{15 f \cos \left(f x +e \right)} \frac{1}{\sqrt{a +a \sin \left(f x +e \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x)
```

```
[Out] -2/15*a^3*(1+sin(f*x+e))*(3*sin(f*x+e)^3+11*sin(f*x+e)^2+44*sin(f*x+e)-88)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [A] time = 1.66292, size = 258, normalized size = 2.19

$$-\frac{8 \left(22 a^{\frac{5}{2}} - \frac{22 a^{\frac{5}{2}} \sin\left(fx+e\right)}{\cos\left(fx+e\right)+1} + \frac{55 a^{\frac{5}{2}} \sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}} - \frac{50 a^{\frac{5}{2}} \sin\left(fx+e\right)^{3}}{\left(\cos\left(fx+e\right)+1\right)^{3}} + \frac{55 a^{\frac{5}{2}} \sin\left(fx+e\right)^{4}}{\left(\cos\left(fx+e\right)+1\right)^{4}} - \frac{22 a^{\frac{5}{2}} \sin\left(fx+e\right)^{5}}{\left(\cos\left(fx+e\right)+1\right)^{5}} + \frac{22 a^{\frac{5}{2}} \sin\left(fx+e\right)^{6}}{\left(\cos\left(fx+e\right)+1\right)^{6}}\right)}{15 f \left(\frac{\sin\left(fx+e\right)}{\cos\left(fx+e\right)+1} - 1\right) \left(\frac{\sin\left(fx+e\right)^{2}}{\left(\cos\left(fx+e\right)+1\right)^{2}} + 1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] -8/15*(22*a^(5/2) - 22*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 55*a^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 50*a^(5/2)*sin(f*x + e)^3/(cos(f*x +
 e) + 1)^3 + 55*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22*a^(5/2)*si
n(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22*a^(5/2)*sin(f*x + e)^6/(cos(f*x + e)
 + 1)^6)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 1)^(5/2))
```

Fricas [A] time = 1.42272, size = 173, normalized size = 1.47

$$\frac{2 \left(11 a^{2} \cos\left(fx+e\right)^{2} + 77 a^{2} + \left(3 a^{2} \cos\left(fx+e\right)^{2} - 47 a^{2}\right) \sin\left(fx+e\right)\right) \sqrt{a \sin\left(fx+e\right) + a}}{15 f \cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 2/15*(11*a^2*cos(f*x + e)^2 + 77*a^2 + (3*a^2*cos(f*x + e)^2 - 47*a^2)*sin(
f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*tan(f*x + e)^2, x)
```

3.101 $\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=151

$$\frac{49a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}+\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{7a\cos(e+fx)(a\sin(e+$$

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (
49*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (7*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^
(3/2))/(5*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/f
```

Rubi [A] time = 0.428696, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2716, 2976, 2981, 2773, 206}

$$\frac{49a^3\cos(e+fx)}{15f\sqrt{a\sin(e+fx)+a}}+\frac{31a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}-\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{7a\cos(e+fx)(a\sin(e+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-5*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (
49*a^3*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) + (31*a^2*Cos[e + f*x]
*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (7*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^
(3/2))/(5*f) - (Cot[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/f
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
```

```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
 b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
 && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = -\frac{\cot(e+fx)(a+a\sin(e+fx))^{5/2}}{f} + \frac{\int \csc(e+fx)\left(\frac{5a}{2}-\frac{7}{2}a\sin(e+fx)\right)(a+$$

$$= \frac{7a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} - \frac{\cot(e+fx)(a+a\sin(e+fx))^{5/2}}{f} + \frac{2}{}$$

$$= \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{7a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} -$$

$$= \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{7a\cos(e+f}{}$$

$$= \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{7a\cos(e+f}{}$$

$$= -\frac{5a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{}}{15}$$

Mathematica [A] time = 1.19308, size = 261, normalized size = 1.73

$$-\frac{a^2\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(-125\sin\left(\frac{1}{2}(e+fx)\right)-93\sin\left(\frac{3}{2}(e+fx)\right)-25\sin\left(\frac{5}{2}(e+fx)\right)+3\sin\left(\frac{7}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a^2*Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(125*Cos[(e + f*x)/2] -
 93*Cos[(3*(e + f*x))/2] + 25*Cos[(5*(e + f*x))/2] + 3*Cos[(7*(e + f*x))/2]
 - 125*Sin[(e + f*x)/2] + 150*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*
Sin[e + f*x] - 150*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x
] - 93*Sin[(3*(e + f*x))/2] - 25*Sin[(5*(e + f*x))/2] + 3*Sin[(7*(e + f*x))
/2]))/(30*f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f*x)/4])*(C
sc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

Maple [A] time = 0.733, size = 162, normalized size = 1.1

$$\frac{1+\sin\left(fx+e\right)}{15\sin\left(fx+e\right)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(\sin\left(fx+e\right)\left(90\sqrt{a-a\sin\left(fx+e\right)}a^{5/2}-40\,a^{3/2}\left(a-a\sin\left(fx+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/15*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*(90*(a-a*sin(f*x
+e))^(1/2)*a^(5/2)-40*a^(3/2)*(a-a*sin(f*x+e))^(3/2)+6*a^(1/2)*(a-a*sin(f*x
+e))^(5/2)-75*arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^3)-15*(a-a*sin(f*x+
e))^(1/2)*a^(5/2))/sin(f*x+e)/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(5/2)*cot(f*x + e)^2, x)
```

Fricas [B] time = 1.57082, size = 934, normalized size = 6.19

$$75\left(a^{2}\cos\left(fx+e\right)^{2}-a^{2}-\left(a^{2}\cos\left(fx+e\right)+a^{2}\right)\sin\left(fx+e\right)\right)\sqrt{a}\log\left(\frac{a\cos\left(fx+e\right)^{3}-7\,a\cos\left(fx+e\right)^{2}-4\left(\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+3\right)\sin\right.}{\cos\left(fx+e\right)^{3}\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/60*(75*(a^2*cos(f*x + e)^2 - a^2 - (a^2*cos(f*x + e) + a^2)*sin(f*x + e))
*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (
```

```
cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) +
a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*s
in(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*si
n(f*x + e) - cos(f*x + e) - 1)) + 4*(6*a^2*cos(f*x + e)^4 + 28*a^2*cos(f*x
+ e)^3 - 40*a^2*cos(f*x + e)^2 - 13*a^2*cos(f*x + e) + 49*a^2 + (6*a^2*cos(
f*x + e)^3 - 22*a^2*cos(f*x + e)^2 - 62*a^2*cos(f*x + e) - 49*a^2)*sin(f*x
+ e))*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e) + f)*si
n(f*x + e) - f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 2.90585, size = 714, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/30*(150*a^3*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x +
1/2*e)^2 + a))/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/sqrt(-a) - 75*a^(5/2
)*log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a
)))*sgn(tan(1/2*f*x + 1/2*e) + 1) + 30*a^(7/2)*sgn(tan(1/2*f*x + 1/2*e) + 1
)/((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 -
a) - (150*sqrt(2)*a^3*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 75*sqr
t(2)*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 150*a^3*arctan((sqrt
(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 75*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a)
+ sqrt(a)) + 83*sqrt(2)*sqrt(-a)*a^(5/2) + 181*sqrt(-a)*a^(5/2))*sgn(tan(1/
2*f*x + 1/2*e) + 1)/(sqrt(2)*sqrt(-a) + sqrt(-a)) + (127*a^5*sgn(tan(1/2*f*
x + 1/2*e) + 1) + (205*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) - (160*a^5*sgn(tan
(1/2*f*x + 1/2*e) + 1) - (45*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) + (15*a^5*sg
```

```
n(tan(1/2*f*x + 1/2*e) + 1)*tan(1/2*f*x + 1/2*e) - 112*a^5*sgn(tan(1/2*f*x
+ 1/2*e) + 1))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/
2*e))*tan(1/2*f*x + 1/2*e)^2)/(a*tan(1/2*f*x + 1/2*e)^2 + a)^(5/2))/f
```

3.102 $\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

Optimal. Leaf size=227

$$-\frac{9a^3\cos(e+fx)}{40f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{17a^2\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{24f}+\frac{55a^{5/2}\tanh^{-1}}{8}$$

```
[Out] (55*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (9*a^3*Cos[e + f*x])/(40*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (17*a^2*Cot[e + f*x]*Sqrt[a + a*Sin[
e + f*x]])/(24*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (
5*a*Cot[e + f*x]*Csc[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(12*f) - (Cot[e +
 f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(3*f)
```

Rubi [A] time = 0.627151, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2718, 2647, 2646, 3044, 2975, 2981, 2773, 206}

$$-\frac{9a^3\cos(e+fx)}{40f\sqrt{a\sin(e+fx)+a}}-\frac{16a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{15f}+\frac{17a^2\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{24f}+\frac{55a^{5/2}\tanh^{-1}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (55*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*f)
 - (9*a^3*Cos[e + f*x])/(40*f*Sqrt[a + a*Sin[e + f*x]]) - (16*a^2*Cos[e + f
*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) + (17*a^2*Cot[e + f*x]*Sqrt[a + a*Sin[
e + f*x]])/(24*f) - (2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (
5*a*Cot[e + f*x]*Csc[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(12*f) - (Cot[e +
 f*x]*Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(3*f)
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2975

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Si
mp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e
+ f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a
*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*
A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b
*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
 b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = \int (a+a\sin(e+fx))^{5/2}\,dx + \int \csc^4(e+fx)(a+a\sin(e+fx))^{5/2}\left(1-2\sin^2(\right.$$

$$= -\frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} - \frac{\cot(e+fx)\csc^2(e+fx)(a+a\sin(e}{3f}$$

$$= -\frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f}$$

$$= -\frac{64a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{17a^2\cot(e}{}$$

$$= -\frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{17a^2\cot(e}{}$$

$$= -\frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{17a^2\cot(e}{}$$

$$= \frac{55a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{}}{15}$$

Mathematica [A] time = 1.89914, size = 360, normalized size = 1.59

$$-\frac{a^2\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(\sin(e+fx)+1)}\left(-108\sin\left(\frac{1}{2}(e+fx)\right)+706\sin\left(\frac{3}{2}(e+fx)\right)+450\sin\left(\frac{5}{2}(e+fx)\right)-156\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] -(a^2*Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(108*Cos[(e + f*x)/2]
+ 706*Cos[(3*(e + f*x))/2] - 450*Cos[(5*(e + f*x))/2] - 156*Cos[(7*(e + f*x
))/2] + 100*Cos[(9*(e + f*x))/2] + 12*Cos[(11*(e + f*x))/2] - 108*Sin[(e +
f*x)/2] - 2475*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] +
2475*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 706*Sin[(3
*(e + f*x))/2] + 450*Sin[(5*(e + f*x))/2] + 825*Log[1 + Cos[(e + f*x)/2] -
Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 825*Log[1 - Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2]]*Sin[3*(e + f*x)] - 156*Sin[(7*(e + f*x))/2] - 100*Sin[(9*(e + f*
x))/2] + 12*Sin[(11*(e + f*x))/2]))/(120*f*(1 + Cot[(e + f*x)/2])*(Csc[(e +
 f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

Maple [A] time = 0.694, size = 222, normalized size = 1.

$$-\frac{1+\sin\left(fx+e\right)}{120\left(\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(48\left(-a\left(-1+\sin\left(fx+e\right)\right)\right)^{5/2}\left(\sin\left(fx+e\right)\right)^{3}\sqrt{a}-320\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/120*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(48*(-a*(-1+sin(f*x+e)))^(
5/2)*sin(f*x+e)^3*a^(1/2)-320*(-a*(-1+sin(f*x+e)))^(3/2)*sin(f*x+e)^3*a^(3/
2)+480*(-a*(-1+sin(f*x+e)))^(1/2)*a^(5/2)*sin(f*x+e)^3-825*arctanh((-a*(-1+
sin(f*x+e)))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+135*(-a*(-1+sin(f*x+e)))^(5/2)
*a^(1/2)-440*(-a*(-1+sin(f*x+e)))^(3/2)*a^(3/2)+345*(-a*(-1+sin(f*x+e)))^(1
/2)*a^(5/2))/sin(f*x+e)^3/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [B] time = 1.73164, size = 1249, normalized size = 5.5

$$\frac{825\left(a^2\cos\left(fx+e\right)^4-2\,a^2\cos\left(fx+e\right)^2+a^2-\left(a^2\cos\left(fx+e\right)^3+a^2\cos\left(fx+e\right)^2-a^2\cos\left(fx+e\right)-a^2\right)\sin\left(fx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(825*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2 - (a^2*cos(f*x
+ e)^3 + a^2*cos(f*x + e)^2 - a^2*cos(f*x + e) - a^2)*sin(f*x + e))*sqrt(a)
*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x
+ e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(
a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x +
 e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x +
e) - cos(f*x + e) - 1)) - 4*(48*a^2*cos(f*x + e)^6 + 224*a^2*cos(f*x + e)^5
 - 128*a^2*cos(f*x + e)^4 - 583*a^2*cos(f*x + e)^3 + 147*a^2*cos(f*x + e)^2
 + 399*a^2*cos(f*x + e) - 27*a^2 + (48*a^2*cos(f*x + e)^5 - 176*a^2*cos(f*x
 + e)^4 - 304*a^2*cos(f*x + e)^3 + 279*a^2*cos(f*x + e)^2 + 426*a^2*cos(f*x
 + e) + 27*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 -
 2*f*cos(f*x + e)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e)
 - f)*sin(f*x + e) + f)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.1945, size = 1135, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] -1/240*(1650*a^3*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x
 + 1/2*e)^2 + a))/sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/sqrt(-a) - 825*a^
(5/2)*log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)^2
 + a)))*sgn(tan(1/2*f*x + 1/2*e) + 1) - (8250*sqrt(2)*a^3*arctan((sqrt(2)*s
qrt(a) + sqrt(a))/sqrt(-a)) - 4125*sqrt(2)*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqr
t(a) + sqrt(a)) + 11550*a^3*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) -
5775*sqrt(-a)*a^(5/2)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 728*sqrt(2)*sqrt(-a)
*a^(5/2) + 1030*sqrt(-a)*a^(5/2))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(5*sqrt(2)*
sqrt(-a) + 7*sqrt(-a)) + (346*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) + (405*a^5*
sgn(tan(1/2*f*x + 1/2*e) + 1) + (100*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) - (5
45*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) + (720*a^5*sgn(tan(1/2*f*x + 1/2*e) +
1) + (641*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) + 5*(20*a^5*sgn(tan(1/2*f*x + 1
/2*e) + 1) + (2*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1)*tan(1/2*f*x + 1/2*e) + 15
*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2
*e))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))*tan(
1/2*f*x + 1/2*e))*tan(1/2*f*x + 1/2*e))/(a*tan(1/2*f*x + 1/2*e)^2 + a)^(5/2
) - 10*(15*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 +
a))^5*a^3*sgn(tan(1/2*f*x + 1/2*e) + 1) + 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)
- sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2)*sgn(tan(1/2*f*x + 1/2*e) +
1) - 24*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))
^2*a^(9/2)*sgn(tan(1/2*f*x + 1/2*e) + 1) - 15*(sqrt(a)*tan(1/2*f*x + 1/2*e)
 - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^5*sgn(tan(1/2*f*x + 1/2*e) + 1) +
14*a^(11/2)*sgn(tan(1/2*f*x + 1/2*e) + 1))/((sqrt(a)*tan(1/2*f*x + 1/2*e) -
 sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^3)/f
```

3.103 $\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=150

$$\frac{\tan^3(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{a\sin(e+fx)\tan(e+fx)}{24f(a\sin(e+fx)+a)^{3/2}}-\frac{(127\sin(e+fx)+53)\sec(e+fx)}{192f\sqrt{a\sin(e+fx)+a}}-\frac{67\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{64\sqrt{2}\sqrt{a}f}$$

```
[Out] (-67*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(6
4*Sqrt[2]*Sqrt[a]*f) - (Sec[e + f*x]*(53 + 127*Sin[e + f*x]))/(192*f*Sqrt[a
 + a*Sin[e + f*x]]) + (a*Sin[e + f*x]*Tan[e + f*x])/(24*f*(a + a*Sin[e + f*
x])^(3/2)) + Tan[e + f*x]^3/(3*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.933351, antiderivative size = 241, normalized size of antiderivative = 1.61, number of steps used = 17, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2714, 2649, 206, 4401, 2687, 2681, 2650, 2877, 2855}

$$\frac{61a\cos(e+fx)}{64f(a\sin(e+fx)+a)^{3/2}}+\frac{7\sec^3(e+fx)\sqrt{a\sin(e+fx)+a}}{12af}-\frac{5\sec^3(e+fx)}{6f\sqrt{a\sin(e+fx)+a}}-\frac{61\sec(e+fx)}{48f\sqrt{a\sin(e+fx)+a}}+\frac{}{24f(}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (61*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(64
*Sqrt[2]*Sqrt[a]*f) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt
[a + a*Sin[e + f*x]])])/(Sqrt[a]*f) + (61*a*Cos[e + f*x])/(64*f*(a + a*Sin[
e + f*x])^(3/2)) + (7*a*Sec[e + f*x])/(24*f*(a + a*Sin[e + f*x])^(3/2)) - (
61*Sec[e + f*x])/(48*f*Sqrt[a + a*Sin[e + f*x]]) - (5*Sec[e + f*x]^3)/(6*f*
Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x]])/(12
*a*f)
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
```

```
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx - \int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} - \int \left(\frac{\sec^4(e+fx)}{\sqrt{a(1+\sin(e+fx))}} - \frac{2\sec^2(e+fx)\tan^2(e+}{\sqrt{a(1+\sin(e+fx))}}\right.$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + 2\int \frac{\sec^2(e+fx)\tan^2(e+fx)}{\sqrt{a(1+\sin(e+fx))}}\,dx - \int \frac{\sec^4(e+fx)}{\sqrt{a(1+\sin(e+fx))}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{5\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} + \frac{\int \sec^4(e+fx)\sqrt{a+a\sin(e+fx)}\,(}{2a^2}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{5\sec^3(e+fx)}{6f\sqrt{a+a\sin(e+fx)}} + \frac{7\sec^3}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{61\sec(e+fx)}{48f\sqrt{a+a\sin(e+fx)}} - \frac{5}{6f\sqrt{a}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{}{48}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}} + \frac{7a\sec(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} - \frac{}{48}$$

$$= \frac{61\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{64\sqrt{2}\sqrt{a}f} - \frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{61a\cos(e+fx)}{64f(a+a\sin(e+fx))^{3/2}} + \frac{}{2}$$

Mathematica [C] time = 0.673361, size = 118, normalized size = 0.79

$$\frac{-\sec^3(e+fx)(-41\sin(e+fx)+183\sin(3(e+fx))+122\cos(2(e+fx))+90)+(804+804i)(-1)^{3/4}\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}{768f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ((804 + 804*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x
)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - Sec[e + f*x]^3*(90 + 122*Cos
[2*(e + f*x)] - 41*Sin[e + f*x] + 183*Sin[3*(e + f*x)]))/(768*f*Sqrt[a*(1 +
 Sin[e + f*x])])
```

Maple [A] time = 0.766, size = 231, normalized size = 1.5

$$\frac{1}{\left(-384+384\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(366\,a^{7/2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{2}+\left(402\,\sqrt{2}\mathrm{Artanh}\left(1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/384*(366*a^(7/2)*sin(f*x+e)*cos(f*x+e)^2+(402*2^(1/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin(f*x+e))^(3/2)-112*a^(7/2))*si
n(f*x+e)+(-201*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^2*(a-a*sin(f*x+e))^(3/2)+122*a^(7/2))*cos(f*x+e)^2+402*2^(1/2)*arctanh(1/
2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*(a-a*sin(f*x+e))^(3/2)-16*a^(
7/2))/a^(7/2)/(-1+sin(f*x+e))/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1
/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.63851, size = 624, normalized size = 4.16

$$\frac{201\sqrt{2}\left(\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)+\cos\left(fx+e\right)^{3}\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)+3\,a\cos\left(f\right.}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(\right.}\right.}{768\left(af\cos\left(fx+e\right)^{3}\sin\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/768*(201*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(a)*l
og(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x
 + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f
*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x
+ e) - 2)) - 4*(61*cos(f*x + e)^2 + (183*cos(f*x + e)^2 - 56)*sin(f*x + e)
- 8)*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e)^3*sin(f*x + e) + a*f*cos(f
*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 5.88912, size = 1056, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(201*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt
(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*f
*x + 1/2*e) + 1)) - 16*(9*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*
x + 1/2*e)^2 + a))^5 - 39*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*
x + 1/2*e)^2 + a))^4*sqrt(a) - 26*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*ta
n(1/2*f*x + 1/2*e)^2 + a))^3*a + 78*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*
tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) + 69*(sqrt(a)*tan(1/2*f*x + 1/2*e) -
```

```
 sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 + 13*a^(5/2))/(((sqrt(a)*tan(1/2*f
*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*
x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - a)^3*sgn(tan(1/2
*f*x + 1/2*e) + 1)) - 6*(43*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*
f*x + 1/2*e)^2 + a))^7 + 237*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2
*f*x + 1/2*e)^2 + a))^6*sqrt(a) + 161*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(
a*tan(1/2*f*x + 1/2*e)^2 + a))^5*a - 221*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sq
rt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2) + 25*(sqrt(a)*tan(1/2*f*x + 1/2
*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2 + 103*(sqrt(a)*tan(1/2*f*x
+ 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) - 93*(sqrt(a)*tan(
1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 + 17*a^(7/2))/((
(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(
sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a)
- a)^4*sgn(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.104 $\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=107

$$\frac{3\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{4af}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4\sqrt{2}\sqrt{a}f}$$

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4*S
qrt[2]*Sqrt[a]*f) - Sec[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*Sec[e
+ f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*a*f)
```

Rubi [A] time = 0.195246, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2712, 2855, 2649, 206}

$$\frac{3\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{4af}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}+\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4\sqrt{2}\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4*S
qrt[2]*Sqrt[a]*f) - Sec[e + f*x]/(2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*Sec[e
+ f*x]*Sqrt[a + a*Sin[e + f*x]])/(4*a*f)
```

Rule 2712

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]),
x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
```

```
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx &= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}}+\frac{\int \sec^2(e+fx)\sqrt{a+a\sin(e+fx)}\left(-\frac{a}{2}+2a\sin(e+fx)\right)dx}{2a^2}\\
&= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}}+\frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af}-\frac{5}{8}\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx\\
&= -\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}}+\frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af}+\frac{5\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(}{\sqrt{a+a\,\mathrm{si}}}\right.}{4f}\\
&= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}f}-\frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}}+\frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af}
\end{aligned}$$

Mathematica [C] time = 0.269125, size = 118, normalized size = 1.1

$$-\frac{\sec(e+fx)\left(-3\sin(e+fx)+(5+5i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}{4f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] -(Sec[e + f*x]*(-1 + (5 + 5*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-
1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^2 - 3*Sin[e + f*x]))/(4*f*Sqrt[a*(1 + Sin[e + f*x])
])
```

Maple [A] time = 0.462, size = 130, normalized size = 1.2

$$\frac{1}{8f\cos\left(fx+e\right)}\left(\sin\left(fx+e\right)\left(5\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)a\sqrt{a-a\sin\left(fx+e\right)}+6\,a^{3/2}\right)+5\sqrt{2}\mathrm{Artanh}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/8*(sin(f*x+e)*(5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/
2))*a*(a-a*sin(f*x+e))^(1/2)+6*a^(3/2))+5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+
e))^(1/2)*2^(1/2)/a^(1/2))*a*(a-a*sin(f*x+e))^(1/2)+2*a^(3/2))/a^(3/2)/cos(
f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 1.54293, size = 549, normalized size = 5.13

$$\frac{5\sqrt{2}\left(\cos\left(fx+e\right)\sin\left(fx+e\right)+\cos\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^2+2\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(fx+e\right)-\sin\left(fx+e\right)+1\right)+3\,a\cos\left(fx+e\right)-(}{\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+2\right)\sin\left(fx+e\right)-\cos\left(fx+e\right)-}\right.}{16\left(af\cos\left(fx+e\right)\sin\left(fx+e\right)+af\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(5*sqrt(2)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*sqrt(a)*log(-(a*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) -
 sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e)
 + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
2)) + 4*sqrt(a*sin(f*x + e) + a)*(3*sin(f*x + e) + 1))/(a*f*cos(f*x + e)*si
n(f*x + e) + a*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 2.82308, size = 594, normalized size = 5.55

$$-\frac{\frac{5\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}-\frac{4\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}+\sqrt{a}\right)}{\left(\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}\right)^2-2\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right.}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*
tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*f*x
```

```
+ 1/2*e) + 1)) - 4*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2
*e)^2 + a) + sqrt(a))/(((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x
+ 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x +
 1/2*e)^2 + a))*sqrt(a) - a)*sgn(tan(1/2*f*x + 1/2*e) + 1)) - 2*(3*(sqrt(a)
*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^3 + (sqrt(a)*ta
n(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a) - (sqrt(
a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a + a^(3/2))/
(((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2
*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a
) - a)^2*sgn(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.105 $\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}} \, dx$

Optimal. Leaf size=62

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(Sqrt[a]*f) - Cot[
e + f*x]/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.110095, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2716, 21, 2773, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(Sqrt[a]*f) - Cot[
e + f*x]/(f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2716

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Dist[1/a,
 Int[((a + b*Sin[e + f*x])^m*(b*m - a*(m + 1)*Sin[e + f*x]))/Sin[e + f*x],
x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1/
2] &&  !LtQ[m, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx &= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+\frac{\int \frac{\csc(e+fx)\left(-\frac{a}{2}-\frac{1}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}\\
&= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}-\frac{\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{2a}\\
&= -\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}+\frac{\operatorname{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}
\end{aligned}$$

Mathematica [B] time = 0.307822, size = 138, normalized size = 2.23

$$\frac{\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)\csc\left(\frac{1}{4}(e+fx)\right)\sec\left(\frac{1}{4}(e+fx)\right)\left(2\sin\left(\frac{1}{2}(e+fx)\right)-2\cos\left(\frac{1}{2}(e+fx)\right)+\sin(e+fx)\left(\log\left(-\sin\left(\frac{1}{2}\right.\right.\right.\right.}{8f\sqrt{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/4]*Sec[(e + f*x)/4]*(-2*Cos[(e + f*x)/2] + 2*Sin[(e + f*x)/2
] + (Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[1 - Cos[(e + f*x)/2
] + Sin[(e + f*x)/2]])*Sin[e + f*x])*(1 + Tan[(e + f*x)/2]))/(8*f*Sqrt[a*(1
```

```
+ Sin[e + f*x])])
```

Maple [A] time = 0.59, size = 103, normalized size = 1.7

$$-\frac{1+\sin\left(fx+e\right)}{\sin\left(fx+e\right)\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(-\mathrm{Artanh}\left(\sqrt{a-a\sin\left(fx+e\right)}\frac{1}{\sqrt{a}}\right)\sin\left(fx+e\right)a+\sqrt{a-a\sin\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] -(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-arctanh((a-a*sin(f*x+e))^(1/2)
/a^(1/2))*sin(f*x+e)*a+(a-a*sin(f*x+e))^(1/2)*a^(1/2))/sin(f*x+e)/a^(3/2)/c
os(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{2}}{\sqrt{a\sin\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)
```

Fricas [B] time = 1.60944, size = 706, normalized size = 11.39

$$\frac{\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\log\left(\frac{a\cos\left(fx+e\right)^{3}-7a\cos\left(fx+e\right)^{2}+4\left(\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+3\right)\sin\left(fx+e\right)-2\right.}{\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)}\right.}{4\left(af\cos\left(fx+e\right)^{2}-af\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a*
cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3
)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a
*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)
/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos
(f*x + e) - 1)) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) +
 1))/(a*f*cos(f*x + e)^2 - a*f - (a*f*cos(f*x + e) + a*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 2.21903, size = 502, normalized size = 8.1

$$\frac{\frac{\left(2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{2}\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)+2\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{a}+\sqrt{a}}{\sqrt{-a}}\right)-\sqrt{-a}\log\left(\sqrt{2}\sqrt{a}+\sqrt{a}\right)-\sqrt{2}\sqrt{-a}-3\sqrt{-a}\right)\mathrm{sgn}\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{2}\sqrt{-a}\sqrt{a}+\sqrt{-a}\sqrt{a}}-\frac{2\arctan\left(-\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - sqrt
(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 2*sqrt(a)*arctan((sqrt(2)*sqr
t(a) + sqrt(a))/sqrt(-a)) - sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - sqrt(
2)*sqrt(-a) - 3*sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt(2)*sqrt(-a)*s
```

```
qrt(a) + sqrt(-a)*sqrt(a)) - 2*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt
(a*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*f*x + 1/2*e
) + 1)) + log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*
e)^2 + a)))/(sqrt(a)*sgn(tan(1/2*f*x + 1/2*e) + 1)) + sqrt(a*tan(1/2*f*x +
1/2*e)^2 + a)/(a*sgn(tan(1/2*f*x + 1/2*e) + 1)) + 2*sqrt(a)/(((sqrt(a)*tan(
1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)*sgn(tan(1/2*f
*x + 1/2*e) + 1)))/f
```

3.106 $\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=135

$$\frac{9\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*Sqrt[a]*f)
 + (9*Cot[e + f*x])/(8*f*Sqrt[a + a*Sin[e + f*x]]) + (Cot[e + f*x]*Csc[e +
f*x])/(12*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*f*
Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.621613, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2718, 2649, 206, 3044, 2984, 2985, 2773}

$$\frac{9\cot(e+fx)}{8f\sqrt{a\sin(e+fx)+a}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (-7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*Sqrt[a]*f)
 + (9*Cot[e + f*x])/(8*f*Sqrt[a + a*Sin[e + f*x]]) + (Cot[e + f*x]*Csc[e +
f*x])/(12*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*f*
Sqrt[a + a*Sin[e + f*x]])
```

Rule 2718

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Sin[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
```

```
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx + \int \frac{\csc^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}-\frac{7}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3a} - \frac{2\,\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cos(e+}{\sqrt{a+a\sin(}}\right.}{f}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \int \frac{\csc}{}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+}{3f\sqrt{a}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+}{3f\sqrt{a}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+}{3f\sqrt{a}}$$

$$= -\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{a}f} + \frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} + \frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)}{3f\sqrt{a+a\,s}}$$

Mathematica [B] time = 0.579591, size = 292, normalized size = 2.16

$$\frac{\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)\left(-36\sin\left(\frac{1}{2}(e+fx)\right)-46\sin\left(\frac{3}{2}(e+fx)\right)+54\sin\left(\frac{5}{2}(e+fx)\right)+36\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(36*Cos[(e + f*x)
/2] - 46*Cos[(3*(e + f*x))/2] - 54*Cos[(5*(e + f*x))/2] - 36*Sin[(e + f*x)/
2] - 63*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 63*Log[
1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 46*Sin[(3*(e + f*x)
)/2] + 54*Sin[(5*(e + f*x))/2] + 21*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]]*Sin[3*(e + f*x)] - 21*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Si
n[3*(e + f*x)]))/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*Sqrt[a*(
1 + Sin[e + f*x])])
```

Maple [A] time = 0.638, size = 144, normalized size = 1.1

$$\frac{1+\sin\left(fx+e\right)}{24\left(\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(-21\,\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}}{\sqrt{a}}\right)\left(\sin\left(fx+e\right)\right)^{3}a^{3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-21*arctanh((-a*(-1+sin(f*x
+e)))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+27*(-a*(-1+sin(f*x+e)))^(5/2)*a^(1/2)
-56*(-a*(-1+sin(f*x+e)))^(3/2)*a^(3/2)+21*(-a*(-1+sin(f*x+e)))^(1/2)*a^(5/2
))/sin(f*x+e)^3/a^(7/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.46294, size = 987, normalized size = 7.31

$$\frac{21\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)^3+\cos\left(fx+e\right)^2-\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(fx}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e
)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7
*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2
*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a
*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 +
 cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) -
4*(27*cos(f*x + e)^3 + 25*cos(f*x + e)^2 - (27*cos(f*x + e)^2 + 2*cos(f*x +
 e) - 17)*sin(f*x + e) - 19*cos(f*x + e) - 17)*sqrt(a*sin(f*x + e) + a))/(a
*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f - (a*f*cos(f*x + e)^3 + a*f*
cos(f*x + e)^2 - a*f*cos(f*x + e) - a*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)
```

Giac [B] time = 2.37322, size = 819, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)*((2*tan(1/2*f*x + 1/2*e)/(a*sgn(ta
n(1/2*f*x + 1/2*e) + 1)) - 3/(a*sgn(tan(1/2*f*x + 1/2*e) + 1)))*tan(1/2*f*x
 + 1/2*e) - 22/(a*sgn(tan(1/2*f*x + 1/2*e) + 1))) - (210*sqrt(2)*sqrt(a)*ar
ctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 105*sqrt(2)*sqrt(-a)*log(sqrt(
2)*sqrt(a) + sqrt(a)) + 294*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt
(-a)) - 147*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) - 128*sqrt(2)*sqrt(-a)
- 186*sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(5*sqrt(2)*sqrt(-a)*sqrt(a) +
 7*sqrt(-a)*sqrt(a)) + 42*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*ta
n(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(tan(1/2*f*x + 1/2*e) + 1
)) - 21*log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)
^2 + a)))/(sqrt(a)*sgn(tan(1/2*f*x + 1/2*e) + 1)) - 2*(3*(sqrt(a)*tan(1/2*f
*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^5 + 18*(sqrt(a)*tan(1/2*f
*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a) - 48*(sqrt(a)*t
an(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) - 3*(sq
rt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 + 22*a
^(5/2))/(((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a
))^2 - a)^3*sgn(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.107 $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=177

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{256\sqrt{2}a^{3/2}f}+\frac{\tan^3(e+fx)}{3f(a\sin(e+fx)+a)^{3/2}}+\frac{a\sin(e+fx)\tan(e+fx)}{12f(a\sin(e+fx)+a)^{5/2}}+\frac{7\cos(e+fx)}{256f(a\sin(e+fx)+a)^{3/2}}-\frac{(87\sin\ldots}{19\ldots}$$

```
[Out] (7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(256
*Sqrt[2]*a^(3/2)*f) + (7*Cos[e + f*x])/(256*f*(a + a*Sin[e + f*x])^(3/2)) -
 (Sec[e + f*x]*(65 + 87*Sin[e + f*x]))/(192*f*(a + a*Sin[e + f*x])^(3/2)) +
 (a*Sin[e + f*x]*Tan[e + f*x])/(12*f*(a + a*Sin[e + f*x])^(5/2)) + Tan[e +
f*x]^3/(3*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 1.19927, antiderivative size = 195, normalized size of antiderivative = 1.1, number of steps used = 20, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2714, 2650, 2649, 206, 4401, 2681, 2687, 2877, 2855}

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{256\sqrt{2}a^{3/2}f}+\frac{7\cos(e+fx)}{256f(a\sin(e+fx)+a)^{3/2}}+\frac{\sec^3(e+fx)}{4af\sqrt{a\sin(e+fx)+a}}-\frac{\sec^3(e+fx)}{6f(a\sin(e+fx)+a)^{3/2}}-\frac{45\,\mathrm{s}\ldots}{64af\sqrt{a\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(256
*Sqrt[2]*a^(3/2)*f) + (7*Cos[e + f*x])/(256*f*(a + a*Sin[e + f*x])^(3/2)) +
 (9*Sec[e + f*x])/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f*x]^3/(6*f*(
a + a*Sin[e + f*x])^(3/2)) - (45*Sec[e + f*x])/(64*a*f*Sqrt[a + a*Sin[e + f
*x]]) + Sec[e + f*x]^3/(4*a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
```

```
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

<u>Rule 2855</u>

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

<u>Rubi steps</u>

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx - \int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a} - \int \left(\frac{\sec^4(e+fx)}{(a(1+\sin(e+fx)))^{3/2}} - \frac{2\sec^2(e+ \cdots}{(a(1+\mathrm{s} \cdots}\right.$$

$$= -\frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + 2\int \frac{\sec^2(e+fx)\tan^2(e+fx)}{(a(1+\sin(e+fx)))^{3/2}}\,dx - \frac{\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{\cdots}{\sqrt{a \cdots}}\right.}{2af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\sec^3(e+fx)}{6f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\sec \cdots}{\cdots}}{\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} - \frac{\sec^3(e+fx)}{6f(a+a\sin(e+fx))^{3/2}} + \frac{\cdots}{4af\sqrt{\cdots}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\cdots}{6f(a \cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\cdots}{6f(a \cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\cdots}{6 \cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\cdots}{6 \cdots}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{256\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} + \frac{9\sec(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\cdots}{6 \cdots}$$

Mathematica [C] time = 0.355786, size = 334, normalized size = 1.89

$$\frac{-\frac{192\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} + \frac{32\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - 171\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2 + 342\sin\left(\frac{1}{2}(e+ \cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (124 + (64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 32/(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (248*Sin[(e + f*x)/2])/(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2]) + 342*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(
e + f*x)/2]) - 171*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (21 + 21*I)*(-
1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2])^3 + (32*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3
)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (192*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])^3)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(768*f*(a*(1 + Sin[e
+ f*x]))^(3/2))
```

Maple [A] time = 0.619, size = 289, normalized size = 1.6

$$-\frac{1}{\left(-1536+1536\,\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)f}\left(\left(\left(-1080\,a^{9/2}-21\,\left(a-a\sin\left(fx+e\right)\right)^{3/2}\sqrt{2}\mathrm{Artanh}\left(1/2\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/1536/a^(11/2)*((-1080*a^(9/2)-21*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)*cos(f*x+e)^2+(3
84*a^(9/2)+84*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)+42*a^(9/2)*cos(f*x+e)^4+(-648*a^(9/2)
-63*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^3)*cos(f*x+e)^2+128*a^(9/2)+84*(a-a*sin(f*x+e))^(3/2)*2^(1/2)
*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(-1+sin(f*x+e))/(
1+sin(f*x+e))^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.62036, size = 726, normalized size = 4.1

$$\frac{21\,\sqrt{2}\left(\cos\left(fx+e\right)^{5}-2\,\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)-2\,\cos\left(fx+e\right)^{3}\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\,\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(fx+e\right)-\cdots\right.}{\cos\left(fx+e\right)^{2}-\left(\cos\left(f\cdots\right.\right.}\right.}{3072\left(a^{2}f\cos\left(fx+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(21*sqrt(2)*(cos(f*x + e)^5 - 2*cos(f*x + e)^3*sin(f*x + e) - 2*cos(
f*x + e)^3)*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e)
+ a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(
f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*si
n(f*x + e) - cos(f*x + e) - 2)) - 4*(21*cos(f*x + e)^4 - 324*cos(f*x + e)^2
 - 12*(45*cos(f*x + e)^2 - 16)*sin(f*x + e) + 64)*sqrt(a*sin(f*x + e) + a))
/(a^2*f*cos(f*x + e)^5 - 2*a^2*f*cos(f*x + e)^3*sin(f*x + e) - 2*a^2*f*cos(
f*x + e)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^{4}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

sage_2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.108 $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{32\sqrt{2}a^{3/2}f}+\frac{\cos(e+fx)}{32f(a\sin(e+fx)+a)^{3/2}}+\frac{5\sec(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(32*Sqrt
[2]*a^(3/2)*f) + Cos[e + f*x]/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f
*x]/(4*f*(a + a*Sin[e + f*x])^(3/2)) + (5*Sec[e + f*x])/(8*a*f*Sqrt[a + a*S
in[e + f*x]])
```

Rubi [A] time = 0.223266, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2712, 2855, 2650, 2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{32\sqrt{2}a^{3/2}f}+\frac{\cos(e+fx)}{32f(a\sin(e+fx)+a)^{3/2}}+\frac{5\sec(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(32*Sqrt
[2]*a^(3/2)*f) + Cos[e + f*x]/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f
*x]/(4*f*(a + a*Sin[e + f*x])^(3/2)) + (5*Sec[e + f*x])/(8*a*f*Sqrt[a + a*S
in[e + f*x]])
```

Rule 2712

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]),
x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\sec^2(e+fx)\left(-\frac{3a}{2}+4a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{4a^2}$$

$$= -\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} - \frac{1}{16}\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx$$

$$= \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} - \frac{\int \frac{}{\sqrt{a+a\,\mathrm{s}}}}{6}$$

$$= \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{\mathrm{Subst}\Big(}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{32\sqrt{2}a^{3/2}f} + \frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}} + \frac{5}{8af\sqrt{}}$$

Mathematica [C] time = 0.443707, size = 128, normalized size = 0.96

$$-\frac{\sec(e+fx)\left(-40\sin(e+fx)-\cos(2(e+fx))+(2+2i)(-1)^{3/4}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\right.}{64f(a(\sin(e+fx)+1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -(Sec[e + f*x]*(-25 - Cos[2*(e + f*x)] + (2 + 2*I)*(-1)^(3/4)*ArcTanh[(1/2
+ I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 40*Sin[e + f*x]))/(64*f*(a*
(1 + Sin[e + f*x]))^(3/2))
```

Maple [A] time = 0.607, size = 202, normalized size = 1.5

$$\frac{1}{\left(64+64\sin\left(fx+e\right)\right)\cos\left(fx+e\right)f}\left(\sin\left(fx+e\right)\left(2\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)a^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/64/a^(7/2)*(sin(f*x+e)*(2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a
*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+40*a^(5/2))+(-(a-a*sin(f*x+e))^(1/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+2*a^(5/2)
)*cos(f*x+e)^2+2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^2+24*a^(5/2))/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin
(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.63547, size = 644, normalized size = 4.81

$$\frac{\sqrt{2}\left(\cos\left(fx+e\right)^{3}-2\,\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,\cos\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}+2\,\sqrt{2}\sqrt{a\sin\left(fx+e\right)+a}\sqrt{a}\left(\cos\left(fx+e\right)-\sin\left(fx+\right.\right.}{\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+2\right)}\right.}{128\left(a^{2}f\cos\left(fx+e\right)^{3}-2\,a^{2}f\cos\left(fx+e\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*(cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqr
t(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e)
 - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
e) - cos(f*x + e) - 2)) - 4*(cos(f*x + e)^2 + 20*sin(f*x + e) + 12)*sqrt(a*
sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^3 - 2*a^2*f*cos(f*x + e)*sin(f*x + e
) - 2*a^2*f*cos(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.109 $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(3/2)*f) -
(2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(a^(3/2)*f) - Cot[e + f*x]/(a*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.228806, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2715, 2985, 2649, 206, 2773}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```


```
[Out] (3*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(3/2)*f) -
(2*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]]
)])/(a^(3/2)*f) - Cot[e + f*x]/(a*f*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2715

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Di
st[1/b^2, Int[((a + b*Sin[e + f*x])^(m + 1)*(b*m - a*(m + 1)*Sin[e + f*x]))
/Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
```

```
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx &= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}+\frac{\int \frac{\csc(e+fx)\left(-\frac{3a}{2}+\frac{1}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}\\
&= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}-\frac{3\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}\,dx}{2a^2}+\frac{2\int \frac{1}{\sqrt{a+a\sin(e+fx)}}\,dx}{a}\\
&= -\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{a-x^2}\,dx,x,\frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{af}-\frac{4\,\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\right.}{af}\\
&= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f}-\frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}
\end{aligned}
$$

Mathematica [C] time = 2.1984, size = 206, normalized size = 1.82

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-\cot\left(\frac{1}{4}(e+fx)\right)+(16+16i)(-1)^{3/4}\tanh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(\tan\left(\frac{1}{4}(e+fx)\right)-1\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((16 + 16*I)*(-1)^(3/4)*ArcTanh[(1
/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - Cot[(e + f*x)/4] + 2*(3*Log
[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 3*Log[1 - Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]] + Sec[(e + f*x)/2] + Csc[e + f*x]*Sin[(e + f*x)/4]^2 - Csc[
e + f*x]*Sin[(e + f*x)/4]*Sin[(3*(e + f*x))/4])))/(4*f*(a*(1 + Sin[e + f*x]
))^(3/2))
```

Maple [A] time = 0.591, size = 134, normalized size = 1.2

$$-\frac{1+\sin(fx+e)}{\sin(fx+e)\cos(fx+e)f}\sqrt{-a\left(-1+\sin(fx+e)\right)}\left(\sin(fx+e)a^2\left(2\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{\sqrt{a}}\right)-3\,\mathrm{Art}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/a^(7/2)*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(sin(f*x+e)*a^2*(2*2^(
1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-3*arctanh((a-a*sin
(f*x+e))^(1/2)/a^(1/2)))+(a-a*sin(f*x+e))^(1/2)*a^(3/2))/sin(f*x+e)/cos(f*x
+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.76445, size = 1133, normalized size = 10.03

$$\frac{3\left(\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\log\left(\frac{a\cos\left(fx+e\right)^{3}-7a\cos\left(fx+e\right)^{2}+4\left(\cos\left(fx+e\right)^{2}+\left(\cos\left(fx+e\right)+3\right)\sin\left(fx+e\right)-\cdots}{\cos\left(fx+e\right)^{3}+\cos\left(fx+\cdots\right.}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*(cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((
a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) +
 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9
*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) -
a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - c
os(f*x + e) - 1)) + 4*sqrt(2)*(a*cos(f*x + e)^2 - (a*cos(f*x + e) + a)*sin(
f*x + e) - a)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sq
rt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) +
3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos
(f*x + e) - 2))/sqrt(a) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*
x + e) + 1))/(a^2*f*cos(f*x + e)^2 - a^2*f - (a^2*f*cos(f*x + e) + a^2*f)*s
in(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{2}\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [B] time = 2.48466, size = 656, normalized size = 5.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((6*sqrt(2)*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 8*sq
rt(2)*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 3*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqr
t(a) + sqrt(a)) + 6*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) -
16*sqrt(a)*arctan(sqrt(a)/sqrt(-a)) - 3*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt
(a)) - sqrt(2)*sqrt(-a) - 3*sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(sqrt(2
)*sqrt(-a)*a^(3/2) + sqrt(-a)*a^(3/2)) + 8*sqrt(2)*arctan(-1/2*sqrt(2)*(sqr
t(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/s
qrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*f*x + 1/2*e) + 1)) - 6*arctan(-(sqrt(a)*ta
n(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a
)*a*sgn(tan(1/2*f*x + 1/2*e) + 1)) + 3*log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e
) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)))/(a^(3/2)*sgn(tan(1/2*f*x + 1/2*e)
+ 1)) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)/(a^2*sgn(tan(1/2*f*x + 1/2*e) +
1)) + 2/(((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a
))^2 - a)*sqrt(a)*sgn(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.110 $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3a^2f}-\frac{\cot(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}+\frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a\sin(e+fx)+a}}$$

```
[Out] -ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(8*a^(3/2)*f) - C
ot[e + f*x]/(8*a*f*Sqrt[a + a*Sin[e + f*x]]) + (11*Cot[e + f*x]*Csc[e + f*x
])/(12*a*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a
+ a*Sin[e + f*x]])/(3*a^2*f)
```

Rubi [A] time = 0.553107, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2717, 2772, 2773, 206, 3044, 2980}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3a^2f}-\frac{\cot(e+fx)}{8af\sqrt{a\sin(e+fx)+a}}+\frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(8*a^(3/2)*f) - C
ot[e + f*x]/(8*a*f*Sqrt[a + a*Sin[e + f*x]]) + (11*Cot[e + f*x]*Csc[e + f*x
])/(12*a*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a
+ a*Sin[e + f*x]])/(3*a^2*f)
```

Rule 2717

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^
2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2772

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e
```

```
+ f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dis
t[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e +
 f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -
1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rule 2980

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Sim
p[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*
(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c
- 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(
c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] &
& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\int \csc^4(e+fx)\sqrt{a+a\sin(e+fx)}\left(1+\sin^2(e+fx)\right)dx}{a^2} - \frac{2\int \csc^3(e+fx)\sqrt{a+a\sin(e+}}{a^2}$$

$$= \frac{\cot(e+fx)\csc(e+fx)}{af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3a^2f} + \frac{\int \csc^3(e+fx)}{}$$

$$= \frac{3\cot(e+fx)}{2af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\,\mathrm{si}}}{3a^2f}$$

$$= -\frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a}}{3a^2f}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a^{3/2}f} - \frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{3/2}f} - \frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e}{}$$

Mathematica [B] time = 0.760534, size = 294, normalized size = 2.04

$$\frac{\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(132\sin\left(\frac{1}{2}(e+fx)\right)+62\sin\left(\frac{3}{2}(e+fx)\right)-6\sin\left(\frac{5}{2}(e+fx)\right)-13\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

```
[Out] (Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-132*Cos[(e +
f*x)/2] + 62*Cos[(3*(e + f*x))/2] + 6*Cos[(5*(e + f*x))/2] + 132*Sin[(e + f
*x)/2] - 9*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 9*Lo
g[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 62*Sin[(3*(e + f*
x))/2] - 6*Sin[(5*(e + f*x))/2] + 3*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x
)/2]]*Sin[3*(e + f*x)] - 3*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin
[3*(e + f*x)]))/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*(a*(1 + S
in[e + f*x]))^(3/2))
```

Maple [A] time = 0.621, size = 144, normalized size = 1.

$$\frac{1+\sin\left(fx+e\right)}{24\left(\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(3\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}a^{7/2}-8\left(-a\left(-1+\sin\left(fx+e\right)\right)\right)^{3/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/24*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(3*(-a*(-1+sin(f*x+e)))^(1/2
)*a^(7/2)-8*(-a*(-1+sin(f*x+e)))^(3/2)*a^(5/2)-3*(-a*(-1+sin(f*x+e)))^(5/2)
*a^(3/2)-3*arctanh((-a*(-1+sin(f*x+e)))^(1/2)/a^(1/2))*a^4*sin(f*x+e)^3)/a^
(11/2)/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.57176, size = 1003, normalized size = 6.97

$$\frac{3\left(\cos\left(fx+e\right)^{4}-2\cos\left(fx+e\right)^{2}-\left(\cos\left(fx+e\right)^{3}+\cos\left(fx+e\right)^{2}-\cos\left(fx+e\right)-1\right)\sin\left(fx+e\right)+1\right)\sqrt{a}\log\left(\frac{a\cos(fx+\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e)
^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*
```

```
a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*
cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*
cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 +
cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4
*(3*cos(f*x + e)^3 + 17*cos(f*x + e)^2 - (3*cos(f*x + e)^2 - 14*cos(f*x + e
) - 25)*sin(f*x + e) - 11*cos(f*x + e) - 25)*sqrt(a*sin(f*x + e) + a))/(a^2
*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f - (a^2*f*cos(f*x + e)^3
+ a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - a^2*f)*sin(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

Giac [B] time = 3.13146, size = 828, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)*((2*tan(1/2*f*x + 1/2*e)/(a^2*sgn(
tan(1/2*f*x + 1/2*e) + 1)) - 9/(a^2*sgn(tan(1/2*f*x + 1/2*e) + 1)))*tan(1/2
*f*x + 1/2*e) + 14/(a^2*sgn(tan(1/2*f*x + 1/2*e) + 1))) - (30*sqrt(2)*sqrt(
a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 15*sqrt(2)*sqrt(-a)*log(s
qrt(2)*sqrt(a) + sqrt(a)) + 42*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/s
qrt(-a)) - 21*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a)) + 280*sqrt(2)*sqrt(-a
) + 402*sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(5*sqrt(2)*sqrt(-a)*a^(3/2)
 + 7*sqrt(-a)*a^(3/2)) + 6*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*t
an(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a*sgn(tan(1/2*f*x + 1/2*e)
+ 1)) - 3*log(abs(-sqrt(a)*tan(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*
e)^2 + a)))/(a^(3/2)*sgn(tan(1/2*f*x + 1/2*e) + 1)) - 2*(9*(sqrt(a)*tan(1/2
```

```
*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^5 - 18*(sqrt(a)*tan(1/2
*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a) + 24*(sqrt(a)
*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) - 9*(
sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - 14
*a^(5/2))/(((sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 +
 a))^2 - a)^3*a*sgn(tan(1/2*f*x + 1/2*e) + 1)))/f
```

3.111 $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=207

$$\frac{317\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4096\sqrt{2}a^{5/2}f}+\frac{\tan^3(e+fx)}{3f(a\sin(e+fx)+a)^{5/2}}+\frac{5a\sin(e+fx)\tan(e+fx)}{48f(a\sin(e+fx)+a)^{7/2}}+\frac{317\cos(e+fx)}{4096af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] (317*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4
096*Sqrt[2]*a^(5/2)*f) + (317*Cos[e + f*x])/(3072*f*(a + a*Sin[e + f*x])^(5
/2)) - (Sec[e + f*x]*(115 + 129*Sin[e + f*x]))/(384*f*(a + a*Sin[e + f*x])^
(5/2)) + (317*Cos[e + f*x])/(4096*a*f*(a + a*Sin[e + f*x])^(3/2)) + (5*a*Si
n[e + f*x]*Tan[e + f*x])/(48*f*(a + a*Sin[e + f*x])^(7/2)) + Tan[e + f*x]^3
/(3*f*(a + a*Sin[e + f*x])^(5/2))
```

Rubi [A] time = 1.43374, antiderivative size = 260, normalized size of antiderivative = 1.26, number of steps used = 23, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2714, 2650, 2649, 206, 4401, 2681, 2687, 2877, 2859}

$$-\frac{31\sec^3(e+fx)}{192a^2f\sqrt{a\sin(e+fx)+a}}-\frac{1085\sec(e+fx)}{3072a^2f\sqrt{a\sin(e+fx)+a}}+\frac{317\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4096\sqrt{2}a^{5/2}f}+\frac{317\cos(e+fx)}{4096af(a\sin(e+fx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (317*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(4
096*Sqrt[2]*a^(5/2)*f) - Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)) - Se
c[e + f*x]^3/(8*f*(a + a*Sin[e + f*x])^(5/2)) + (317*Cos[e + f*x])/(4096*a*
f*(a + a*Sin[e + f*x])^(3/2)) + (217*Sec[e + f*x])/(1536*a*f*(a + a*Sin[e +
 f*x])^(3/2)) + (53*Sec[e + f*x]^3)/(96*a*f*(a + a*Sin[e + f*x])^(3/2)) - (
1085*Sec[e + f*x])/(3072*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (31*Sec[e + f*x]
^3)/(192*a^2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rule 2714

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[((a + b*Sin[e + f*x])^m*(
1 - 2*Sin[e + f*x]^2))/Cos[e + f*x]^4, x] /; FreeQ[{a, b, e, f, m}, x] && E
```

```
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2681

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x
])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(
g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] &&
 IntegersQ[2*m, 2*p]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2877

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^
(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] - Dist[1/(a^2*(2*
m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(
2*m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a
^2 - b^2, 0] && LeQ[m, -2^(-1)] && NeQ[2*m + p + 1, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{1}{(a+a\sin(e+fx))^{5/2}}\,dx - \int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{(a+a\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} + \frac{3\int \frac{1}{(a+a\sin(e+fx))^{3/2}}\,dx}{8a} - \int \left(\frac{\sec^4(e+fx)}{(a(1+\sin(e+fx)))^{5/2}} - \frac{2\sec^2(e}{(a(1+}\right.$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + 2\int \frac{\sec^2(e+fx)\tan^2(e+fx)}{(a(1+\sin(e+fx)))^{5/2}}\,dx$$

$$= -\frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{3\cos(e+fx)}{16af(a+a\sin(e+fx))^{3/2}} + \frac{\int \sec^4}{}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{}{16af(}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{}{16af(}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{}{16af(}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} - \frac{}{16af(}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} + \frac{}{4096}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{16\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} + \frac{}{4096}$$

$$= \frac{317\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4096\sqrt{2}a^{5/2}f} - \frac{\cos(e+fx)}{4f(a+a\sin(e+fx))^{5/2}} - \frac{\sec^3(e+fx)}{8f(a+a\sin(e+fx))^{5/2}} + \frac{}{4096}$$

Mathematica [C] time = 0.546479, size = 394, normalized size = 1.9

$$\frac{-\frac{1152\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} + \frac{256\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - 201\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4 + 402\sin\left(\frac{1}{2}(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (1312 + (768*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 38
4/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (2624*Sin[(e + f*x)/2])/(Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]) + 2584*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2]) - 1292*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 402*Sin[
(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 201*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])^4 - (951 + 951*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)
^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + (
256*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(Cos[(e + f*x)/2] - Sin[(e + f
*x)/2])^3 - (1152*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(Cos[(e + f*x)/2
] - Sin[(e + f*x)/2]))/(12288*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A] time = 0.843, size = 353, normalized size = 1.7

$$-\frac{1}{\left(-24576+24576\sin\left(fx+e\right)\right)\left(1+\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)f}\left(1902\,a^{11/2}\sin\left(fx+e\right)\left(\cos\left(fx+e\right)\right)^{4}+\left(-13888\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/24576/a^(15/2)*(1902*a^(11/2)*sin(f*x+e)*cos(f*x+e)^4+(-13888*a^(11/2)-3
804*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/
2)/a^(1/2))*a^4)*cos(f*x+e)^2*sin(f*x+e)+(5632*a^(11/2)+7608*(a-a*sin(f*x+e
))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*s
in(f*x+e)+(4438*a^(11/2)+951*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)*cos(f*x+e)^4+(-9920*a^(11/2)-7608
*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^4)*cos(f*x+e)^2+2560*a^(11/2)+7608*(a-a*sin(f*x+e))^(3/2)*2^(1/2
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)/(-1+sin(f*x+e))/
(1+sin(f*x+e))^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maximima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.90076, size = 830, normalized size = 4.01

$$\frac{951 \sqrt{2}\left(3 \cos\left(fx+e\right)^5 - 4 \cos\left(fx+e\right)^3 + \left(\cos\left(fx+e\right)^5 - 4 \cos\left(fx+e\right)^3\right) \sin\left(fx+e\right)\right)\sqrt{a} \log\left(-\frac{a \cos\left(fx+e\right)^2 + 2 \sqrt{2}\sqrt{a \sin \cdots}}{\cdots}\right. \cdots}{49152\left(3 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(951*sqrt(2)*(3*cos(f*x + e)^5 - 4*cos(f*x + e)^3 + (cos(f*x + e)^5
 - 4*cos(f*x + e)^3)*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(
2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a
*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2
- (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(2219*cos(f*x +
e)^4 - 4960*cos(f*x + e)^2 + (951*cos(f*x + e)^4 - 6944*cos(f*x + e)^2 + 28
16)*sin(f*x + e) + 1280)*sqrt(a*sin(f*x + e) + a))/(3*a^3*f*cos(f*x + e)^5
- 4*a^3*f*cos(f*x + e)^3 + (a^3*f*cos(f*x + e)^5 - 4*a^3*f*cos(f*x + e)^3)*
sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.112 $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=167

$$\frac{11\sec(e+fx)}{96a^2f\sqrt{a\sin(e+fx)+a}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{128\sqrt{2}a^{5/2}f}-\frac{11\cos(e+fx)}{128af(a\sin(e+fx)+a)^{3/2}}+\frac{17\sec(e+fx)}{48af(a\sin(e+fx)+a)^{3/2}}-\frac{}{6f}$$

```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(1
28*Sqrt[2]*a^(5/2)*f) - Sec[e + f*x]/(6*f*(a + a*Sin[e + f*x])^(5/2)) - (11
*Cos[e + f*x])/(128*a*f*(a + a*Sin[e + f*x])^(3/2)) + (17*Sec[e + f*x])/(48
*a*f*(a + a*Sin[e + f*x])^(3/2)) + (11*Sec[e + f*x])/(96*a^2*f*Sqrt[a + a*S
in[e + f*x]])
```

Rubi [A] time = 0.30282, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2712, 2859, 2687, 2650, 2649, 206}

$$\frac{11\sec(e+fx)}{96a^2f\sqrt{a\sin(e+fx)+a}}-\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{128\sqrt{2}a^{5/2}f}-\frac{11\cos(e+fx)}{128af(a\sin(e+fx)+a)^{3/2}}+\frac{17\sec(e+fx)}{48af(a\sin(e+fx)+a)^{3/2}}-\frac{}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```



```
[Out] (-11*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(1
28*Sqrt[2]*a^(5/2)*f) - Sec[e + f*x]/(6*f*(a + a*Sin[e + f*x])^(5/2)) - (11
*Cos[e + f*x])/(128*a*f*(a + a*Sin[e + f*x])^(3/2)) + (17*Sec[e + f*x])/(48
*a*f*(a + a*Sin[e + f*x])^(3/2)) + (11*Sec[e + f*x])/(96*a^2*f*Sqrt[a + a*S
in[e + f*x]])
```


Rule 2712

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]),
x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
 f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2687

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sq
rt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Co
s[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} + \frac{\int \frac{\sec^2(e+fx)\left(-\frac{5a}{2}+6a\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx}{6a^2}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))^{3/2}} + \frac{11\int \frac{\sec^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{96a^2}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))^{3/2}} + \frac{11\sec(e+fx)}{96a^2f\sqrt{a+a\sin(e+fx)}} + \frac{11\int}{}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{11\cos(e+fx)}{128af(a+a\sin(e+fx))^{3/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))^{3/2}} + \frac{}{96}$$

$$= -\frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{11\cos(e+fx)}{128af(a+a\sin(e+fx))^{3/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))^{3/2}} + \frac{}{96}$$

$$= -\frac{11\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{128\sqrt{2}a^{5/2}f} - \frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{11\cos(e+fx)}{128af(a+a\sin(e+fx))^{3/2}} + \frac{}{4}$$

Mathematica [C] time = 0.408413, size = 284, normalized size = 1.7

$$\frac{\frac{48\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5}{\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)} + 15\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4 - 30\sin\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (-32 + (64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 104*Si
n[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 52*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2])^2 - 30*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])^3 + 15*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (33 + 33*I)*(-1)^(
3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2])^5 + (48*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(C
os[(e + f*x)/2] - Sin[(e + f*x)/2]))/(384*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A] time = 0.648, size = 266, normalized size = 1.6

$$-\frac{1}{768\left(1+\sin\left(fx+e\right)\right)^{2}\cos\left(fx+e\right)f}\left(\left(\left(66\,a^{7/2}-33\,\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{a-a\sin\left(fx+e\right)}\sqrt{2}}{\sqrt{a}}\right)a^{3}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/768/a^(11/2)*((66*a^(7/2)-33*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)*cos(f*x+e)^2+(-448*
a^(7/2)+132*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/
2)*2^(1/2)/a^(1/2))*a^3)*sin(f*x+e)+(154*a^(7/2)-99*(a-a*sin(f*x+e))^(1/2)*
2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3)*cos(f*x+e)
^2-320*a^(7/2)+132*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+
e))^(1/2)*2^(1/2)/a^(1/2))*a^3)/(1+sin(f*x+e))^2/cos(f*x+e)/(a+a*sin(f*x+e)
)^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 1.69739, size = 751, normalized size = 4.5

$$\frac{33\sqrt{2}\left(3\cos\left(fx+e\right)^{3}+\left(\cos\left(fx+e\right)^{3}-4\cos\left(fx+e\right)\right)\sin\left(fx+e\right)-4\cos\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a\cos\left(fx+e\right)^{2}-2\sqrt{2}\sqrt{a\sin(}}{}\right.}{1536\left(3\,a^{3}f\cos\left(fx+e\right)^{3}-4\,a^{3}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(33*sqrt(2)*(3*cos(f*x + e)^3 + (cos(f*x + e)^3 - 4*cos(f*x + e))*si
n(f*x + e) - 4*cos(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqr
t(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f
*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos
(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(77*cos(f*x + e)^2 + (
33*cos(f*x + e)^2 - 224)*sin(f*x + e) - 160)*sqrt(a*sin(f*x + e) + a))/(3*a
^3*f*cos(f*x + e)^3 - 4*a^3*f*cos(f*x + e) + (a^3*f*cos(f*x + e)^3 - 4*a^3*
f*cos(f*x + e))*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.113 $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=141

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}a^{5/2}f}-\frac{2\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(5/2)*f) -
(7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqr
t[2]*a^(5/2)*f) - (2*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)) - Cot[e
 + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rubi [A] time = 0.346435, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2715, 2978, 2985, 2649, 206, 2773}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}a^{5/2}f}-\frac{2\cos(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] (5*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(a^(5/2)*f) -
(7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqr
t[2]*a^(5/2)*f) - (2*Cos[e + f*x])/(a*f*(a + a*Sin[e + f*x])^(3/2)) - Cot[e
 + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2))
```

Rule 2715

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Di
st[1/b^2, Int[((a + b*Sin[e + f*x])^(m + 1)*(b*m - a*(m + 1)*Sin[e + f*x]))
/Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\csc(e+fx)\left(-\frac{5a}{2}+\frac{3}{2}a\sin(e+fx)\right)}{(a+a\sin(e+fx))^{3/2}}\,dx}{a^2}$$

$$= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{\int \frac{\csc(e+fx)\left(-5a^2+2a^2\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{2a^4}$$

$$= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{5\int \csc(e+fx)\sqrt{a+a\sin(e+fx)}}{2a^3}$$

$$= -\frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}} + \frac{5\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \frac{a\cos(e+fx)}{\sqrt{a+a\sin(e+f}}\right.}{a^2 f}$$

$$= \frac{5\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}a^{5/2}f} - \frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{}{af(a}$$

Mathematica [C] time = 0.71848, size = 451, normalized size = 3.2

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^3\left(8\sin\left(\frac{1}{2}(e+fx)\right)+\frac{2\sin\left(\frac{1}{4}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2}{\cos\left(\frac{1}{4}(e+fx)\right)-\sin\left(\frac{1}{4}(e+fx)\right)}-\frac{2\sin\left(\frac{1}{4}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx\right.\right.}{\sin\left(\frac{1}{4}(e+fx)\right)+\cos}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(8*Sin[(e + f*x)/2] - 4*(Cos[(e +
f*x)/2] + Sin[(e + f*x)/2]) + 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (
28 + 28*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4]
)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - Cot[(e + f*x)/4]*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^2 + 10*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 10*Log[1 - Cos[(e + f*x)/2] + S
in[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (2*Sin[(e + f*x)
/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/4] - Sin[(e + f
*x)/4]) - (2*Sin[(e + f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos
[(e + f*x)/4] + Sin[(e + f*x)/4]) - (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2
*Tan[(e + f*x)/4]))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A] time = 0.57, size = 219, normalized size = 1.6

$$-\frac{1}{2\sin\left(fx+e\right)\cos\left(fx+e\right)f}\left(7\sqrt{2}\mathrm{Artanh}\left(1/2\,\frac{\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\sqrt{2}}{\sqrt{a}}\right)\left(\sin\left(fx+e\right)\right)^{2}a-10\,\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1\right.}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/2/a^(7/2)*(7*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1
/2))*sin(f*x+e)^2*a-10*arctanh((-a*(-1+sin(f*x+e)))^(1/2)/a^(1/2))*sin(f*x+
e)^2*a+7*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/2))*a*
sin(f*x+e)+4*(-a*(-1+sin(f*x+e)))^(1/2)*a^(1/2)*sin(f*x+e)-10*arctanh((-a*(
-1+sin(f*x+e)))^(1/2)/a^(1/2))*sin(f*x+e)*a+2*(-a*(-1+sin(f*x+e)))^(1/2)*a^
(1/2))*(-a*(-1+sin(f*x+e)))^(1/2)/sin(f*x+e)/cos(f*x+e)/(a+a*sin(f*x+e))^(1
/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.8593, size = 1436, normalized size = 10.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(5*(cos(f*x + e)^3 + 2*cos(f*x + e)^2 + (cos(f*x + e)^2 - cos(f*x + e)
- 2)*sin(f*x + e) - cos(f*x + e) - 2)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*c
os(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos
```

```
(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos
(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos
(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) + 7*sq
rt(2)*(a*cos(f*x + e)^3 + 2*a*cos(f*x + e)^2 - a*cos(f*x + e) + (a*cos(f*x
+ e)^2 - a*cos(f*x + e) - 2*a)*sin(f*x + e) - 2*a)*log(-(cos(f*x + e)^2 - (
cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*
x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 -
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(2*cos(f*x
 + e)^2 + (2*cos(f*x + e) + 1)*sin(f*x + e) + cos(f*x + e) - 1)*sqrt(a*sin(
f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 2*a^3*f*cos(f*x + e)^2 - a^3*f*cos(f
*x + e) - 2*a^3*f + (a^3*f*cos(f*x + e)^2 - a^3*f*cos(f*x + e) - 2*a^3*f)*s
in(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] sage2
```

3.114 $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=191

$$-\frac{19\cot(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{5/2}f}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3a^2f\sqrt{a\sin(e+fx)+a}}+\frac{13}{12}$$

```
[Out] (45*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*a^(5/2)*f)
 - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(a^(5/2)*f) - (19*Cot[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]) +
 (13*Cot[e + f*x]*Csc[e + f*x])/(12*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[
e + f*x]*Csc[e + f*x]^2)/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]])
```

Rubi [A] time = 0.958734, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2717, 2779, 2984, 2985, 2649, 206, 2773, 3044}

$$-\frac{19\cot(e+fx)}{8a^2f\sqrt{a\sin(e+fx)+a}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{8a^{5/2}f}-\frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{a^{5/2}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3a^2f\sqrt{a\sin(e+fx)+a}}+\frac{13}{12}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```


```
[Out] (45*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/(8*a^(5/2)*f)
 - (4*Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*
x]])])/(a^(5/2)*f) - (19*Cot[e + f*x])/(8*a^2*f*Sqrt[a + a*Sin[e + f*x]]) +
 (13*Cot[e + f*x]*Csc[e + f*x])/(12*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[
e + f*x]*Csc[e + f*x]^2)/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]])
```


Rule 2717

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Dist[-2/(a*b), Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^3
, x], x] + Dist[1/a^2, Int[((a + b*Sin[e + f*x])^(m + 2)*(1 + Sin[e + f*x]^
2))/Sin[e + f*x]^4, x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] &
& IntegerQ[m - 1/2] && LtQ[m, -1]
```

Rule 2779

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(c + d*Sin[e + f*x])^(
n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] - Dist[1/(2*b*
(n + 1)*(c^2 - d^2)), Int[((c + d*Sin[e + f*x])^(n + 1)*Simp[a*d - 2*b*c*(n
 + 1) + b*d*(2*n + 3)*Sin[e + f*x], x])/Sqrt[a + b*Sin[e + f*x]], x], x] /;
 FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2984

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A
*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c -
 A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x
] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2773

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x
], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3044

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a*d*
m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*(c^
2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2
, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\int \frac{\csc^4(e+fx)\left(1+\sin^2(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2} - \frac{2\int \frac{\csc^3(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx}{a^2}$$

$$= \frac{\cot(e+fx)\csc(e+fx)}{a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2 f\sqrt{a+a\sin(e+fx)}} + \frac{\int \frac{\csc^3(e+fx)\left(-\frac{a}{2}+\frac{11}{2}a\sin(e+fx)\right)}{\sqrt{a+a\sin(e+fx)}}\,dx}{3a^3} + \cdots$$

$$= -\frac{\cot(e+fx)}{2a^2 f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2 f\sqrt{a+a\sin(e+fx)}} + \cdots$$

$$= -\frac{19\cot(e+fx)}{8a^2 f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2 f\sqrt{a+a\sin(e+fx)}} + \cdots$$

$$= -\frac{19\cot(e+fx)}{8a^2 f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2 f\sqrt{a+a\sin(e+fx)}} - \frac{17}{\cdots}$$

$$= \frac{7\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{2a^{5/2}f} - \frac{2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{19\cot(e+fx)}{8a^2 f\sqrt{a+a\sin(e+fx)}} + \frac{1\cdots}{1\cdots}$$

$$= \frac{45\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{5/2}f} - \frac{4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{19\cot(e+fx)}{8a^2 f\sqrt{a+a\sin(e+fx)}} + \cdots$$

Mathematica [C] time = 2.38875, size = 332, normalized size = 1.74

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^5\left(-\frac{8\csc^9\left(\frac{1}{2}(e+fx)\right)\left(-396\sin\left(\frac{1}{2}(e+fx)\right)-218\sin\left(\frac{3}{2}(e+fx)\right)+114\sin\left(\frac{5}{2}(e+fx)\right)+396\cos\left(\frac{1}{2}(e+fx)\right)-218\cos\cdots\right.}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*((1536 + 1536*I)*(-1)^(3/4)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - (8*Csc[(e + f*x)/2]^9*(
396*Cos[(e + f*x)/2] - 218*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/2]
- 396*Sin[(e + f*x)/2] - 405*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*S
in[e + f*x] + 405*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x]
```

```
 - 218*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] + 135*Log[1 + Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 135*Log[1 - Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)]))/(Csc[(e + f*x)/4]^2 - Sec[(e + f
*x)/4]^2)^3))/(192*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

Maple [A] time = 0.722, size = 182, normalized size = 1.

$$-\frac{1+\sin\left(fx+e\right)}{24\left(\sin\left(fx+e\right)\right)^{3}\cos\left(fx+e\right)f}\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}\left(-135\,a^{5}\mathrm{Artanh}\left(\frac{\sqrt{-a\left(-1+\sin\left(fx+e\right)\right)}}{\sqrt{a}}\right)\left(\sin\left(fx+e\right)\right)^{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/24/a^(15/2)*(1+sin(f*x+e))*(-a*(-1+sin(f*x+e)))^(1/2)*(-135*a^5*arctanh(
(-a*(-1+sin(f*x+e)))^(1/2)/a^(1/2))*sin(f*x+e)^3+57*(-a*(-1+sin(f*x+e)))^(5
/2)*a^(5/2)+96*2^(1/2)*arctanh(1/2*(-a*(-1+sin(f*x+e)))^(1/2)*2^(1/2)/a^(1/
2))*a^5*sin(f*x+e)^3-88*(-a*(-1+sin(f*x+e)))^(3/2)*a^(7/2)+39*(-a*(-1+sin(f
*x+e)))^(1/2)*a^(9/2))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 1.74451, size = 1504, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/96*(135*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x +
e)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 -
7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) -
2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (
a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3
+ cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1)) +
 192*sqrt(2)*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 - (a*cos(f*x + e)^3 + a
*cos(f*x + e)^2 - a*cos(f*x + e) - a)*sin(f*x + e) + a)*log(-(cos(f*x + e)^
2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(c
os(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)
^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4*(57*c
os(f*x + e)^3 + 83*cos(f*x + e)^2 - (57*cos(f*x + e)^2 - 26*cos(f*x + e) -
91)*sin(f*x + e) - 65*cos(f*x + e) - 91)*sqrt(a*sin(f*x + e) + a))/(a^3*f*c
os(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f - (a^3*f*cos(f*x + e)^3 + a^
3*f*cos(f*x + e)^2 - a^3*f*cos(f*x + e) - a^3*f)*sin(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.15976, size = 975, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)*((2*tan(1/2*f*x + 1/2*e)/(a^3*sgn(
tan(1/2*f*x + 1/2*e) + 1)) - 15/(a^3*sgn(tan(1/2*f*x + 1/2*e) + 1)))*tan(1/
2*f*x + 1/2*e) + 74/(a^3*sgn(tan(1/2*f*x + 1/2*e) + 1))) + (1890*sqrt(2)*sq
```

```
rt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 3840*sqrt(2)*sqrt(a)*a
rctan(sqrt(a)/sqrt(-a)) - 945*sqrt(2)*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a
)) + 2700*sqrt(a)*arctan((sqrt(2)*sqrt(a) + sqrt(a))/sqrt(-a)) - 5376*sqrt(
a)*arctan(sqrt(a)/sqrt(-a)) - 1350*sqrt(-a)*log(sqrt(2)*sqrt(a) + sqrt(a))
- 1302*sqrt(2)*sqrt(-a) - 1808*sqrt(-a))*sgn(tan(1/2*f*x + 1/2*e) + 1)/(7*s
qrt(2)*sqrt(-a)*a^(5/2) + 10*sqrt(-a)*a^(5/2)) + 384*sqrt(2)*arctan(-1/2*sq
rt(2)*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a) +
sqrt(a))/sqrt(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*f*x + 1/2*e) + 1)) - 270*arcta
n(-(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt
(-a))/(sqrt(-a)*a^2*sgn(tan(1/2*f*x + 1/2*e) + 1)) + 135*log(abs(-sqrt(a)*t
an(1/2*f*x + 1/2*e) + sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a)))/(a^(5/2)*sgn(tan
(1/2*f*x + 1/2*e) + 1)) - 2*(15*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(
1/2*f*x + 1/2*e)^2 + a))^5 - 78*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sqrt(a*tan(
1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a) + 144*(sqrt(a)*tan(1/2*f*x + 1/2*e) - sq
rt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) - 15*(sqrt(a)*tan(1/2*f*x + 1/2
*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - 74*a^(5/2))/(((sqrt(a)*tan(
1/2*f*x + 1/2*e) - sqrt(a*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^3*a^2*sgn(tan
(1/2*f*x + 1/2*e) + 1)))/f
```

3.115 $\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$

Optimal. Leaf size=982

result too large to display

```
[Out] (-361*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1/3))/(126*f) + (361*Sec[e + f*x]*
(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(1/3))/(63*f) - (Sec[e + f*x]*(65*a
^2 - 142*a^2*Sin[e + f*x]))/(42*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])
^(2/3)) + (361*(1 + Sqrt[3])*Sec[e + f*x]*(1 - Sin[e + f*x])*(a + a*Sin[e +
 f*x])^(2/3))/(63*f*(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(
1/3))) - (361*2^(1/3)*EllipticE[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a
+ a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*
x])^(1/3))], (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1
/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a
^(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^
(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(21*3^(3/4)*a^(2/3)*f
*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])
^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2)])
- (361*(1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a +
 a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x
])^(1/3))], (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/
3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^
(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(
1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(63*2^(2/3)*3^(1/4)*a^
(2/3)*f*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e
+ f*x])^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3)
)^2)]) + (3*a^2*Sin[e + f*x]*Tan[e + f*x])/(2*f*(a - a*Sin[e + f*x])*(a + a
*Sin[e + f*x])^(2/3)) - (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e
 + f*x])*(a + a*Sin[e + f*x])^(2/3))
```

Rubi [A] time = 1.2706, antiderivative size = 982, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {2719, 100, 153, 144, 51, 63, 308, 225, 1881}

$$-\frac{3\sin^2(e+fx)\tan(e+fx)a^2}{f(a-a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}+\frac{3\sin(e+fx)\tan(e+fx)a^2}{2f(a-a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}-\frac{\sec(e+fx)\left(65a^2-142a^2\text{s}\right.}{42f(a-a\sin(e+fx))(\sin(e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] (-361*Sec[e + f*x]*(a + a*Sin[e + f*x])^(1/3))/(126*f) + (361*Sec[e + f*x]*
(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(1/3))/(63*f) - (Sec[e + f*x]*(65*a
^2 - 142*a^2*Sin[e + f*x]))/(42*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])
^(2/3)) + (361*(1 + Sqrt[3])*Sec[e + f*x]*(1 - Sin[e + f*x])*(a + a*Sin[e +
 f*x])^(2/3))/(63*f*(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(
1/3))) - (361*2^(1/3)*EllipticE[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a
+ a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*
x])^(1/3))], (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1
/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a
^(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^
(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2]])/(21*3^(3/4)*a^(2/3)*f
*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])
^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2)])
- (361*(1 - Sqrt[3])*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a +
 a*Sin[e + f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x
])^(1/3))], (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/
3)*a^(1/3) - (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^
(1/3)*(a + a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(
1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2]])/(63*2^(2/3)*3^(1/4)*a^
(2/3)*f*Sqrt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e
+ f*x])^(1/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3)
)^2)]) + (3*a^2*Sin[e + f*x]*Tan[e + f*x])/(2*f*(a - a*Sin[e + f*x])*(a + a
*Sin[e + f*x])^(2/3)) - (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e
 + f*x])*(a + a*Sin[e + f*x])^(2/3))
```

<u>Rule 2719</u>

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

<u>Rule 100</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 144

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*
h*(n + 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n
 + 2)) + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*
(m + 1) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)*(a + b*x)^(m +
 1)*(c + d*x)^(n + 1))/(b*d*(b*c - a*d)^2*(m + 1)*(n + 1)), x] - Dist[(a^2*
d^2*f*h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
 + n + 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n
+ 3) + d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m
+ 1)*(n + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^4}{(a-x)^{5/2}(a+x)^{13/6}}\,dx\right.}{af}$$

$$= -\frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}} - \frac{\left(3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\right.}{}$$

$$= \frac{3a^2\sin(e+fx)\tan(e+fx)}{2f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}} - \frac{3a^2\sin^2(e+fx)\tan(e+f}{f(a-a\sin(e+fx))(a+a\sin(e+}$$

$$= -\frac{\sec(e+fx)\left(65a^2-142a^2\sin(e+fx)\right)}{42f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}} + \frac{3a^2\sin(e+fx)\tan(e+}{2f(a-a\sin(e+fx))(a+a\sin}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} - \frac{\sec(e+fx)\left(65a^2-142a^2\sin(e+fx\right.}{42f(a-a\sin(e+fx))(a+a\sin(e+fx}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin(e+fx))\sqrt[3]{a+a}}{63f}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin(e+fx))\sqrt[3]{a+a}}{63f}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin(e+fx))\sqrt[3]{a+a}}{63f}$$

$$= -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f} + \frac{361\sec(e+fx)(1-\sin(e+fx))\sqrt[3]{a+a}}{63f}$$

Mathematica [C] time = 3.273, size = 318, normalized size = 0.32

$$\frac{\sqrt[3]{a(\sin(e+fx)+1)}\left(3\left(-172\tan(e+fx)-3\sec^3(e+fx)+86\sec(e+fx)+24\tan(e+fx)\sec^2(e+fx)+361\right)+\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1/3)*(((1083/10 + (1083*I)/10)*(-1)^(3/4)*(20*E^(I
*(e + f*x))*Sqrt[Cos[(2*e + Pi + 2*f*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3,
2/3, (-I)/E^(I*(e + f*x))] - 2*(1 + I/E^(I*(e + f*x)))^(2/3)*(1 + E^((2*I)*
(e + f*x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*e + Pi + 2*f*x)/4]^2]
+ (5*I)*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(e + f*x))]*Sqrt[2 - 2*S
in[e + f*x]]))/(Sqrt[2]*E^(I*(e + f*x))*(1 + I/E^(I*(e + f*x)))^(2/3)*Sqrt[
(I*(-I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]) + 3*(361 + 86*Sec[e + f*x] -
 3*Sec[e + f*x]^3 - 172*Tan[e + f*x] + 24*Sec[e + f*x]^2*Tan[e + f*x])))/(1
89*f)
```

Maple [F] time = 0.128, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + a\sin\left(fx + e\right)}\left(\tan\left(fx + e\right)\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx + e\right) + a\right)^{\frac{1}{3}} \tan\left(fx + e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx + e\right) + a\right)^{\frac{1}{3}} \tan\left(fx + e\right)^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

3.116 $\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$

Optimal. Leaf size=123

$$-\frac{5a\sqrt[6]{\sin(e+fx)+1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3\sqrt[6]{2}f(a\sin(e+fx)+a)^{2/3}}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}+\frac{7\sec(e+fx)\sqrt[3]{\ldots}}{\ldots}$$

```
[Out] (-5*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - Sin[e + f*x])/2]*(
1 + Sin[e + f*x])^(1/6))/(3*2^(1/6)*f*(a + a*Sin[e + f*x])^(2/3)) + (7*Sec[
e + f*x]*(a + a*Sin[e + f*x])^(1/3))/f - (3*Sec[e + f*x]*(a + a*Sin[e + f*x
])^(4/3))/(a*f)
```

Rubi [A] time = 0.19337, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2713, 2855, 2652, 2651}

$$-\frac{5a\sqrt[6]{\sin(e+fx)+1}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{3\sqrt[6]{2}f(a\sin(e+fx)+a)^{2/3}}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}+\frac{7\sec(e+fx)\sqrt[3]{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] (-5*a*Cos[e + f*x]*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - Sin[e + f*x])/2]*(
1 + Sin[e + f*x])^(1/6))/(3*2^(1/6)*f*(a + a*Sin[e + f*x])^(2/3)) + (7*Sec[
e + f*x]*(a + a*Sin[e + f*x])^(1/3))/f - (3*Sec[e + f*x]*(a + a*Sin[e + f*x
])^(4/3))/(a*f)
```

Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
```

```
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx = -\frac{3\sec(e+fx)(a+a\sin(e+fx))^{4/3}}{af}+\frac{3\int \sec^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\left(\frac{4a}{3}\right.}{a}$$

$$=\frac{7\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{f}-\frac{3\sec(e+fx)(a+a\sin(e+fx))^{4/3}}{af}+\frac{1}{3}(5a)$$

$$=\frac{7\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{f}-\frac{3\sec(e+fx)(a+a\sin(e+fx))^{4/3}}{af}+\frac{\left(5a(1\right.}{}$$

$$=-\frac{5a\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sqrt[6]{1+\sin(e+fx)}}{3\sqrt[6]{2}f(a+a\sin(e+fx))^{2/3}}+\frac{7\sec(e+}{}$$

Mathematica [C] time = 2.746, size = 290, normalized size = 2.36

$$\frac{\sqrt[3]{a(\sin(e+fx)+1)}\left(-3(-2\tan(e+fx)+\sec(e+fx)+5)+\frac{\left(\frac{3}{2}+\frac{3i}{2}\right)(-1)^{3/4}e^{-i(e+fx)}\left(2\left(1+ie^{-i(e+fx)}\right)^{2/3}\left(1+e^{2i(e+fx)}\right){}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sin^2\left(\frac{1}{4}(\right.\right.\right.}{}\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(1/3)*(((3/2 + (3*I)/2)*(-1)^(3/4)*(-20*E^(I*(e + f
*x))*Sqrt[Cos[(2*e + Pi + 2*f*x)/4]^2]*Hypergeometric2F1[-1/3, 1/3, 2/3, (-
I)/E^(I*(e + f*x))] + 2*(1 + I/E^(I*(e + f*x)))^(2/3)*(1 + E^((2*I)*(e + f*
x)))*Hypergeometric2F1[1/2, 5/6, 11/6, Sin[(2*e + Pi + 2*f*x)/4]^2] - (5*I)
*Hypergeometric2F1[1/3, 2/3, 5/3, (-I)/E^(I*(e + f*x))]*Sqrt[2 - 2*Sin[e +
f*x]]))/(Sqrt[2]*E^(I*(e + f*x))*(1 + I/E^(I*(e + f*x)))^(2/3)*Sqrt[(I*(-I
+ E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]) - 3*(5 + Sec[e + f*x] - 2*Tan[e + f
*x])))/(3*f)
```

Maple [F] time = 0.104, size = 0, normalized size = 0.

$$\int \sqrt[3]{a+a\sin\left(fx+e\right)}\left(\tan\left(fx+e\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**2,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*tan(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\tan\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

3.117 $\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=80

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{7/3}F_1\left(\frac{11}{6};-\frac{1}{2},2;\frac{17}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{11a^2f}$$

```
[Out] (6*Sqrt[2]*AppellF1[11/6, -1/2, 2, 17/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(7/3))/(11*a
^2*f)
```

Rubi [A] time = 0.101394, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2719, 137, 136}

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{7/3}F_1\left(\frac{11}{6};-\frac{1}{2},2;\frac{17}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{11a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]
```


```
[Out] (6*Sqrt[2]*AppellF1[11/6, -1/2, 2, 17/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(7/3))/(11*a
^2*f)
```


Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a-x}(a+x)^{5/6}}{x^2}\,dx,x,\right.}{af}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)^{5/6}\sqrt{\frac{1}{2}-\frac{x}{2a}}}{x^2}\,d\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{6\sqrt{2}F_1\left(\frac{11}{6};-\frac{1}{2},2;\frac{17}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(}}{11a^2f}$$

Mathematica [C] time = 23.8982, size = 2692, normalized size = 33.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] ((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f
*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3))/
(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2
]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])] + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/
2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])] + I*Ap
pellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*
(1 + Cot[(e + f*x)/2])])*(1 + Cot[(e + f*x)/2]))) + ((-4 - Cot[e + f*x])*(a
*(1 + Sin[e + f*x]))^(1/3))/f + ((5/2 + (5*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/
```

```
3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]*
(a*(1 + Sin[e + f*x]))^(1/3))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1
/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])] + (Ap
pellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*
(1 + Tan[(e + f*x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + T
an[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])])*(1 + Tan[(e + f*x)/2
]))) + (Cos[(3*(e + f*x))/2]*Csc[(e + f*x)/2]*Sec[(e + f*x)/2]*(a*(1 + Sin[
e + f*x]))^(1/3)*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8
 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2])
)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2
])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/
3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/
2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e +
 f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2])))/(f*(Cos[(e + f*x)/2] + Sin[(e + f
*x)/2])*(1 + Tan[(e + f*x)/2])*((-3*Sec[(e + f*x)/2]^2*((1 + Tan[(e + f*x)/
2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot
[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5
/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan
[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x
)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e +
f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2
])))/(4*(1 + Tan[(e + f*x)/2])^2) + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Co
t[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3,
5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Ta
n[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*
x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e +
 f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/
2]))*(Sqrt[Sec[(e + f*x)/2]^2]/2 - (Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2])
)/(2*Sqrt[Sec[(e + f*x)/2]^2])))/((1 + Tan[(e + f*x)/2])*((1 + Tan[(e + f*x
)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(1/3)) + (3*((1 + Tan[(e + f*x)/2])/Sqrt[Se
c[(e + f*x)/2]^2])^(2/3)*(-(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + C
ot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I
)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Sec[(e +
f*x)/2]^2)/2 + ((1 + I)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x
)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f
*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*Sec[(e + f*x)/2]^2)/2^(1/3) + ((1/3 + I/3
)*2^(2/3)*(((1/2 - I/2)*(I + Cot[(e + f*x)/2])*Csc[(e + f*x)/2]^2)/(1 + Cot
[(e + f*x)/2])^2 - ((1/2 - I/2)*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2]))
*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 +
 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]))/(((1 - I)*(I + Cot[(e + f*x)/
2]))/(1 + Cot[(e + f*x)/2]))^(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3, 5
/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]
*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*Csc[(e + f*x)/2]^2*(1 + Tan
[(e + f*x)/2]))/((-1 - I)*(I + Cot[(e + f*x)/2]))^(2/3) - ((1/3 - I/3)*Appe
llF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1
```

```
 + Cot[(e + f*x)/2])]*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Csc[(e + f*x)
/2]^2*(1 + Tan[(e + f*x)/2]))/((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(2/3
) - ((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e +
f*x)/2]))^(1/3)*((-1/30 + I/30)*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1
 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^
2 - (1/30 + I/30)*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*
x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^2)*(1 + Tan[(e
 + f*x)/2]) + ((2/3 + (2*I)/3)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1
 + Cot[(e + f*x)/2]))^(1/3)*(I + Tan[(e + f*x)/2])*(2 + 2*Tan[(e + f*x)/2])
*(-((Sec[(e + f*x)/2]^2*((1 + I) + (1 - I)*Tan[(e + f*x)/2]))/(2 + 2*Tan[(e
 + f*x)/2])^2) + ((1/2 - I/2)*Sec[(e + f*x)/2]^2)/(2 + 2*Tan[(e + f*x)/2]))
*(-Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2
 + 2*Tan[(e + f*x)/2])] + (1 - ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*
Tan[(e + f*x)/2]))^(-1/3)))/((1 + I) + (1 - I)*Tan[(e + f*x)/2])))/(2*(1 +
Tan[(e + f*x)/2])))
```

Maple [F] time = 0.095, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^{2}\sqrt[3]{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}\cot^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(1/3)*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)
```

3.118 $\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=80

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{10/3}F_1\left(\frac{17}{6};-\frac{3}{2},4;\frac{23}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{17a^3f}$$

```
[Out] (12*Sqrt[2]*AppellF1[17/6, -3/2, 4, 23/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
 f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(10/3))/(17
*a^3*f)
```

Rubi [A] time = 0.0938648, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2719, 137, 136}

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{10/3}F_1\left(\frac{17}{6};-\frac{3}{2},4;\frac{23}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{17a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (12*Sqrt[2]*AppellF1[17/6, -3/2, 4, 23/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
 f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(10/3))/(17
*a^3*f)
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a-x)^{3/2}(a+x)^{11/6}}{x^4}\,dx, x\right.}{af}$$

$$= \frac{\left(2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)^{11/6}\left(\frac{1}{2}-\frac{x}{2a}\right)^{3/}}{x^4}\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{12\sqrt{2}F_1\left(\frac{17}{6};-\frac{3}{2},4;\frac{23}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e}}{17a^3f}$$

Mathematica [C] time = 22.5988, size = 2796, normalized size = 34.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] ((239/54 + (77*Cot[e + f*x])/54 - (Cot[e + f*x]*Csc[e + f*x])/18 - (Cot[e +
 f*x]*Csc[e + f*x]^2)/3)*(a*(1 + Sin[e + f*x]))^(1/3))/f - ((70/9 + (70*I)/
9)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 -
I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3)*(1 + Tan[(e + f*x
)/2]))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e +
 f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Sec[(e + f*x)/2] + AppellF1[
5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Co
t[(e + f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]) + I*AppellF1[5/3, 4/
```

3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e +
f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]))*(Cos[(e + f*x)/2] + Sin[(e
 + f*x)/2])) - ((355/108 + (355*I)/108)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 +
 I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]*(a*(1 + S
in[e + f*x]))^(1/3))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)
*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])] + (AppellF1[5/
3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[
(e + f*x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f
x)/2]), (1/2 - I/2)(1 + Tan[(e + f*x)/2])])*(1 + Tan[(e + f*x)/2]))) - (2
39*Cos[(3*(e + f*x))/2]*Csc[e + f*x]*(a*(1 + Sin[e + f*x]))^(1/3)*((1 + Tan
[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 -
I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[
1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2]
)]*(I + Tan[(e + f*x)/2]) - AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + C
ot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I
)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[
(e + f*x)/2])))/(216*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(1 + Tan[(e +
f*x)/2])*((-3*Sec[(e + f*x)/2]^2*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)
/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + C
ot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)
*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - Appel
lF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1
+ Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 -
I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2])))/(8*(1 + Tan[(e +
f*x)/2])^2) + ((8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 +
Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I
)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - Appe
llF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1
 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 -
 I)*(I + Cot[(e + f*x)/2]))^(1/3)*(1 + Tan[(e + f*x)/2]))*(Sqrt[Sec[(e + f*
x)/2]^2]/2 - (Tan[(e + f*x)/2]*(1 + Tan[(e + f*x)/2]))/(2*Sqrt[Sec[(e + f*x
)/2]^2])))/(2*(1 + Tan[(e + f*x)/2])*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e +
f*x)/2]^2])^(1/3)) + (3*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(
2/3)*(-(AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1
/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^
(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Sec[(e + f*x)/2]^2)/2 + ((1 +
 I)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(
e + f*x)/2])]*Sec[(e + f*x)/2]^2)/2^(1/3) + ((1/3 + I/3)*2^(2/3)*(((1/2 - I
/2)*(I + Cot[(e + f*x)/2])*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2])^2 - (
(1/2 - I/2)*Csc[(e + f*x)/2]^2)/(1 + Cot[(e + f*x)/2]))*Hypergeometric2F1[1
/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])
]*(I + Tan[(e + f*x)/2]))/(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f
*x)/2]))^(2/3) - ((1/6 + I/6)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 +
 Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*((2 + 2*I) - (2 - 2

```
*I)*Cot[(e + f*x)/2])^(1/3)*Csc[(e + f*x)/2]^2*(1 + Tan[(e + f*x)/2]))/((-1
 - I)*(I + Cot[(e + f*x)/2]))^(2/3) - ((1/3 - I/3)*AppellF1[2/3, 1/3, 1/3,
5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])
]*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*Csc[(e + f*x)/2]^2*(1 + Tan[(e +
f*x)/2]))/((2 + 2*I) - (2 - 2*I)*Cot[(e + f*x)/2])^(2/3) - ((2 + 2*I) - (2
- 2*I)*Cot[(e + f*x)/2])^(1/3)*((-1 - I)*(I + Cot[(e + f*x)/2]))^(1/3)*((-1
/30 + I/30)*AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2])
, (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^2 - (1/30 + I/30)*Ap
pellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*
(1 + Cot[(e + f*x)/2])]*Csc[(e + f*x)/2]^2)*(1 + Tan[(e + f*x)/2]) + ((2/3
+ (2*I)/3)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2])
)^(1/3)*(I + Tan[(e + f*x)/2])*(2 + 2*Tan[(e + f*x)/2])*(-((Sec[(e + f*x)/2
]^2*((1 + I) + (1 - I)*Tan[(e + f*x)/2]))/(2 + 2*Tan[(e + f*x)/2])^2) + ((1
/2 - I/2)*Sec[(e + f*x)/2]^2)/(2 + 2*Tan[(e + f*x)/2]))*(-Hypergeometric2F1
[1/3, 2/3, 5/3, ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2
])] + (1 - ((1 + I) + (1 - I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2]))^(
-1/3)))/((1 + I) + (1 - I)*Tan[(e + f*x)/2])))/(4*(1 + Tan[(e + f*x)/2])))
```

Maple [F] time = 0.102, size = 0, normalized size = 0.

$$\int \left(\cot \left(f x+e\right)\right)^{4} \sqrt[3]{a+a \sin \left(f x+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(f x+e\right)+a\right)^{\frac{1}{3}} \cot \left(f x+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)
```

3.119 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=551

$$\frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a\sin(e+fx)+a)^{4/3}}+\frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a\sin(e+fx)+a)^{4/3}}+\frac{973\sec(e+fx)(a\sin(e+fx)+}{}$$

```
[Out] (973*Sec[e + f*x])/(396*f*(a + a*Sin[e + f*x])^(1/3)) - (973*Sec[e + f*x]*(
1 - Sin[e + f*x]))/(495*f*(a + a*Sin[e + f*x])^(1/3)) - (Sec[e + f*x]*(95*a
 + 356*a*Sin[e + f*x]))/(132*f*(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(4/3
)) + (973*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a + a*Sin[e +
f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))],
 (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/3)*a^(1/3)
- (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a +
a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(1/3) - (1 +
 Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(495*2^(1/3)*3^(1/4)*a^(4/3)*f*Sq
rt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1
/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2)]) + (
3*a^2*Sin[e + f*x]*Tan[e + f*x])/(4*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f
*x])^(4/3)) + (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e + f*x])*(
a + a*Sin[e + f*x])^(4/3))
```

Rubi [A] time = 0.493573, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {2719, 100, 153, 144, 51, 63, 225}

$$\frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a\sin(e+fx)+a)^{4/3}}+\frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a\sin(e+fx)+a)^{4/3}}+\frac{973\sec(e+fx)(a\sin(e+fx)+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (973*Sec[e + f*x])/(396*f*(a + a*Sin[e + f*x])^(1/3)) - (973*Sec[e + f*x]*(
1 - Sin[e + f*x]))/(495*f*(a + a*Sin[e + f*x])^(1/3)) - (Sec[e + f*x]*(95*a
 + 356*a*Sin[e + f*x]))/(132*f*(1 - Sin[e + f*x])*(a + a*Sin[e + f*x])^(4/3
)) + (973*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt[3])*(a + a*Sin[e +
f*x])^(1/3))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))],
 (2 + Sqrt[3])/4]*Sec[e + f*x]*(a + a*Sin[e + f*x])^(2/3)*(2^(1/3)*a^(1/3)
- (a + a*Sin[e + f*x])^(1/3))*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a +
a*Sin[e + f*x])^(1/3) + (a + a*Sin[e + f*x])^(2/3))/(2^(1/3)*a^(1/3) - (1 +
 Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2])/(495*2^(1/3)*3^(1/4)*a^(4/3)*f*Sq
rt[-(((a + a*Sin[e + f*x])^(1/3)*(2^(1/3)*a^(1/3) - (a + a*Sin[e + f*x])^(1
/3)))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*(a + a*Sin[e + f*x])^(1/3))^2)]) + (
3*a^2*Sin[e + f*x]*Tan[e + f*x])/(4*f*(a - a*Sin[e + f*x])*(a + a*Sin[e + f
*x])^(4/3)) + (3*a^2*Sin[e + f*x]^2*Tan[e + f*x])/(f*(a - a*Sin[e + f*x])*(
a + a*Sin[e + f*x])^(4/3))
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 144

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*
h*(n + 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n
 + 2)) + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*
(m + 1) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)*(a + b*x)^(m +
 1)*(c + d*x)^(n + 1))/(b*d*(b*c - a*d)^2*(m + 1)*(n + 1)), x] - Dist[(a^2*
d^2*f*h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
 + n + 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n
+ 3) + d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m
+ 1)*(n + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a,
 b, c, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^4}{(a-x)^{5/2}(a+x)^{17/6}}\,dx, x, a\sin(e+\right.}{af}$$

$$= \frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{\left(3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+\right.}{}$$

$$= \frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + -$$

$$= -\frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2\sin(e+}{4f(a-a\sin(e+f}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\,\text{s}}{132f(1-\sin(e+fx))(a+a}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\,\text{s}}{132f(1-\sin(e+fx))(a+a}$$

$$= \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\,\text{s}}{132f(1-\sin(e+fx))(a+a}$$

Mathematica [C] time = 0.796308, size = 128, normalized size = 0.23

$$\frac{973\sqrt{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)+\sqrt{1-\sin(e+fx)}\sec^3(e+fx)(22\sin(e+fx)-128\sin(3(e+}{495f\sqrt{1-\sin(e+fx)}\sqrt[3]{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (973*Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi +
2*f*x)/4]^2] + Sec[e + f*x]^3*Sqrt[1 - Sin[e + f*x]]*(-49 - 64*Cos[2*(e + f
```

```
*x)] + 22*Sin[e + f*x] - 128*Sin[3*(e + f*x)]))/(495*f*Sqrt[1 - Sin[e + f*x
]]*(a*(1 + Sin[e + f*x]))^(1/3))
```

Maple [F] time = 0.119, size = 0, normalized size = 0.

$$\int \left(\tan \left(fx+e\right)\right)^{4} \frac{1}{\sqrt[3]{a+a\sin \left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

```
[Out] int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\tan \left(fx+e\right)^{4}}{\left(a\sin \left(fx+e\right)+a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx+e)^4}{(a\sin(fx+e)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

3.120 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=126

$$\frac{11\sqrt[6]{2}\cos(e+fx)\,_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{\sin(e+fx)+1}\sqrt[3]{a\sin(e+fx)+a}}+\frac{4\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{5af}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

```
[Out] (-3*Sec[e + f*x])/(5*f*(a + a*Sin[e + f*x])^(1/3)) + (11*2^(1/6)*Cos[e + f*
x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[e + f*x])/2])/(15*f*(1 + Sin[e
 + f*x])^(1/6)*(a + a*Sin[e + f*x])^(1/3)) + (4*Sec[e + f*x]*(a + a*Sin[e +
 f*x])^(2/3))/(5*a*f)
```

Rubi [A] time = 0.216494, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2712, 2855, 2652, 2651}

$$\frac{11\sqrt[6]{2}\cos(e+fx)\,_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{\sin(e+fx)+1}\sqrt[3]{a\sin(e+fx)+a}}+\frac{4\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{5af}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-3*Sec[e + f*x])/(5*f*(a + a*Sin[e + f*x])^(1/3)) + (11*2^(1/6)*Cos[e + f*
x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[e + f*x])/2])/(15*f*(1 + Sin[e
 + f*x])^(1/6)*(a + a*Sin[e + f*x])^(1/3)) + (4*Sec[e + f*x]*(a + a*Sin[e +
 f*x])^(2/3))/(5*a*f)
```

Rule 2712

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(b*(a + b*Sin[e + f*x])^m)/(a*f*(2*m - 1)*Cos[e + f*x]),
x] - Dist[1/(a^2*(2*m - 1)), Int[((a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*
m - 1)*Sin[e + f*x]))/Cos[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*
```

```
c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)),
x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x
])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}}+\frac{3\int \sec^2(e+fx)(a+a\sin(e+fx))^{2/3}\left(-\frac{a}{3}+\frac{5}{3}a\sin(e+fx)\right)dx}{5a^2}$$

$$= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}}+\frac{4\sec(e+fx)(a+a\sin(e+fx))^{2/3}}{5af}-\frac{11}{15}\int \frac{1}{\sqrt[3]{a+a\sin(e+fx)}}\,d$$

$$= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}}+\frac{4\sec(e+fx)(a+a\sin(e+fx))^{2/3}}{5af}-\frac{\left(11\sqrt[3]{1+\sin(e+fx)}\right)\int \frac{}{\sqrt[3]{1}}}{15\sqrt[3]{a+a\sin(e+f}}$$

$$= -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}}+\frac{11\sqrt[6]{2}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{1+\sin(e+fx)}\sqrt[3]{a+a\sin(e+fx)}}+\frac{4\sec(e+fx}{}$$

Mathematica [A] time = 0.513245, size = 100, normalized size = 0.79

$$\frac{\sqrt{2-2\sin(e+fx)}(4\tan(e+fx)+\sec(e+fx))-22\cos(e+fx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)\right)}{5f\sqrt{2-2\sin(e+fx)}\sqrt[3]{a(\sin(e+fx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (-22*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi + 2*f*x)/4
]^2] + Sqrt[2 - 2*Sin[e + f*x]]*(Sec[e + f*x] + 4*Tan[e + f*x]))/(5*f*Sqrt[
2 - 2*Sin[e + f*x]]*(a*(1 + Sin[e + f*x]))^(1/3))
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \left(\tan \left(fx + e\right)\right)^{2} \frac{1}{\sqrt[3]{a + a \sin \left(fx + e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

```
[Out] int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan \left(fx + e\right)^{2}}{\left(a \sin \left(fx + e\right) + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tan \left(fx + e\right)^{2}}{\left(a \sin \left(fx + e\right) + a\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] integral(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a(\sin(e+fx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(fx+e)^2}{(a\sin(fx+e)+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

3.121 $\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{5/3}F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{7a^2f}$$

```
[Out] (6*Sqrt[2]*AppellF1[7/6, -1/2, 2, 13/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f
*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(5/3))/(7*a^2
*f)
```

Rubi [A] time = 0.0949979, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2719, 137, 136}

$$\frac{6\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{5/3}F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{7a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (6*Sqrt[2]*AppellF1[7/6, -1/2, 2, 13/6, (1 + Sin[e + f*x])/2, 1 + Sin[e + f
*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(5/3))/(7*a^2
*f)
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
```

```
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a-x}\sqrt[6]{a+x}}{x^2}\,dx, x, a\sin(e+fx)\right)}{af}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt[6]{a+x}\sqrt{\frac{1}{2}-\frac{x}{2a}}}{x^2}\,dx, x, a\sin(e+\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{6\sqrt{2}F_1\left(\frac{7}{6};-\frac{1}{2},2;\frac{13}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}(a+a}{7a^2f}$$

Mathematica [F] time = 12.7663, size = 0, normalized size = 0.

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3), x]
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^{2} \frac{1}{\sqrt[3]{a+a\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{2}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{2}\left(e+fx\right)}{\sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

3.122 $\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

Optimal. Leaf size=80

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{8/3}F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{13a^3f}$$

```
[Out] (12*Sqrt[2]*AppellF1[13/6, -3/2, 4, 19/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
 f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(8/3))/(13*
a^3*f)
```

Rubi [A] time = 0.0935749, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2719, 137, 136}

$$\frac{12\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{8/3}F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{13a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] (12*Sqrt[2]*AppellF1[13/6, -3/2, 4, 19/6, (1 + Sin[e + f*x])/2, 1 + Sin[e +
 f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(8/3))/(13*
a^3*f)
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
```

```
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a-x)^{3/2}(a+x)^{7/6}}{x^4}\,dx, x, a\sin(e+f}{af}$$

$$= \frac{\left(2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)^{7/6}\left(\frac{1}{2}-\frac{x}{2a}\right)^{3/2}}{x^4}\,dx, x, a\sin}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{12\sqrt{2}F_1\left(\frac{13}{6};-\frac{3}{2},4;\frac{19}{6};\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}(a+}{13a^3f}$$

Mathematica [F] time = 9.12866, size = 0, normalized size = 0.

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

```
[Out] Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3), x]
```

Maple [F] time = 0.105, size = 0, normalized size = 0.

$$\int \left(\cot \left(fx+e\right)\right)^{4} \frac{1}{\sqrt[3]{a+a\sin \left(fx+e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

```
[Out] int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^{4}\left(e+fx\right)}{\sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(fx+e\right)^{4}}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

3.123 $\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=269

$$\frac{3a^3(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+a^3\sin^3$$

```
[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^3*(Cos[e + f*x]^2)^((1 + p)/2)*H
ypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e + f
*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a^3*Hypergeometric2F1[2,
 (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3
 + p))
```

Rubi [A] time = 0.345108, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2710, 3476, 364, 2602, 2577, 2591}

$$\frac{3a^3(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+a^3\sin^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^3*(Cos[e + f*x]^2)^((1 + p)/2)*H
ypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e + f
*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a^3*Hypergeometric2F1[2,
 (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3
 + p))
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
```

```
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx = \int \left(a^3(g\tan(e+fx))^p + 3a^3\sin(e+fx)(g\tan(e+fx))^p + 3a^3\sin^2(e+fx)(g$$

$$= a^3\int (g\tan(e+fx))^p\,dx + a^3\int \sin^3(e+fx)(g\tan(e+fx))^p\,dx + \left(3a^3\right)\int \mathrm{s}$$

$$= \frac{\left(a^3g\right)\mathrm{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{\left(3a^3g\right)\mathrm{Subst}\left(\int \frac{x^{2+p}}{\left(g^2+x^2\right)^2}\,dx, x, g\right.}{f}$$

$$= \frac{a^3\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{3a^3\cos^2(e+fx)^{\frac{1+p}{2}}}{}$$

Mathematica [C] time = 58.8952, size = 5199, normalized size = 19.33

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.709, size = 0, normalized size = 0.

$$\int \left(a + a\sin\left(fx+e\right)\right)^3 \left(g\tan\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right) + a\right)^3 \left(g\tan\left(fx+e\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)\right)\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(
f*x + e))*(g*tan(f*x + e))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{3}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

3.124 $\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=187

$$\frac{a^2(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2a^2\sin(}{}$$

```
[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (2*a^2*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^2*Hypergeometric2F1[2, (3 + p)/2
, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))
```

Rubi [A] time = 0.226524, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2710, 3476, 364, 2602, 2577, 2591}

$$\frac{a^2(g\tan(e+fx))^{p+3}\,{}_2F_1\left(2,\frac{p+3}{2};\frac{p+5}{2};-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2a^2\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```


```
[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (2*a^2*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^2*Hypergeometric2F1[2, (3 + p)/2
, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))
```


Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \int \left(a^2(g\tan(e+fx))^p + 2a^2\sin(e+fx)(g\tan(e+fx))^p + a^2\sin^2(e+fx)(g\right.$$
$$= a^2\int (g\tan(e+fx))^p\,dx + a^2\int \sin^2(e+fx)(g\tan(e+fx))^p\,dx + \left(2a^2\right)\int \mathrm{s}$$
$$= \frac{\left(a^2g\right)\mathrm{Subst}\left(\int \frac{x^{2+p}}{\left(g^2+x^2\right)^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{\left(a^2g\right)\mathrm{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\right.}{f}$$
$$= \frac{a^2\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{2a^2\cos^2(e+fx)^{\frac{1+p}{2}}}{}$$

Mathematica [C] time = 18.053, size = 2054, normalized size = 10.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(a + a*Sin[e + f*x])^2*Tan[(e + f*x)
/2]*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f
*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[1 + p/2,
 p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])*
(g*Tan[e + f*x])^p*(Cos[(e + f*x)/2]^4*Tan[e + f*x]^p + 4*Cos[(e + f*x)/2]^
3*Sin[(e + f*x)/2]*Tan[e + f*x]^p + 6*Cos[(e + f*x)/2]^2*Sin[(e + f*x)/2]^2
*Tan[e + f*x]^p + 4*Cos[(e + f*x)/2]*Sin[(e + f*x)/2]^3*Tan[e + f*x]^p + Si
n[(e + f*x)/2]^4*Tan[e + f*x]^p))/(f*(1 + p)*(2 + p)*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2])^4*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Sec[e + f*x]^2*
Tan[(e + f*x)/2]*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x
)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2,
 p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*Appe
llF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(
e + f*x)/2])*Tan[e + f*x]^(-1 + p))/((1 + p)*(2 + p)) + (Sec[(e + f*x)/2]^2
*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3
+ p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 +
p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)
*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2] + 4*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(
```

```
e + f*x)/2]^2]*Tan[(e + f*x)/2])*Tan[e + f*x]^p)/((1 + p)*(2 + p)) + (2*p*(
Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + p)*Tan[(e + f*x)/2]*((2 + p)*AppellF
1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*
(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[
(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])*(-(Sec[(e + f*x)/2]^
2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])*Tan[e +
 f*x]^p)/((1 + p)*(2 + p)) + (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e
+ f*x)/2]*(2*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2 + 4*(1 + p)*Tan[(e + f*x)/2]*((-2*(1
 + p/2)*AppellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*Appell
F1[2 + p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2)) + (2 + p)*(-(((1 + p)*AppellF1
[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 + p)*AppellF1[1 +
(1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 4*(2 + p)*((-2*(1 + p)*Ap
pellF1[1 + (1 + p)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*
x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1
[1 + (1 + p)/2, 1 + p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)
/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - 4*(2 + p)*((-3*(1 +
p)*AppellF1[1 + (1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*App
ellF1[1 + (1 + p)/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)))*Tan[e + f*x]^p)/
((1 + p)*(2 + p))))
```

Maple [F] time = 1.421, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{2}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(a^{2}\cos\left(fx+e\right)^{2}-2\,a^{2}\sin\left(fx+e\right)-2\,a^{2}\right)\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(g*tan(f*x + e)
)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int\left(g\tan\left(e+fx\right)\right)^{p}\,dx+\int 2\left(g\tan\left(e+fx\right)\right)^{p}\sin\left(e+fx\right)dx+\int\left(g\tan\left(e+fx\right)\right)^{p}\sin^{2}\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)
```

```
[Out] a**2*(Integral((g*tan(e + f*x))**p, x) + Integral(2*(g*tan(e + f*x))**p*sin
(e + f*x), x) + Integral((g*tan(e + f*x))**p*sin(e + f*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{2}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

3.125 $\int (a + a\sin(e + fx))(g\tan(e + fx))^p\,dx$

Optimal. Leaf size=129

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{a\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2}\right.}{fg(p+2)}$$

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f
*x])^(1 + p))/(f*g*(1 + p)) + (a*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometri
c2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e
 + f*x])^(1 + p))/(f*g*(2 + p))
```

Rubi [A] time = 0.138781, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2710, 3476, 364, 2602, 2577}

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{a\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2}\right.}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f
*x])^(1 + p))/(f*g*(1 + p)) + (a*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometri
c2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e
 + f*x])^(1 + p))/(f*g*(2 + p))
```

Rule 2710

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx &= \int \left(a(g\tan(e+fx))^p + a\sin(e+fx)(g\tan(e+fx))^p\right)dx\\ &= a\int (g\tan(e+fx))^p\,dx + a\int \sin(e+fx)(g\tan(e+fx))^p\,dx\\ &= \frac{(ag)\,\text{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{\left(a\cos^{1+p}(e+fx)\sin^{-1-p}(e+fx\right.}{}\\ &= \frac{a\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{a\cos^2(e+fx)^{\frac{1+p}{2}}\,{}_2F}{}\end{aligned}$$

Mathematica [F] time = 2.02497, size = 0, normalized size = 0.

$$\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p, x]
```

Maple [F] time = 0.907, size = 0, normalized size = 0.

$$\int \left(a + a \sin \left(fx + e\right)\right) \left(g \tan \left(fx + e\right)\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx + e\right) + a\right) \left(g \tan \left(fx + e\right)\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a \sin \left(fx + e\right) + a\right) \left(g \tan \left(fx + e\right)\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\left(g\tan\left(e+fx\right)\right)^{p}\,dx+\int\left(g\tan\left(e+fx\right)\right)^{p}\sin\left(e+fx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))**p,x)
```

```
[Out] a*(Integral((g*tan(e + f*x))**p, x) + Integral((g*tan(e + f*x))**p*sin(e +
f*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

3.126 $\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx$

Optimal. Leaf size=108

$$\frac{(g\tan(e+fx))^{p+1}}{afg(p+1)}-\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+3}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{afg^2(p+2)}$$

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a*f*g*(1 + p)) - ((Cos[e + f*x]^2)^((3 + p)/2)*Hy
pergeometric2F1[(2 + p)/2, (3 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*
x]*(g*Tan[e + f*x])^(2 + p))/(a*f*g^2*(2 + p))
```

Rubi [A] time = 0.12612, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2706, 2607, 32, 2617}

$$\frac{(g\tan(e+fx))^{p+1}}{afg(p+1)}-\frac{\sec(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+3}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{afg^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]
```


```
[Out] (g*Tan[e + f*x])^(1 + p)/(a*f*g*(1 + p)) - ((Cos[e + f*x]^2)^((3 + p)/2)*Hy
pergeometric2F1[(2 + p)/2, (3 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e + f*
x]*(g*Tan[e + f*x])^(2 + p))/(a*f*g^2*(2 + p))
```


Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx = \frac{\int \sec^2(e+fx)(g\tan(e+fx))^p\,dx}{a} - \frac{\int \sec(e+fx)(g\tan(e+fx))^{1+p}\,dx}{ag}$$

$$= -\frac{\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{3+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec(e+fx)(g\tan(e+fx))^{2+p}}{afg^2(2+p)} + \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{afg(1+p)} - \frac{\cos^2(e+fx)^{\frac{3+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{3+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec(e+fx)(g\tan(e+}{afg^2(2+p)}$$

Mathematica [B] time = 4.03293, size = 232, normalized size = 2.15

$$\frac{2\tan\left(\frac{1}{2}(e+fx)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^2(g\tan(e+fx))^p\left(\left(p^2+5p+6\right)\,{}_2F_1\left(\frac{p+1}{2},p+2;\frac{p+3}{2};\tan^2\left(\frac{1}{2}(e+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2*Tan[(e + f*x)/2]*((6 + 5*p + p^2)*Hypergeometric2F1[(1 + p)/2, 2 + p, (
3 + p)/2, Tan[(e + f*x)/2]^2] - (1 + p)*Tan[(e + f*x)/2]*(2*(3 + p)*Hyperge
ometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan[(e + f*x)/2]^2] - (2 + p)*Hyper
geometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)
/2]))*(g*Tan[e + f*x])^p)/(f*(1 + p)*(2 + p)*(3 + p)*(a + a*Sin[e + f*x]))
```

Maple [F] time = 0.178, size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{a+a\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)
```

```
[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(g\tan\left(fx+e\right)\right)^{p}}{a\sin\left(fx+e\right)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(g\tan\left(e+fx\right)\right)^p}{\sin\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e)),x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^p}{a\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

3.127 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx$

Optimal. Leaf size=138

$$-\frac{2\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+5}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+5}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{a^2fg^2(p+2)}+\frac{2(g\tan(e+fx))^{p+3}}{a^2fg^3(p+3)}+\frac{(g\tan(e+f}{a^2fg(p+}$$

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a^2*f*g*(1 + p)) - (2*(Cos[e + f*x]^2)^((5 + p)/2
)*Hypergeometric2F1[(2 + p)/2, (5 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e
+ f*x]^3*(g*Tan[e + f*x])^(2 + p))/(a^2*f*g^2*(2 + p)) + (2*(g*Tan[e + f*x]
)^(3 + p))/(a^2*f*g^3*(3 + p))
```

Rubi [A] time = 0.274754, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2711, 2607, 14, 16, 2617, 32}

$$-\frac{2\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+5}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+5}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{a^2fg^2(p+2)}+\frac{2(g\tan(e+fx))^{p+3}}{a^2fg^3(p+3)}+\frac{(g\tan(e+f}{a^2fg(p+}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a^2*f*g*(1 + p)) - (2*(Cos[e + f*x]^2)^((5 + p)/2
)*Hypergeometric2F1[(2 + p)/2, (5 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e
+ f*x]^3*(g*Tan[e + f*x])^(2 + p))/(a^2*f*g^2*(2 + p)) + (2*(g*Tan[e + f*x]
)^(3 + p))/(a^2*f*g^3*(3 + p))
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \left(a^2\sec^4(e+fx)(g\tan(e+fx))^p - 2a^2\sec^3(e+fx)\tan(e+fx)(g\tan(e+fx))^p + a^2\sec\right.}{a^4}$$

$$= \frac{\int \sec^4(e+fx)(g\tan(e+fx))^p\,dx}{a^2} + \frac{\int \sec^2(e+fx)\tan^2(e+fx)(g\tan(e+fx))^p\,dx}{a^2} - \frac{2\int}{}$$

$$= \frac{\mathrm{Subst}\left(\int (gx)^p\left(1+x^2\right)dx, x, \tan(e+fx)\right)}{a^2f} + \frac{\int \sec^2(e+fx)(g\tan(e+fx))^{2+p}\,dx}{a^2g^2} - \frac{2\int \mathrm{s}}{}$$

$$= -\frac{2\cos^2(e+fx)^{\frac{5+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{5+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^3(e+fx)(g\tan(e+fx))^{2+p}}{a^2fg^2(2+p)} + \frac{\mathrm{Sub}}{}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{a^2fg(1+p)} - \frac{2\cos^2(e+fx)^{\frac{5+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{5+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^3(e+fx)(g\tan}{a^2fg^2(2+p)}$$

Mathematica [B] time = 14.1776, size = 667, normalized size = 4.83

$$2^{p+1}\tan\left(\frac{1}{2}(e+fx)\right)\left(1-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)^p\left(-\frac{\tan\left(\frac{1}{2}(e+fx)\right)}{\tan^2\left(\frac{1}{2}(e+fx)\right)-1}\right)^p\tan^{-p}(e+fx)\left(\sin\left(\frac{1}{2}(e+fx)\right)+\cos\left(\frac{1}{2}(e+fx)\right)\right)^4(g\,\mathrm{ta}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (2^(1 + p)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Tan[(e + f*x)/2]*(1 - Ta
n[(e + f*x)/2]^2)^p*(-(Tan[(e + f*x)/2]/(-1 + Tan[(e + f*x)/2]^2)))^p*(Hype
rgeometric2F1[(1 + p)/2, 2 + p, (3 + p)/2, Tan[(e + f*x)/2]^2]/(1 + p) - (2
*Hypergeometric2F1[(1 + p)/2, 3 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/(1 + p
) + (2*Hypergeometric2F1[(1 + p)/2, 4 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/
(1 + p) - (2*Hypergeometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan[(e + f*x)/2
]^2]*Tan[(e + f*x)/2])/(2 + p) + (6*Hypergeometric2F1[(2 + p)/2, 3 + p, (4
+ p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) - (8*Hypergeometric2F
1[(2 + p)/2, 4 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p
) + (Hypergeometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan
[(e + f*x)/2]^2)/(3 + p) - (6*Hypergeometric2F1[(3 + p)/2, 3 + p, (5 + p)/2
, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (12*Hypergeometric2F1[(
3 + p)/2, 4 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p)
 + (2*Hypergeometric2F1[3 + p, (4 + p)/2, (6 + p)/2, Tan[(e + f*x)/2]^2]*Ta
n[(e + f*x)/2]^3)/(4 + p) - (8*Hypergeometric2F1[(4 + p)/2, 4 + p, (6 + p)/
2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) + (2*Hypergeometric2F1[4
 + p, (5 + p)/2, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p)
)*(g*Tan[e + f*x])^p)/(f*(a + a*Sin[e + f*x])^2*Tan[e + f*x]^p)
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^p}{\left(a+a\sin\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)
```

```
[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g \tan\left(fx+e\right)\right)^p}{\left(a \sin\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(g \tan\left(fx+e\right)\right)^p}{a^2 \cos\left(fx+e\right)^2 - 2\,a^2 \sin\left(fx+e\right) - 2\,a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(-(g*tan(f*x + e))^p/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a
^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\left(g \tan\left(e+fx\right)\right)^p}{\sin^2\left(e+fx\right)+2 \sin\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(a\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)
```

3.128 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx$

Optimal. Leaf size=248

$$-\frac{3\sec^5(e+fx)\cos^2(e+fx)^{\frac{p+7}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+7}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{a^3fg^2(p+2)}-\frac{\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+7}{2}}(g\tan}{}$$

```
[Out] (g*Tan[e + f*x])^(1 + p)/(a^3*f*g*(1 + p)) - (3*(Cos[e + f*x]^2)^((7 + p)/2
)*Hypergeometric2F1[(2 + p)/2, (7 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e
+ f*x]^5*(g*Tan[e + f*x])^(2 + p))/(a^3*f*g^2*(2 + p)) + (5*(g*Tan[e + f*x]
)^(3 + p))/(a^3*f*g^3*(3 + p)) - ((Cos[e + f*x]^2)^((7 + p)/2)*Hypergeometr
ic2F1[(4 + p)/2, (7 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(g*Ta
n[e + f*x])^(4 + p))/(a^3*f*g^4*(4 + p)) + (4*(g*Tan[e + f*x])^(5 + p))/(a^
3*f*g^5*(5 + p))
```

Rubi [A] time = 0.421813, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2711, 2607, 270, 16, 2617, 14}

$$-\frac{3\sec^5(e+fx)\cos^2(e+fx)^{\frac{p+7}{2}}(g\tan(e+fx))^{p+2}\,{}_2F_1\left(\frac{p+2}{2},\frac{p+7}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{a^3fg^2(p+2)}-\frac{\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+7}{2}}(g\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^3,x]
```


```
[Out] (g*Tan[e + f*x])^(1 + p)/(a^3*f*g*(1 + p)) - (3*(Cos[e + f*x]^2)^((7 + p)/2
)*Hypergeometric2F1[(2 + p)/2, (7 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e
+ f*x]^5*(g*Tan[e + f*x])^(2 + p))/(a^3*f*g^2*(2 + p)) + (5*(g*Tan[e + f*x]
)^(3 + p))/(a^3*f*g^3*(3 + p)) - ((Cos[e + f*x]^2)^((7 + p)/2)*Hypergeometr
ic2F1[(4 + p)/2, (7 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sec[e + f*x]^3*(g*Ta
n[e + f*x])^(4 + p))/(a^3*f*g^4*(4 + p)) + (4*(g*Tan[e + f*x])^(5 + p))/(a^
3*f*g^5*(5 + p))
```


Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
```

```
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2617

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[((a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*(Cos[e
+ f*x]^2)^((m + n + 1)/2)*Hypergeometric2F1[(n + 1)/2, (m + n + 1)/2, (n +
3)/2, Sin[e + f*x]^2])/(b*f*(n + 1)), x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \left(a^3\sec^6(e+fx)(g\tan(e+fx))^p - 3a^3\sec^5(e+fx)\tan(e+fx)(g\tan(e+fx))^p + 3a^3 \mathrm{se}\right.}{}$$

$$= \frac{\int \sec^6(e+fx)(g\tan(e+fx))^p\,dx}{a^3} - \frac{\int \sec^3(e+fx)\tan^3(e+fx)(g\tan(e+fx))^p\,dx}{a^3} - \frac{3\int}{}$$

$$= \frac{\mathrm{Subst}\left(\int (gx)^p\left(1+x^2\right)^2\,dx, x, \tan(e+fx)\right)}{a^3 f} - \frac{\int \sec^3(e+fx)(g\tan(e+fx))^{3+p}\,dx}{a^3 g^3} + \frac{3\int}{}$$

$$= -\frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e+fx)(g\tan(e+fx))^{2+p}}{a^3 f g^2(2+p)} - \frac{\cos^2}{}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{a^3 f g(1+p)} - \frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e+fx)(g\tan}{a^3 f g^2(2+p)}$$

$$= \frac{(g\tan(e+fx))^{1+p}}{a^3 f g(1+p)} - \frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,{}_2F_1\left(\frac{2+p}{2},\frac{7+p}{2};\frac{4+p}{2};\sin^2(e+fx)\right)\sec^5(e+fx)(g\tan}{a^3 f g^2(2+p)}$$

Mathematica [B] time = 31.2325, size = 1276, normalized size = 5.15

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] (2^(1 + p)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Tan[(e + f*x)/2]*(1 - Ta
n[(e + f*x)/2]^2)^p*(-(Tan[(e + f*x)/2]/(-1 + Tan[(e + f*x)/2]^2)))^p*(Hype
rgeometric2F1[(1 + p)/2, 2 + p, (3 + p)/2, Tan[(e + f*x)/2]^2]/(1 + p) - (4
*Hypergeometric2F1[(1 + p)/2, 3 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/(1 + p
) + (8*Hypergeometric2F1[(1 + p)/2, 4 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/
(1 + p) - (8*Hypergeometric2F1[(1 + p)/2, 5 + p, (3 + p)/2, Tan[(e + f*x)/2
]^2])/(1 + p) + (4*Hypergeometric2F1[(1 + p)/2, 6 + p, (3 + p)/2, Tan[(e +
f*x)/2]^2])/(1 + p) - (2*Hypergeometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan
[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) + (12*Hypergeometric2F1[(2 + p)/
2, 3 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) - (32*Hy
pergeometric2F1[(2 + p)/2, 4 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f
*x)/2])/(2 + p) + (40*Hypergeometric2F1[(2 + p)/2, 5 + p, (4 + p)/2, Tan[(e
 + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) - (24*Hypergeometric2F1[(2 + p)/2,
6 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) + (Hypergeo
metric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]
^2)/(3 + p) - (12*Hypergeometric2F1[(3 + p)/2, 3 + p, (5 + p)/2, Tan[(e + f
```

```
*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (48*Hypergeometric2F1[(3 + p)/2, 4
+ p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) - (80*Hyper
geometric2F1[(3 + p)/2, 5 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)
/2]^2)/(3 + p) + (60*Hypergeometric2F1[(3 + p)/2, 6 + p, (5 + p)/2, Tan[(e
+ f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (4*Hypergeometric2F1[3 + p, (4 +
 p)/2, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) - (32*Hyp
ergeometric2F1[(4 + p)/2, 4 + p, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*
x)/2]^3)/(4 + p) + (80*Hypergeometric2F1[(4 + p)/2, 5 + p, (6 + p)/2, Tan[(
e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) - (80*Hypergeometric2F1[(4 + p)/
2, 6 + p, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) + (8*H
ypergeometric2F1[4 + p, (5 + p)/2, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e +
f*x)/2]^4)/(5 + p) - (40*Hypergeometric2F1[(5 + p)/2, 5 + p, (7 + p)/2, Tan
[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p) + (60*Hypergeometric2F1[(5 + p
)/2, 6 + p, (7 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p) + (8
*Hypergeometric2F1[3 + p/2, 5 + p, 4 + p/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*
x)/2]^5)/(6 + p) - (24*Hypergeometric2F1[(6 + p)/2, 6 + p, (8 + p)/2, Tan[(
e + f*x)/2]^2]*Tan[(e + f*x)/2]^5)/(6 + p) + (4*Hypergeometric2F1[6 + p, (7
 + p)/2, (9 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^6)/(7 + p))*(g*Tan
[e + f*x])^p)/(f*(a + a*Sin[e + f*x])^3*Tan[e + f*x]^p)
```

Maple [F] time = 0.451, size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(a+a\sin\left(fx+e\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)

[Out] int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(g\tan\left(fx+e\right)\right)^{p}}{3\,a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}+\left(a^{3}\cos\left(fx+e\right)^{2}-4\,a^{3}\right)\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(g*tan(f*x + e))^p/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x +
 e)^2 - 4*a^3)*sin(f*x + e)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(a\sin\left(fx+e\right)+a\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)
```

3.129 $\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=111

$$\frac{(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(g\tan(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F_1\left(p+1;\frac{p+1}{2},\frac{1}{2}(-2m+p+1);p+\right.}{fg(p+1)}$$

```
[Out] (AppellF1[1 + p, (1 + p)/2, (1 - 2*m + p)/2, 2 + p, Sin[e + f*x], -Sin[e +
f*x]]*(1 - Sin[e + f*x])^((1 + p)/2)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(
a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p))
```

Rubi [A] time = 0.12401, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2720, 135, 133}

$$\frac{(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^m(g\tan(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}F_1\left(p+1;\frac{p+1}{2},\frac{1}{2}(-2m+p+1);p+\right.}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] (AppellF1[1 + p, (1 + p)/2, (1 - 2*m + p)/2, 2 + p, Sin[e + f*x], -Sin[e +
f*x]]*(1 - Sin[e + f*x])^((1 + p)/2)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)*(
a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p))
```

Rule 2720

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_), x_Symbol] :> Dist[((g*Tan[e + f*x])^(p + 1)*(a - b*Sin[e + f*x])
^((p + 1)/2)*(a + b*Sin[e + f*x])^((p + 1)/2))/(f*g*(b*Sin[e + f*x])^(p + 1
)), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&
  !IntegerQ[m] &&  !IntegerQ[p]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
```

```
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (a+a\sin(e+fx))^m (g\tan(e+fx))^p\,dx = \frac{\left((a\sin(e+fx))^{-1-p}(a-a\sin(e+fx))^{\frac{1+p}{2}}(a+a\sin(e+fx))^{\frac{1+p}{2}}(g\tan(e+ \dots\right.}{fg \dots}$$

$$= \frac{\left((1-\sin(e+fx))^{\frac{1}{2}+\frac{p}{2}}(a\sin(e+fx))^{-1-p}(a-a\sin(e+fx))^{-\frac{1}{2}-\frac{p}{2}+\frac{1+p}{2}}(a+a \dots\right.}{\dots}$$

$$= \frac{\left((1-\sin(e+fx))^{\frac{1}{2}+\frac{p}{2}}(a\sin(e+fx))^{-1-p}(1+\sin(e+fx))^{\frac{1}{2}-m+\frac{p}{2}}(a-a\sin(\dots\right.}{\dots}$$

$$= \frac{F_1\left(1+p;\frac{1+p}{2},\frac{1}{2}(1-2m+p);2+p;\sin(e+fx),-\sin(e+fx)\right)(1-\sin(e+ \dots}{fg(1 \dots}$$

Mathematica [B] time = 2.19787, size = 367, normalized size = 3.31

$$-\frac{2(p-3)\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^3\left(\frac{1}{4}(2e+2 \dots\right.}{f(p-1)\left((p-3)\cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)F_1\left(\frac{1-p}{2};-p,m+1;\frac{3-p}{2};\cot^2\left(\frac{1}{4}(2e+2fx+\pi)\right),-\tan^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)+ \dots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] (-2*(-3 + p)*AppellF1[(1 - p)/2, -p, 1 + m, (3 - p)/2, Cot[(2*e + Pi + 2*f*
x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[(2*e - Pi + 2*f*x)/4]^3*(a*(1 +
Sin[e + f*x]))^m*Sin[(2*e - Pi + 2*f*x)/4]*(g*Tan[e + f*x])^p)/(f*(-1 + p)*
((-3 + p)*AppellF1[(1 - p)/2, -p, 1 + m, (3 - p)/2, Cot[(2*e + Pi + 2*f*x)/
4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Cos[(2*e - Pi + 2*f*x)/4]^2 + 2*(p*Appe
llF1[(3 - p)/2, 1 - p, 1 + m, (5 - p)/2, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[
```

```
(2*e - Pi + 2*f*x)/4]^2] + (1 + m)*AppellF1[(3 - p)/2, -p, 2 + m, (5 - p)/2
, Cot[(2*e + Pi + 2*f*x)/4]^2, -Tan[(2*e - Pi + 2*f*x)/4]^2])*Sin[(2*e - Pi
 + 2*f*x)/4]^2))
```

Maple [F] time = 0.847, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(g*tan(f*x+e))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

3.130 $\int (a+a\sin(e+fx))^m \tan^3(e+fx)\,dx$

Optimal. Leaf size=163

$$-\frac{a^2\sin^2(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a(m+4)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}+\frac{(2am\sin\ldots}{}$$

```
[Out] (a*(4 + m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(a + a*Sin
[e + f*x])^(-1 + m))/(4*f*(1 - m)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x
])^(-1 + m))/(f*m*(a - a*Sin[e + f*x])) + ((a + a*Sin[e + f*x])^(-1 + m)*(a
*(2 - 3*m - m^2) + 2*a*m*Sin[e + f*x]))/(2*f*(1 - m)*m*(1 - Sin[e + f*x]))
```

Rubi [A] time = 0.148753, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2707, 100, 146, 68}

$$-\frac{a^2\sin^2(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a(m+4)(a\sin(e+fx)+a)^{m-1}\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)}{4f(1-m)}+\frac{(2am\sin\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```


```
[Out] (a*(4 + m)*Hypergeometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(a + a*Sin
[e + f*x])^(-1 + m))/(4*f*(1 - m)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x
])^(-1 + m))/(f*m*(a - a*Sin[e + f*x])) + ((a + a*Sin[e + f*x])^(-1 + m)*(a
*(2 - 3*m - m^2) + 2*a*m*Sin[e + f*x]))/(2*f*(1 - m)*m*(1 - Sin[e + f*x]))
```


Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^{-2+m}}{(a-x)^2}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= -\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}}{fm(a-a\sin(e+fx))} - \frac{\text{Subst}\left(\int \frac{x(a+x)^{-2+m}\left(-2a^2-amx\right)}{(a-x)^2}\,dx, x \ldots\right.}{fm}$$

$$= -\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}}{fm(a-a\sin(e+fx))} + \frac{(a+a\sin(e+fx))^{-1+m}\left(a\left(2-3m \ldots\right.\right.}{2f(1-m)m(1-\sin \ldots}$$

$$= \frac{a(4+m)\,{}_2F_1\left(1,-1+m;m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{-1+m}}{4f(1-m)} - \frac{a^2 \ldots}{\ldots}$$

Mathematica [A] time = 0.267321, size = 105, normalized size = 0.64

$$\frac{a(a(\sin(e+fx)+1))^{m-1}\left(-m(m+4)(\sin(e+fx)-1)\,{}_2F_1\left(1,m-1;m;\frac{1}{2}(\sin(e+fx)+1)\right)+4(m-1)\sin^2(e+fx)+4m\right.}{4f(m-1)m(\sin(e+fx)-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

```
[Out] (a*(a*(1 + Sin[e + f*x]))^(-1 + m)*(-2*(-2 + 3*m + m^2) - m*(4 + m)*Hyperge
ometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]) + 4*m*Si
n[e + f*x] + 4*(-1 + m)*Sin[e + f*x]^2))/(4*f*(-1 + m)*m*(-1 + Sin[e + f*x]
))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(\tan\left(fx+e\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

3.131 $\int (a+a\sin(e+fx))^m \tan(e+fx)\,dx$

Optimal. Leaf size=72

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}-\frac{(a\sin(e+fx)+a)^m}{2fm}$$

```
[Out] -(a + a*Sin[e + f*x])^m/(2*f*m) + (Hypergeometric2F1[1, 1 + m, 2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(1 + m))/(4*a*f*(1 + m))
```

Rubi [A] time = 0.0504618, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2707, 79, 68}

$$\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)}{4af(m+1)}-\frac{(a\sin(e+fx)+a)^m}{2fm}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] -(a + a*Sin[e + f*x])^m/(2*f*m) + (Hypergeometric2F1[1, 1 + m, 2 + m, (1 +
Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^(1 + m))/(4*a*f*(1 + m))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^{-1+m}}{a-x}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= -\frac{(a+a\sin(e+fx))^m}{2fm} + \frac{\text{Subst}\left(\int \frac{(a+x)^m}{a-x}\,dx, x, a\sin(e+fx)\right)}{2f}$$

$$= -\frac{(a+a\sin(e+fx))^m}{2fm} + \frac{{}_2F_1\left(1,1+m;2+m;\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+}{4af(1+m)}$$

Mathematica [A] time = 0.0706524, size = 63, normalized size = 0.88

$$\frac{(a(\sin(e+fx)+1))^m\left(m(\sin(e+fx)+1)\,{}_2F_1\left(1,m+1;m+2;\frac{1}{2}(\sin(e+fx)+1)\right)-2(m+1)\right)}{4fm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^m*(-2*(1 + m) + m*Hypergeometric2F1[1, 1 + m, 2 + m
, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))/(4*f*m*(1 + m))
```

Maple [F] time = 0.76, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m \tan(fx+e)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

3.132 $\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=43

$$-\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(e+fx)+1)}{af(m+1)}$$

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x]
)^(1 + m))/(a*f*(1 + m)))
```

Rubi [A] time = 0.0439523, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2707, 65}

$$-\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(e+fx)+1)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x]
)^(1 + m))/(a*f*(1 + m)))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^m}{x}\,dx, x, a\sin(e+fx)\right)}{f}$$
$$= -\frac{{}_2F_1(1,1+m;2+m;1+\sin(e+fx))(a+a\sin(e+fx))^{1+m}}{af(1+m)}$$

Mathematica [A] time = 0.0589381, size = 43, normalized size = 1.

$$-\frac{(a\sin(e+fx)+a)^{m+1}\,{}_2F_1(1,m+1;m+2;\sin(e+fx)+1)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x]
)^(1 + m))/(a*f*(1 + m)))
```

Maple [F] time = 0.821, size = 0, normalized size = 0.

$$\int \cot\left(fx+e\right)\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

3.133 $\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=83

$$-\frac{(2-m)(a\sin(e+fx)+a)^{m+2}\,{}_2F_1(2,m+2;m+3;\sin(e+fx)+1)}{2a^2f(m+2)}-\frac{\csc^2(e+fx)(a\sin(e+fx)+a)^{m+2}}{2a^2f}$$

```
[Out] -(Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(2 + m))/(2*a^2*f) - ((2 - m)*Hyperge
ometric2F1[2, 2 + m, 3 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m))
/(2*a^2*f*(2 + m))
```

Rubi [A] time = 0.0685841, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2707, 78, 65}

$$-\frac{(2-m)(a\sin(e+fx)+a)^{m+2}\,{}_2F_1(2,m+2;m+3;\sin(e+fx)+1)}{2a^2f(m+2)}-\frac{\csc^2(e+fx)(a\sin(e+fx)+a)^{m+2}}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -(Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(2 + m))/(2*a^2*f) - ((2 - m)*Hyperge
ometric2F1[2, 2 + m, 3 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m))
/(2*a^2*f*(2 + m))
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)(a+x)^{1+m}}{x^3}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= -\frac{\csc^2(e+fx)(a+a\sin(e+fx))^{2+m}}{2a^2f} - \frac{(2-m)\,\text{Subst}\left(\int \frac{(a+x)^{1+m}}{x^2}\,dx, x, a\sin(e-\right.}{2f}$$

$$= -\frac{\csc^2(e+fx)(a+a\sin(e+fx))^{2+m}}{2a^2f} - \frac{(2-m)\,{}_2F_1(2,2+m;3+m;1+\sin(e+}{2a^2f(2+m)}$$

Mathematica [A] time = 0.188158, size = 68, normalized size = 0.82

$$-\frac{(\sin(e+fx)+1)^2(a(\sin(e+fx)+1))^m\left((m+2)\csc^2(e+fx)-(m-2)\,{}_2F_1(2,m+2;m+3;\sin(e+fx)+1)\right)}{2f(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((((2 + m)*Csc[e + f*x]^2 - (-2 + m)*Hypergeometric2F1[2, 2 + m, 3 + m, 1 +
 Sin[e + f*x]])*(1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^m)/(2*f*(2 + m)
)
```

Maple [F] time = 0.268, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^3\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m \cot(fx+e)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sin(fx+e)+a)^m \cot(fx+e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m \cot(fx+e)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

3.134 $\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=123

$$-\frac{(m^2-9m+12)\,(a\sin(e+fx)+a)^{m+3}\,{}_2F_1(3,m+3;m+4;\sin(e+fx)+1)}{12a^3f(m+3)}-\frac{\csc^4(e+fx)(a\sin(e+fx)+a)^{m+3}}{4a^3f}+$$

```
[Out] ((9 - m)*Csc[e + f*x]^3*(a + a*Sin[e + f*x])^(3 + m))/(12*a^3*f) - (Csc[e +
 f*x]^4*(a + a*Sin[e + f*x])^(3 + m))/(4*a^3*f) - ((12 - 9*m + m^2)*Hyperge
ometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m))
/(12*a^3*f*(3 + m))
```

Rubi [A] time = 0.0982222, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2707, 89, 78, 65}

$$-\frac{(m^2-9m+12)\,(a\sin(e+fx)+a)^{m+3}\,{}_2F_1(3,m+3;m+4;\sin(e+fx)+1)}{12a^3f(m+3)}-\frac{\csc^4(e+fx)(a\sin(e+fx)+a)^{m+3}}{4a^3f}+$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]
```


```
[Out] ((9 - m)*Csc[e + f*x]^3*(a + a*Sin[e + f*x])^(3 + m))/(12*a^3*f) - (Csc[e +
 f*x]^4*(a + a*Sin[e + f*x])^(3 + m))/(4*a^3*f) - ((12 - 9*m + m^2)*Hyperge
ometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m))
/(12*a^3*f*(3 + m))
```


Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
```

```
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\text{Subst}\left(\int \frac{(a-x)^2(a+x)^{2+m}}{x^5}\,dx, x, a\sin(e+fx)\right)}{f}$$

$$= -\frac{\csc^4(e+fx)(a+a\sin(e+fx))^{3+m}}{4a^3f} + \frac{\text{Subst}\left(\int \frac{(a+x)^{2+m}\left(-a^2(9-m)+4ax\right)}{x^4}\,dx, x, a\,\text{si}\right.}{4af}$$

$$= \frac{(9-m)\csc^3(e+fx)(a+a\sin(e+fx))^{3+m}}{12a^3f} - \frac{\csc^4(e+fx)(a+a\sin(e+fx))^{3+}}{4a^3f}$$

$$= \frac{(9-m)\csc^3(e+fx)(a+a\sin(e+fx))^{3+m}}{12a^3f} - \frac{\csc^4(e+fx)(a+a\sin(e+fx))^{3+}}{4a^3f}$$

Mathematica [A] time = 0.274734, size = 83, normalized size = 0.67

$$-\frac{(\sin(e+fx)+1)^3(a(\sin(e+fx)+1))^m\left(\left(m^2-9m+12\right)\,{}_2F_1(3,m+3;m+4;\sin(e+fx)+1)+(m+3)\csc^3(e+fx)(3\right.}{12f(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((((3 + m)*Csc[e + f*x]^3*(-9 + m + 3*Csc[e + f*x]) + (12 - 9*m + m^2)*Hype
rgeometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]])*(1 + Sin[e + f*x])^3*(a*(
1 + Sin[e + f*x]))^m)/(12*f*(3 + m))
```

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int \left(\cot \left(fx+e\right)\right)^{5} \left(a+a\sin \left(fx+e\right)\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin \left(fx+e\right)+a\right)^{m} \cot \left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

3.135 $\int (a+a\sin(e+fx))^m\tan^4(e+fx)\,dx$

Optimal. Leaf size=311

$$-\frac{a^2\sin^2(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a^2\sin(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(1-m)(a-a\sin(e+fx))}+\frac{2^{m-\frac{3}{2}}\left(m^4+6\right.}{}$$

```
[Out] (2^(-3/2 + m)*(9 - 12*m - 7*m^2 + 6*m^3 + m^4)*Hypergeometric2F1[1/2, 5/2 -
 m, 3/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 - Sin[e + f*x])*(1 + Sin[e +
 f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(3*f*(1 - m)*m) - (Sec[e + f*x]*(a
 + a*Sin[e + f*x])^(-1 + m)*(a*(6 - m - 7*m^2 - m^3) - a*(9 - 6*m - 8*m^2 -
 m^3)*Sin[e + f*x]))/(3*f*(1 - m)*m*(1 - Sin[e + f*x])) + (a^2*Sin[e + f*x]
*(a + a*Sin[e + f*x])^(-1 + m)*Tan[e + f*x])/(f*(1 - m)*(a - a*Sin[e + f*x]
)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x])^(-1 + m)*Tan[e + f*x])/(f*m*(
a - a*Sin[e + f*x]))
```

Rubi [A] time = 0.355058, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2719, 100, 153, 145, 70, 69}

$$-\frac{a^2\sin^2(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{fm(a-a\sin(e+fx))}+\frac{a^2\sin(e+fx)\tan(e+fx)(a\sin(e+fx)+a)^{m-1}}{f(1-m)(a-a\sin(e+fx))}+\frac{2^{m-\frac{3}{2}}\left(m^4+6\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```


```
[Out] (2^(-3/2 + m)*(9 - 12*m - 7*m^2 + 6*m^3 + m^4)*Hypergeometric2F1[1/2, 5/2 -
 m, 3/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 - Sin[e + f*x])*(1 + Sin[e +
 f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(3*f*(1 - m)*m) - (Sec[e + f*x]*(a
 + a*Sin[e + f*x])^(-1 + m)*(a*(6 - m - 7*m^2 - m^3) - a*(9 - 6*m - 8*m^2 -
 m^3)*Sin[e + f*x]))/(3*f*(1 - m)*m*(1 - Sin[e + f*x])) + (a^2*Sin[e + f*x]
*(a + a*Sin[e + f*x])^(-1 + m)*Tan[e + f*x])/(f*(1 - m)*(a - a*Sin[e + f*x]
)) - (a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x])^(-1 + m)*Tan[e + f*x])/(f*m*(
a - a*Sin[e + f*x]))
```


Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
```

```
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
```

```
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{x^4(a+x)^{-\frac{5}{2}+m}}{(a-x)^{5/2}}\,dx, x\right.}{af}$$

$$= -\frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}\tan(e+fx)}{fm(a-a\sin(e+fx))} - \frac{\left(\sec(e+fx)\sqrt{a-a\sin\right.}{}$$

$$= \frac{a^2\sin(e+fx)(a+a\sin(e+fx))^{-1+m}\tan(e+fx)}{f(1-m)(a-a\sin(e+fx))} - \frac{a^2\sin^2(e+fx)(a+a\sin}{fm(a-a\,\mathrm{s}}$$

$$= -\frac{\sec(e+fx)(a+a\sin(e+fx))^{-1+m}\left(a\left(6-m-7m^2-m^3\right)-a\left(9-6m-8m\right.\right.}{3f(1-m)m(1-\sin(e+fx))}$$

$$= -\frac{\sec(e+fx)(a+a\sin(e+fx))^{-1+m}\left(a\left(6-m-7m^2-m^3\right)-a\left(9-6m-8m\right.\right.}{3f(1-m)m(1-\sin(e+fx))}$$
$$= \frac{2^{-\frac{3}{2}+m}\left(9-12m-7m^2+6m^3+m^4\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)\sec}{3f(1-m)m}$$

Mathematica [F] time = 1.06385, size = 0, normalized size = 0.

$$\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

```
[Out] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4, x]
```

Maple [F] time = 0.152, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^{m}\left(\tan\left(fx+e\right)\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\tan\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(fx+e)+a)^m \tan(fx+e)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

3.136 $\int (a + a\sin(e + fx))^m \tan^2(e + fx)\, dx$

Optimal. Leaf size=157

$$\frac{2^{m-\frac{1}{2}}\left(-m^2-m+1\right)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)m}+\frac{\sec(e\cdots}{}$$

```
[Out] (Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) + (2^(-1/2 + m)*(1 - m
- m^2)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[e +
f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) - (
Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*m)
```

Rubi [A] time = 0.248016, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2713, 2860, 2689, 70, 69}

$$\frac{2^{m-\frac{1}{2}}\left(-m^2-m+1\right)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)m}+\frac{\sec(e\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```


```
[Out] (Sec[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) + (2^(-1/2 + m)*(1 - m
- m^2)*Hypergeometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[e +
f*x]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m)*m) - (
Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*m)
```


Rule 2713

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> -Simp[(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Dist[1/(b*m), Int[((a + b*Sin[e + f*x])^m*(b*(m + 1) + a*Sin[e + f*x]))/Cos
[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !
IntegerQ[m] &&  !LtQ[m, 0]
```

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
```

```
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*
Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2
 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2689

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Si
n[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b
*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx = -\frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm} + \frac{\int \sec^2(e+fx)(a+a\sin(e+fx))^m(a(1+ \ldots}{am}$$

$$= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m} - \frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm} + \frac{\left(1-m \ldots\right.}{\ldots}$$

$$= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m} - \frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm} + \frac{\left(a^2\left(1- \ldots\right.\right.}{\ldots}$$

$$= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m} - \frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm} + \frac{\left(2^{-\frac{3}{2}+m} \ldots\right.}{\ldots}$$

$$= \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m} + \frac{2^{-\frac{1}{2}+m}\left(1-m-m^2\right)\,{}_2F_1\left(-\frac{1}{2},\frac{3}{2}-m;\frac{1}{2};\frac{1}{2}(1- \ldots\right.}{\ldots}$$

Mathematica [C] time = 34.458, size = 11184, normalized size = 71.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.12, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m \left(\tan\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx + e\right) + a\right)^{m} \tan \left(fx + e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a \sin \left(fx + e\right) + a\right)^{m} \tan \left(fx + e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx + e\right) + a\right)^{m} \tan \left(fx + e\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

3.137 $\int (a + a\sin(e + fx))^m \, dx$

Optimal. Leaf size=74

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

```
[Out] -((2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rubi [A] time = 0.0348217, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2652, 2651}

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] -((2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e
 + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/f)
```

Rule 2652

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n +
 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a+a\sin(e+fx))^m\,dx = \left((1+\sin(e+fx))^{-m}(a+a\sin(e+fx))^m\right)\int (1+\sin(e+fx))^m\,dx$$
$$= -\frac{2^{\frac{1}{2}+m}\cos(e+fx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx))\right)(1+\sin(e+fx))^{-\frac{1}{2}-m}(a+a\sin(e+}{f}$$

Mathematica [A] time = 0.181155, size = 90, normalized size = 1.22

$$\frac{\sqrt{2}\cos(e+fx)(a(\sin(e+fx)+1))^m\,{}_2F_1\left(\frac{1}{2},m+\frac{1}{2};m+\frac{3}{2};\frac{1}{4}\cos^2(e+fx)\csc^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2fm+f)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (Cos[e + f*x
]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4]*(a*(1 + Sin[e + f*x]))^m)/((f + 2*f*m)*
Sqrt[1 - Sin[e + f*x]])
```

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int \left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m,x)
```

```
[Out] int((a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(e+fx\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*sin(e + f*x) + a)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m, x)
```

3.138 $\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+2}F_1\left(m+\frac{3}{2};-\frac{1}{2},2;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{a^2f(2m+3)}$$

```
[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, 2, 5/2 + m, (1 + Sin[e + f*x])/2, 1 + Si
n[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m
))/(a^2*f*(3 + 2*m))
```

Rubi [A] time = 0.10163, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2719, 137, 136}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+2}F_1\left(m+\frac{3}{2};-\frac{1}{2},2;m+\frac{5}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{a^2f(2m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (2*Sqrt[2]*AppellF1[3/2 + m, -1/2, 2, 5/2 + m, (1 + Sin[e + f*x])/2, 1 + Si
n[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m
))/(a^2*f*(3 + 2*m))
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{a-x}(a+x)^{\frac{1}{2}+m}}{x^2}\,dx,\right.}{af}$$

$$= \frac{\left(\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)^{\frac{1}{2}+m}\sqrt{\frac{1}{2}-}}{x^2}\right.}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{2\sqrt{2}F_1\left(\frac{3}{2}+m;-\frac{1}{2},2;\frac{5}{2}+m;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)}{a^2f(3+2m)}$$

Mathematica [C] time = 99.9376, size = 47487, normalized size = 533.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^2\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\left(\sin\left(e+fx\right)+1\right)\right)^{m}\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sin \left(fx+e\right)+a\right)^{m} \cot \left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

3.139 $\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx$

Optimal. Leaf size=89

$$\frac{4\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+3}F_1\left(m+\frac{5}{2};-\frac{3}{2},4;m+\frac{7}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{a^3f(2m+5)}$$

```
[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, 4, 7/2 + m, (1 + Sin[e + f*x])/2, 1 + Si
n[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m
))/(a^3*f*(5 + 2*m))
```

Rubi [A] time = 0.0993711, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2719, 137, 136}

$$\frac{4\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+3}F_1\left(m+\frac{5}{2};-\frac{3}{2},4;m+\frac{7}{2};\frac{1}{2}(\sin(e+fx)+1),\sin(e+fx)+1\right)}{a^3f(2m+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]
```

```
[Out] (4*Sqrt[2]*AppellF1[5/2 + m, -3/2, 4, 7/2 + m, (1 + Sin[e + f*x])/2, 1 + Si
n[e + f*x]]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m
))/(a^3*f*(5 + 2*m))
```

Rule 2719

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]])/(b
*f*Cos[e + f*x]), Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^
2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
```

```
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{\left(\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a-x)^{3/2}(a+x)^{\frac{3}{2}+m}}{x^4}\,d\right.}{af}$$

$$= \frac{\left(2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a+a\sin(e+fx)}\right)\text{Subst}\left(\int \frac{(a+x)^{\frac{3}{2}+m}\left(\frac{1}{2}-\right.}{x^4}\right.}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$= \frac{4\sqrt{2}F_1\left(\frac{5}{2}+m;-\frac{3}{2},4;\frac{7}{2}+m;\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{}}{a^3f(5+2m)}$$

Mathematica [F] time = 0.740835, size = 0, normalized size = 0.

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]

[Out] Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m, x]
```

Maple [F] time = 0.294, size = 0, normalized size = 0.

$$\int \left(\cot\left(fx+e\right)\right)^4\left(a+a\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)
```

```
[Out] int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(fx+e\right)+a\right)^m\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

3.140 $\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{(2a+3b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-3b)\log(\sin(c+dx)+1)}{4d}+\frac{\tan^2(c+dx)(a+b\sin(c+dx))}{2d}+\frac{3b\sin(c+dx)}{2d}$$

```
[Out] ((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 3*b)*Log[1 + Sin[c + d*
x]])/(4*d) + (3*b*Sin[c + d*x])/(2*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^
2)/(2*d)
```

Rubi [A] time = 0.0771489, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2721, 819, 774, 633, 31}

$$\frac{(2a+3b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-3b)\log(\sin(c+dx)+1)}{4d}+\frac{\tan^2(c+dx)(a+b\sin(c+dx))}{2d}+\frac{3b\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] ((2*a + 3*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 3*b)*Log[1 + Sin[c + d*
x]])/(4*d) + (3*b*Sin[c + d*x])/(2*d) + ((a + b*Sin[c + d*x])*Tan[c + d*x]^
2)/(2*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
```

```
!ILtQ[m + 2*p + 3, 0])
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x\left(2ab^2+3b^2x\right)}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{2b^2d}$$

$$= \frac{3b\sin(c+dx)}{2d} + \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d} + \frac{\text{Subst}\left(\int \frac{-3b^4-2ab^2x}{b^2-x^2}\,dx, x, b$$

$$= \frac{3b\sin(c+dx)}{2d} + \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx$$

$$= \frac{(2a+3b)\log(1-\sin(c+dx))}{4d} + \frac{(2a-3b)\log(1+\sin(c+dx))}{4d} + \frac{3b\sin(c+dx$$

Mathematica [A] time = 0.119188, size = 77, normalized size = 0.88

$$\frac{a\left(\tan^2(c+dx)+2\log(\cos(c+dx))\right)}{2d} - \frac{b\sin(c+dx)\tan^2(c+dx)}{d} - \frac{3b\left(\tanh^{-1}(\sin(c+dx))-\tan(c+dx)\sec(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

```
[Out] -((b*Sin[c + d*x]*Tan[c + d*x]^2)/d) - (3*b*(ArcTanh[Sin[c + d*x]] - Sec[c
+ d*x]*Tan[c + d*x]))/(2*d) + (a*(2*Log[Cos[c + d*x]] + Tan[c + d*x]^2))/(2
*d)
```

Maple [A] time = 0.032, size = 96, normalized size = 1.1

$$\frac{(\tan(dx+c))^2 a}{2d} + \frac{a\ln(\cos(dx+c))}{d} + \frac{b(\sin(dx+c))^5}{2d(\cos(dx+c))^2} + \frac{b(\sin(dx+c))^3}{2d} + \frac{3b\sin(dx+c)}{2d} - \frac{3b\ln(\sec(dx+c)+t}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a*tan(d*x+c)^2+1/d*a*ln(cos(d*x+c))+1/2/d*b*sin(d*x+c)^5/cos(d*x+c)^2
+1/2/d*b*sin(d*x+c)^3+3/2*b*sin(d*x+c)/d-3/2/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.90057, size = 99, normalized size = 1.12

$$\frac{(2a-3b)\log(\sin(dx+c)+1) + (2a+3b)\log(\sin(dx+c)-1) + 4b\sin(dx+c) - \frac{2(b\sin(dx+c)+a)}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*a - 3*b)*log(sin(d*x + c) + 1) + (2*a + 3*b)*log(sin(d*x + c) - 1)
+ 4*b*sin(d*x + c) - 2*(b*sin(d*x + c) + a)/(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.68026, size = 236, normalized size = 2.68

$$\frac{(2a-3b)\cos(dx+c)^2\log(\sin(dx+c)+1) + (2a+3b)\cos(dx+c)^2\log(-\sin(dx+c)+1) + 2\left(2b\cos(dx+c)^2 + b\right)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a + 3*b)*cos(d*x
 + c)^2*log(-sin(d*x + c) + 1) + 2*(2*b*cos(d*x + c)^2 + b)*sin(d*x + c) +
2*a)/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sin (c + dx)) \tan^3 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.141 $\int (a+b\sin(c+dx))\tan(c+dx)\,dx$

Optimal. Leaf size=55

$$-\frac{(a+b)\log(1-\sin(c+dx))}{2d}-\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{b\sin(c+dx)}{d}$$

```
[Out] -((a + b)*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)*Log[1 + Sin[c + d*x]])/(2
*d) - (b*Sin[c + d*x])/d
```

Rubi [A] time = 0.0378478, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2721, 774, 633, 31}

$$-\frac{(a+b)\log(1-\sin(c+dx))}{2d}-\frac{(a-b)\log(\sin(c+dx)+1)}{2d}-\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] -((a + b)*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)*Log[1 + Sin[c + d*x]])/(2
*d) - (b*Sin[c + d*x])/d
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 774

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a+b\sin(c+dx))\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\sin(c+dx)}{d} - \frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\sin(c+dx)}{d} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d} + \frac{(a+b)\,\text{Subst}\left(\int \right.}{}$$
$$= -\frac{(a+b)\log(1-\sin(c+dx))}{2d} - \frac{(a-b)\log(1+\sin(c+dx))}{2d} - \frac{b\sin(c+dx)}{d}$$

Mathematica [A] time = 0.01638, size = 38, normalized size = 0.69

$$-\frac{a\log(\cos(c+dx))}{d} - \frac{b\sin(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d - (b*Sin[c + d*x])/d
```

Maple [A] time = 0.025, size = 46, normalized size = 0.8

$$-\frac{b\sin(dx+c)}{d} + \frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{a\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] -b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))-1/d*a*ln(cos(d*x+c))
```

Maxima [A] time = 1.9426, size = 58, normalized size = 1.05

$$-\frac{(a-b)\log(\sin(dx+c)+1)+(a+b)\log(\sin(dx+c)-1)+2\,b\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(sin(d*x + c) - 1) + 2*b*s
in(d*x + c))/d
```

Fricas [A] time = 1.60971, size = 124, normalized size = 2.25

$$-\frac{(a-b)\log(\sin(dx+c)+1)+(a+b)\log(-\sin(dx+c)+1)+2\,b\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(-sin(d*x + c) + 1) + 2*b*
sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\sin(c+dx))\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x), x)
```

Giac [B] time = 2.82687, size = 1966, normalized size = 35.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/
2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x
)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1
/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*log(4*(tan(c)^2 + 1)/(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b*log(2*(tan(1/2*c)^2 + 1)
/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x
)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 - b*log(2*(tan(1/2*c)^2
 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/
2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*
tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 + a*log(4*(tan(c)^
2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan
(1/2*c) + b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*ta
n(1/2*c)^2 - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))
*tan(1/2*c)^2 + a*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*
tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2
*c)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c)^2 + b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2
*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3
 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/
2*d*x) - 2*tan(1/2*c) + 1)) - b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*ta
```

```
n(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)) + a*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x
)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 4
*b*tan(1/2*d*x) + 4*b*tan(1/2*c))/(d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(1/
2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

3.142 $\int \cot(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rubi [A] time = 0.0213609, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2721, 43}

$$\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot(c+dx)(a+b\sin(c+dx))\,dx = \frac{\operatorname{Subst}\left(\int \frac{a+x}{x}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(1+\frac{a}{x}\right)\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{a\log(\sin(c+dx))}{d} + \frac{b\sin(c+dx)}{d}$$

Mathematica [A] time = 0.0331985, size = 43, normalized size = 1.79

$$\frac{a(\log(\tan(c+dx))+\log(\cos(c+dx)))}{d} + \frac{b\sin(c)\cos(dx)}{d} + \frac{b\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

```
[Out] (a*(Log[Cos[c + d*x]] + Log[Tan[c + d*x]]))/d + (b*Cos[d*x]*Sin[c])/d + (b*
Cos[c]*Sin[d*x])/d
```

Maple [A] time = 0.019, size = 25, normalized size = 1.

$$\frac{a\ln(\sin(dx+c))}{d} + \frac{b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

Maxima [A] time = 1.6977, size = 30, normalized size = 1.25

$$\frac{a\log(\sin(dx+c)) + b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(sin(d*x + c)) + b*sin(d*x + c))/d
```

Fricas [A] time = 1.50454, size = 62, normalized size = 2.58

$$\frac{a\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+b\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*log(1/2*sin(d*x + c)) + b*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+dx\right)\right)\cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x), x)
```

Giac [A] time = 2.28168, size = 31, normalized size = 1.29

$$\frac{a\log\left(\left|\sin\left(dx+c\right)\right|\right)+b\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(sin(d*x + c))) + b*sin(d*x + c))/d
```

3.143 $\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=54

$$-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

```
[Out] -((b*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (b*Sin[c + d*x])/d
```

Rubi [A] time = 0.0409958, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 766}

$$-\frac{a\csc^2(c+dx)}{2d}-\frac{a\log(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*Csc[c + d*x]^2)/(2*d) - (a*Log[Sin[c + d*x]])/d
- (b*Sin[c + d*x])/d
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)}{x^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{ab^2}{x^3}+\frac{b^2}{x^2}-\frac{a}{x}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\log(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d}$$

Mathematica [A] time = 0.210205, size = 60, normalized size = 1.11

$$-\frac{a\left(\cot^2(c+dx)+2\log(\tan(c+dx))+2\log(\cos(c+dx))\right)}{2d} - \frac{b\sin(c+dx)}{d} - \frac{b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[Ta
n[c + d*x]]))/(2*d) - (b*Sin[c + d*x])/d
```

Maple [A] time = 0.039, size = 83, normalized size = 1.5

$$-\frac{(\cot(dx+c))^2 a}{2d} - \frac{a\ln(\sin(dx+c))}{d} - \frac{b(\cos(dx+c))^4}{d\sin(dx+c)} - \frac{(\cos(dx+c))^2\sin(dx+c)\,b}{d} - 2\,\frac{b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/2/d*a*cot(d*x+c)^2-a*ln(sin(d*x+c))/d-1/d*b/sin(d*x+c)*cos(d*x+c)^4-1/d*
cos(d*x+c)^2*sin(d*x+c)*b-2*b*sin(d*x+c)/d
```

Maxima [A] time = 1.44042, size = 61, normalized size = 1.13

$$-\frac{2\,a\log(\sin(dx+c)) + 2\,b\sin(dx+c) + \frac{2\,b\sin(dx+c)+a}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*log(sin(d*x + c)) + 2*b*sin(d*x + c) + (2*b*sin(d*x + c) + a)/sin
(d*x + c)^2)/d
```

Fricas [A] time = 1.60132, size = 167, normalized size = 3.09

$$-\frac{2\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+2\left(b\cos\left(dx+c\right)^2-2b\right)\sin\left(dx+c\right)-a}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(b*cos(d*x + c)^2
- 2*b)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+dx\right)\right)\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**3, x)
```

Giac [A] time = 2.00709, size = 81, normalized size = 1.5

$$-\frac{2a\log\left(\left|\sin\left(dx+c\right)\right|\right)+2b\sin\left(dx+c\right)-\frac{3a\sin(dx+c)^2-2b\sin(dx+c)-a}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(sin(d*x + c))) + 2*b*sin(d*x + c) - (3*a*sin(d*x + c)^2 -
 2*b*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

3.144 $\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=81

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

```
[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rubi [A] time = 0.0521741, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 766}

$$-\frac{a\csc^4(c+dx)}{4d}+\frac{a\csc^2(c+dx)}{d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d + (a*Csc[c + d*x]^2)/d - (b*Csc[c + d*x]^3)/(3*d) - (a
*Csc[c + d*x]^4)/(4*d) + (a*Log[Sin[c + d*x]])/d + (b*Sin[c + d*x])/d
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)(b^2-x^2)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{ab^4}{x^5}+\frac{b^4}{x^4}-\frac{2ab^2}{x^3}-\frac{2b^2}{x^2}+\frac{a}{x}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{2b\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin(c}{d}$$

Mathematica [A] time = 0.238635, size = 87, normalized size = 1.07

$$\frac{a\left(-\cot^4(c+dx)+2\cot^2(c+dx)+4\log(\tan(c+dx))+4\log(\cos(c+dx))\right)}{4d}+\frac{b\sin(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{2b\csc}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d) + (a*(2*Cot[c + d*x]^2 - Co
t[c + d*x]^4 + 4*Log[Cos[c + d*x]] + 4*Log[Tan[c + d*x]]))/(4*d) + (b*Sin[c
 + d*x])/d
```

Maple [A] time = 0.04, size = 136, normalized size = 1.7

$$-\frac{a(\cot(dx+c))^4}{4d}+\frac{(\cot(dx+c))^2a}{2d}+\frac{a\ln(\sin(dx+c))}{d}-\frac{b(\cos(dx+c))^6}{3d(\sin(dx+c))^3}+\frac{b(\cos(dx+c))^6}{d\sin(dx+c)}+\frac{8b\sin(dx+c)}{3d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/4/d*a*cot(d*x+c)^4+1/2/d*a*cot(d*x+c)^2+a*ln(sin(d*x+c))/d-1/3/d*b/sin(d
*x+c)^3*cos(d*x+c)^6+1/d*b/sin(d*x+c)*cos(d*x+c)^6+8/3*b*sin(d*x+c)/d+1/d*c
os(d*x+c)^4*sin(d*x+c)*b+4/3/d*cos(d*x+c)^2*sin(d*x+c)*b
```

Maxima [A] time = 1.28052, size = 93, normalized size = 1.15

$$\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,b\sin\left(dx+c\right)+\frac{24\,b\sin(dx+c)^3+12\,a\sin(dx+c)^2-4\,b\sin(dx+c)-3\,a}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*a*log(sin(d*x + c)) + 12*b*sin(d*x + c) + (24*b*sin(d*x + c)^3 + 1
2*a*sin(d*x + c)^2 - 4*b*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.53603, size = 292, normalized size = 3.6

$$-\frac{12\,a\cos\left(dx+c\right)^2-12\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\left(3\,b\cos\left(dx+c\right)^4-12\,b\cos\left(dx+\right.\right.}{12\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)
*log(1/2*sin(d*x + c)) - 4*(3*b*cos(d*x + c)^4 - 12*b*cos(d*x + c)^2 + 8*b)
*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+dx\right)\right)\cot^5\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**5, x)
```

Giac [A] time = 2.09962, size = 111, normalized size = 1.37

$$\frac{12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,b\sin\left(dx+c\right)-\frac{25\,a\sin(dx+c)^{4}-24\,b\sin(dx+c)^{3}-12\,a\sin(dx+c)^{2}+4\,b\sin(dx+c)+3\,a}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(sin(d*x + c))) + 12*b*sin(d*x + c) - (25*a*sin(d*x + c)^
4 - 24*b*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*b*sin(d*x + c) + 3*a)/sin
(d*x + c)^4)/d
```

3.145 $\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+ax-\frac{b\cos(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{2b\sec(c+dx)}{d}$$

```
[Out] a*x - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0778072, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2722, 3473, 8, 2590, 270}

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+ax-\frac{b\cos(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{2b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] a*x - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b*Sec[c + d*x]^3)/(3*d)
- (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx &= \int \left(a\tan^4(c+dx)+b\sin(c+dx)\tan^4(c+dx)\right)dx\\ &= a\int \tan^4(c+dx)\,dx+b\int \sin(c+dx)\tan^4(c+dx)\,dx\\ &= \frac{a\tan^3(c+dx)}{3d}-a\int \tan^2(c+dx)\,dx-\frac{b\,\mathrm{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}+a\int 1\,dx-\frac{b\,\mathrm{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx,x,\text{c}\right.}{d}\\ &= ax-\frac{b\cos(c+dx)}{d}-\frac{2b\sec(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(}{3}\end{aligned}$$

Mathematica [A] time = 0.037818, size = 81, normalized size = 1.12

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan^{-1}(\tan(c+dx))}{d}-\frac{a\tan(c+dx)}{d}-\frac{b\cos(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{2b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

```
[Out] (a*ArcTan[Tan[c + d*x]])/d - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b
*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.032, size = 98, normalized size = 1.4

$$\frac{1}{d}\left(a\left(\frac{(\tan(dx+c))^3}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{(\sin(dx+c))^6}{3(\cos(dx+c))^3}-\frac{(\sin(dx+c))^6}{\cos(dx+c)}-\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4(\sin(}{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+b*(1/3*sin(d*x+c)^6/cos(d*x+c)^3
-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 2.1333, size = 88, normalized size = 1.22

$$\frac{\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a-b\left(\frac{6\,\cos(dx+c)^{2}-1}{\cos(dx+c)^{3}}+3\,\cos\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - b*((6*cos(d*x + c)
^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A] time = 1.81522, size = 182, normalized size = 2.53

$$\frac{3\,adx\cos\left(dx+c\right)^{3}-3\,b\cos\left(dx+c\right)^{4}-6\,b\cos\left(dx+c\right)^{2}-\left(4\,a\cos\left(dx+c\right)^{2}-a\right)\sin\left(dx+c\right)+b}{3\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*d*x*cos(d*x + c)^3 - 3*b*cos(d*x + c)^4 - 6*b*cos(d*x + c)^2 - (4*
a*cos(d*x + c)^2 - a)*sin(d*x + c) + b)/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+dx\right)\right)\tan^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**4, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.146 $\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] -(a*x) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0593762, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2722, 3473, 8, 2590, 14}

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -(a*x) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx &= \int \left(a\tan^2(c+dx)+b\sin(c+dx)\tan^2(c+dx)\right)\,dx \\
&= a\int \tan^2(c+dx)\,dx+b\int \sin(c+dx)\tan^2(c+dx)\,dx \\
&= \frac{a\tan(c+dx)}{d}-a\int 1\,dx-\frac{b\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{a\tan(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0334988, size = 47, normalized size = 1.24

$$-\frac{a\tan^{-1}(\tan(c+dx))}{d}+\frac{a\tan(c+dx)}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

```
[Out] -((a*ArcTan[Tan[c + d*x]])/d) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (
a*Tan[c + d*x])/d
```

Maple [A] time = 0.026, size = 59, normalized size = 1.6

$$\frac{1}{d}\left(a\left(\tan\left(dx+c\right)-dx-c\right)+b\left(\frac{\left(\sin\left(dx+c\right)\right)^4}{\cos\left(dx+c\right)}+\left(2+\left(\sin\left(dx+c\right)\right)^2\right)\cos\left(dx+c\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a*(tan(d*x+c)-d*x-c)+b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d
*x+c)))
```

Maxima [A] time = 2.72276, size = 53, normalized size = 1.39

$$-\frac{(dx+c-\tan(dx+c))a-b\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c - tan(d*x + c))*a - b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.70543, size = 108, normalized size = 2.84

$$-\frac{adx\cos(dx+c)-b\cos(dx+c)^2-a\sin(dx+c)-b}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(a*d*x*cos(d*x + c) - b*cos(d*x + c)^2 - a*sin(d*x + c) - b)/(d*cos(d*x +
c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))*tan(c + d*x)**2, x)
```

Giac [B] time = 8.56166, size = 1361, normalized size = 35.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4*
tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*b*ta
n(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*tan(1
/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3*tan
(1/2*c)^3 + 2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d*x)^4
*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x*tan(d
*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*
c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x
)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*tan(1/2*d
*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c)
^3 - 8*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*b*tan(d*x)*ta
n(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*b
*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*b*tan(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(c) - 8*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*b*ta
n(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a*tan(d*x)*tan(1
/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - a*tan(d*x)*
tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^3*tan(1/2*c)*tan(
c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^4*tan(c) + 2*b*tan
(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*b*tan(1/2*d*x)^3*tan(1/2*
c) + 24*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*b*tan(1/2*d*x)*tan(1/2*c)^3 + 2*b
*tan(1/2*c)^4 + a*d*x*tan(d*x)*tan(c) + 8*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c
)*tan(c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 4*a*tan(1/2*d*x)*tan(1/2*
c)*tan(c) - a*d*x - 8*b*tan(1/2*d*x)*tan(1/2*c) - 2*b*tan(d*x)*tan(c) + a*t
an(d*x) + a*tan(c) + 2*b)/(d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) -
d*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*ta
n(c) + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c) -
 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*
tan(1/2*c)^3*tan(c) - d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4 + 4
*d*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)
^4 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 4*d*tan(1/2*d*x)*tan(1/2
*c) + d*tan(d*x)*tan(c) - d)
```

3.147 $\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=41

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -(a*x) - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rubi [A] time = 0.0533796, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2722, 2592, 321, 206, 3473, 8}

$$-\frac{a\cot(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}-\frac{b\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d - (a*Cot[c + d*x]
)/d
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx &= \int \left(b\cos(c+dx)\cot(c+dx)+a\cot^2(c+dx)\right)\,dx \\
&= a\int \cot^2(c+dx)\,dx+b\int \cos(c+dx)\cot(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)}{d}-a\int 1\,dx-\frac{b\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\cos(c+dx)\right)}{d} \\
&= -ax-\frac{b\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}
\end{aligned}$$

Mathematica [C] time = 0.0312252, size = 75, normalized size = 1.83

$$-\frac{a\cot(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(c+dx)\right)}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[c
 + d*x]^2])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/2]])/d
```

Maple [A] time = 0.029, size = 57, normalized size = 1.4

$$-ax+\frac{b\cos(dx+c)}{d}-\frac{a\cot(dx+c)}{d}+\frac{b\ln(\csc(dx+c)-\cot(dx+c))}{d}-\frac{ca}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)),x)
```

```
[Out] -a*x+b*cos(d*x+c)/d-a*cot(d*x+c)/d+1/d*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*c*a
```

Maxima [A] time = 2.20653, size = 73, normalized size = 1.78

$$-\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a-b(2\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx+c)-1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c + 1/tan(d*x + c))*a - b*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.83606, size = 236, normalized size = 5.76

$$-\frac{b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+2a\cos(dx+c)+2(adx-b\cos(dx}{2d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - b*cos(d*x + c))*sin(d*x
 + c))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sin(c + dx))\cot^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**2, x)
```

Giac [B] time = 2.30761, size = 146, normalized size = 3.56

$$-\frac{6(dx+c)a - 6b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right) - 3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + \frac{2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-10b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(6*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
+ 1/2*c) + (2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*b*
tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))
)/d
```

3.148 $\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=82

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

```
[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rubi [A] time = 0.0798351, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2722, 2592, 288, 321, 206, 3473, 8}

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{3b\tanh^{-1}(\cos(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Cot
[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)/(
3*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx = \int \left(b\cos(c+dx)\cot^3(c+dx)+a\cot^4(c+dx)\right)\,dx$$
$$= a\int \cot^4(c+dx)\,dx + b\int \cos(c+dx)\cot^3(c+dx)\,dx$$
$$= -\frac{a\cot^3(c+dx)}{3d} - a\int \cot^2(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx)}{3d} + a\int 1\,dx + \frac{(3b)\,\text{Su}}{}$$
$$= ax - \frac{3b\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)\cot^2(c+dx)}{2d} - \frac{a\cot^3(c+dx}{3d}$$
$$= ax + \frac{3b\tanh^{-1}(\cos(c+dx))}{2d} - \frac{3b\cos(c+dx)}{2d} + \frac{a\cot(c+dx)}{d} - \frac{b\cos(c+dx)\,\text{co}}{2d}$$

Mathematica [C] time = 0.0419753, size = 125, normalized size = 1.52

$$-\frac{a\cot^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2(c+dx)\right)}{3d} - \frac{b\cos(c+dx)}{d} - \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{3b\log\left(\sin\left(\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) - (b*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*Hy
pergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*b*Log[Cos[(c +
d*x)/2]])/(2*d) - (3*b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2]^2
)/(8*d)
```

Maple [A] time = 0.036, size = 106, normalized size = 1.3

$$-\frac{a(\cot(dx+c))^3}{3d} + \frac{a\cot(dx+c)}{d} + ax + \frac{ca}{d} - \frac{b(\cos(dx+c))^5}{2d(\sin(dx+c))^2} - \frac{b(\cos(dx+c))^3}{2d} - \frac{3b\cos(dx+c)}{2d} - \frac{3b\ln(\csc(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/3*a*cot(d*x+c)^3/d+a*cot(d*x+c)/d+a*x+1/d*c*a-1/2/d*b/sin(d*x+c)^2*cos(d
*x+c)^5-1/2/d*b*cos(d*x+c)^3-3/2*b*cos(d*x+c)/d-3/2/d*b*ln(csc(d*x+c)-cot(d
*x+c))
```

Maxima [A] time = 1.87341, size = 124, normalized size = 1.51

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\left(dx+c\right)-1\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*b*(2*co
s(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1)
- 3*log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.86448, size = 425, normalized size = 5.18

$$\frac{16\,a\cos\left(dx+c\right)^3+9\left(b\cos\left(dx+c\right)^2-b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\left(b\cos\left(dx+c\right)^2-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx\right.\right.}{12\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(16*a*cos(d*x + c)^3 + 9*(b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c) +
 1/2)*sin(d*x + c) - 9*(b*cos(d*x + c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*b*cos(d*x
+ c)^3 - 2*a*d*x + 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*
sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+dx\right)\right)\cot^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

```
[Out] Integral((a + b*sin(c + d*x))*cot(c + d*x)**4, x)
```

Giac [A] time = 2.17653, size = 190, normalized size = 2.32

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+24\,(dx+c)a-36\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{4}{\tan\left(\frac{1}{2}\,dx\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)*
a - 36*b*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*b/
(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1/2*
d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.149 $\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx$

Optimal. Leaf size=122

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-ax+\frac{15b\cos(c+dx)}{8d}-\frac{b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5b\cos(c+dx)}{8d}$$

```
[Out] -(a*x) - (15*b*ArcTanh[Cos[c + d*x]])/(8*d) + (15*b*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rubi [A] time = 0.0972308, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2722, 2592, 288, 321, 206, 3473, 8}

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-ax+\frac{15b\cos(c+dx)}{8d}-\frac{b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5b\cos(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] -(a*x) - (15*b*ArcTanh[Cos[c + d*x]])/(8*d) + (15*b*Cos[c + d*x])/(8*d) - (
a*Cot[c + d*x])/d + (5*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c + d*
x]^3)/(3*d) - (b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)/(5
*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx = \int \left(b\cos(c+dx)\cot^5(c+dx)+a\cot^6(c+dx)\right)dx$$
$$= a\int \cot^6(c+dx)\,dx + b\int \cos(c+dx)\cot^5(c+dx)\,dx$$
$$= -\frac{a\cot^5(c+dx)}{5d} - a\int \cot^4(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{a\cot^3(c+dx)}{3d} - \frac{b\cos(c+dx)\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d} + a\int \cot^2(c+dx$$
$$= -\frac{a\cot(c+dx)}{d} + \frac{5b\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c+dx)}{3d} - \frac{b\cos(c+dx)\,c}{4d}$$
$$= -ax + \frac{15b\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5b\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(}{3}$$
$$= -ax - \frac{15b\tanh^{-1}(\cos(c+dx))}{8d} + \frac{15b\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + 5b\cos(c+$$

Mathematica [C] time = 0.0598431, size = 164, normalized size = 1.34

$$-\frac{a\cot^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\tan^2(c+dx)\right)}{5d} + \frac{b\cos(c+dx)}{d} - \frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{9b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{b\sec^4\left(\frac{1}{2}\right.}{64}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Cos[c + d*x])/d + (9*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x)/2]^
4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d*x
]^2])/(5*d) - (15*b*Log[Cos[(c + d*x)/2]])/(8*d) + (15*b*Log[Sin[(c + d*x)/
2]])/(8*d) - (9*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(64*d
)
```

Maple [A] time = 0.039, size = 159, normalized size = 1.3

$$-\frac{a(\cot(dx+c))^5}{5d} + \frac{a(\cot(dx+c))^3}{3d} - \frac{a\cot(dx+c)}{d} - ax - \frac{ca}{d} - \frac{b(\cos(dx+c))^7}{4d(\sin(dx+c))^4} + \frac{3b(\cos(dx+c))^7}{8d(\sin(dx+c))^2} + \frac{3b(\cos(}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c)),x)
```

```
[Out] -1/5*a*cot(d*x+c)^5/d+1/3*a*cot(d*x+c)^3/d-a*cot(d*x+c)/d-a*x-1/d*c*a-1/4/d
*b/sin(d*x+c)^4*cos(d*x+c)^7+3/8/d*b/sin(d*x+c)^2*cos(d*x+c)^7+3/8/d*b*cos(
d*x+c)^5+5/8/d*b*cos(d*x+c)^3+15/8*b*cos(d*x+c)/d+15/8/d*b*ln(csc(d*x+c)-co
t(d*x+c))
```

Maxima [A] time = 2.70003, size = 169, normalized size = 1.39

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+15\,b\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos\left(dx+c\right)+15\,\log\left(\cos\left(dx+c\right)+\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a + 15*b*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^4
 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 15*
log(cos(d*x + c) - 1)))/d
```

Fricas [B] time = 1.7059, size = 620, normalized size = 5.08

$$-\frac{368\,a\cos\left(dx+c\right)^5-560\,a\cos\left(dx+c\right)^3+225\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(b*cos(d*x + c)^4
 - 2*b*cos(d*x + c)^2 + b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(
b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*b*cos(d*x +
 c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*b*cos(d*x + c)^3 + 8*a*d*x - 15*b*cos(
```

```
d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.22426, size = 269, normalized size = 2.2

$$\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-240\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-960\,(dx+c)a+1800}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*b*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan(
1/2*d*x + 1/2*c)^3 - 240*b*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 1800*
b*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*b/(tan
(1/2*d*x + 1/2*c)^2 + 1) - (4110*b*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1/2*d
*x + 1/2*c)^4 - 240*b*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*c)^2
+ 15*b*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.150 $\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$

Optimal. Leaf size=111

$$\frac{2ab\sin(c+dx)}{d}+\frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d}+\frac{(a-2b)(a-b)\log(\sin(c+dx)+1)}{2d}+\frac{\sec^2(c+dx)(a+b\sin(c+}{2d}$$

```
[Out] ((a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - 2*b)*(a - b)*Log[1
+ Sin[c + d*x]])/(2*d) + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d
) + (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.172935, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2721, 1645, 1629, 633, 31}

$$\frac{2ab\sin(c+dx)}{d}+\frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d}+\frac{(a-2b)(a-b)\log(\sin(c+dx)+1)}{2d}+\frac{\sec^2(c+dx)(a+b\sin(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] ((a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]])/(2*d) + ((a - 2*b)*(a - b)*Log[1
+ Sin[c + d*x]])/(2*d) + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d
) + (Sec[c + d*x]^2*(a + b*Sin[c + d*x])^2)/(2*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
```

```
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^2}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} + \frac{\text{Subst}\left(\int \frac{(a+x)\left(-2b^4-2ab^2x-2b^2x^2\right)}{b^2-x^2}\,dx, x, b\sin(\right.}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} + \frac{\text{Subst}\left(\int \left(4ab^2+2b^2x-\frac{2\left(3ab^4+b^2\left(a^2+2b^2\right)x\right)}{b^2-x^2}\right.\right.}{2b^2d}$$

$$= \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} - \frac{\text{Subst}}{}$$

$$= \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d} - \frac{((a-2}{}$$

$$= \frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d} + \frac{(a-2b)(a-b)\log(1+\sin(c+dx))}{2d} + \frac{2}{}$$

Mathematica [A] time = 0.431531, size = 108, normalized size = 0.97

$$\frac{\frac{(a-b)^2}{\sin(c+dx)+1}+8ab\sin(c+dx)-\frac{(a+b)^2}{\sin(c+dx)-1}+2(a-2b)(a-b)\log(\sin(c+dx)+1)+2(a+b)(a+2b)\log(1-\sin(c+dx))+}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

```
[Out] (2*(a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]] + 2*(a - 2*b)*(a - b)*Log[1 + Si
n[c + d*x]] - (a + b)^2/(-1 + Sin[c + d*x]) + 8*a*b*Sin[c + d*x] + 2*b^2*Si
n[c + d*x]^2 + (a - b)^2/(1 + Sin[c + d*x]))/(4*d)
```

Maple [A] time = 0.049, size = 172, normalized size = 1.6

$$\frac{a^2(\tan(dx+c))^2}{2d}+\frac{a^2\ln(\cos(dx+c))}{d}+\frac{ab(\sin(dx+c))^5}{d(\cos(dx+c))^2}+\frac{ab(\sin(dx+c))^3}{d}+3\frac{ab\sin(dx+c)}{d}-3\frac{ab\ln(\sec(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^2*tan(d*x+c)^2+1/d*a^2*ln(cos(d*x+c))+1/d*a*b*sin(d*x+c)^5/cos(d*x+
c)^2+1/d*a*b*sin(d*x+c)^3+3*a*b*sin(d*x+c)/d-3/d*a*b*ln(sec(d*x+c)+tan(d*x+
c))+1/2/d*b^2*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*b^2*sin(d*x+c)^4+b^2*sin(d*x+
c)^2/d+2/d*b^2*ln(cos(d*x+c))
```

Maxima [A] time = 1.67617, size = 142, normalized size = 1.28

$$\frac{b^2\sin(dx+c)^2+4ab\sin(dx+c)+\left(a^2-3ab+2b^2\right)\log(\sin(dx+c)+1)+\left(a^2+3ab+2b^2\right)\log(\sin(dx+c)-1)-}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 3*a*b + 2*b^2)*log(si
n(d*x + c) + 1) + (a^2 + 3*a*b + 2*b^2)*log(sin(d*x + c) - 1) - (2*a*b*sin(
```

```
d*x + c) + a^2 + b^2)/(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.54829, size = 348, normalized size = 3.14

$$-\frac{2\,b^2\cos\left(dx+c\right)^4-b^2\cos\left(dx+c\right)^2-2\left(a^2-3\,ab+2\,b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-2\left(a^2+3\,ab+2\,b^2\right)\cos}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*(a^2 - 3*a*b + 2*b^2)*c
os(d*x + c)^2*log(sin(d*x + c) + 1) - 2*(a^2 + 3*a*b + 2*b^2)*cos(d*x + c)^
2*log(-sin(d*x + c) + 1) - 2*a^2 - 2*b^2 - 4*(2*a*b*cos(d*x + c)^2 + a*b)*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\tan^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.151 $\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx$

Optimal. Leaf size=78

$$-\frac{2ab\sin(c+dx)}{d}-\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] -((a + b)^2*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)^2*Log[1 + Sin[c + d*x]]
)/(2*d) - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0709252, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2721, 801, 633, 31}

$$-\frac{2ab\sin(c+dx)}{d}-\frac{(a-b)^2\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^2\log(1-\sin(c+dx))}{2d}-\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a + b)^2*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)^2*Log[1 + Sin[c + d*x]]
)/(2*d) - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
```

```
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^2}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a-x+\frac{2ab^2+(a^2+b^2)x}{b^2-x^2}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d} + \frac{\text{Subst}\left(\int \frac{2ab^2+(a^2+b^2)x}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d} + \frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{-b-x}\,dx, x, b\sin(c+dx)\right)}{2d}$$
$$= -\frac{(a+b)^2\log(1-\sin(c+dx))}{2d} - \frac{(a-b)^2\log(1+\sin(c+dx))}{2d} - \frac{2ab\sin(c+dx)}{d}$$

Mathematica [A] time = 0.13574, size = 64, normalized size = 0.82

$$-\frac{4ab\sin(c+dx)+(a-b)^2\log(\sin(c+dx)+1)+(a+b)^2\log(1-\sin(c+dx))+b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

```
[Out] -((a + b)^2*Log[1 - Sin[c + d*x]] + (a - b)^2*Log[1 + Sin[c + d*x]] + 4*a*b
*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.039, size = 82, normalized size = 1.1

$$-\frac{a^2\ln(\cos(dx+c))}{d} - 2\,\frac{ab\sin(dx+c)}{d} + 2\,\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d} - \frac{(\sin(dx+c))^2\,b^2}{2\,d} - \frac{b^2\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c),x)
```

```
[Out] -1/d*a^2*ln(cos(d*x+c))-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c)
)-1/2*b^2*sin(d*x+c)^2/d-1/d*b^2*ln(cos(d*x+c))
```

Maxima [A] time = 1.63094, size = 95, normalized size = 1.22

$$-\frac{b^2\sin(dx+c)^2+4\,ab\sin(dx+c)+\left(a^2-2\,ab+b^2\right)\log(\sin(dx+c)+1)+\left(a^2+2\,ab+b^2\right)\log(\sin(dx+c)-1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 2*a*b + b^2)*log(sin
(d*x + c) + 1) + (a^2 + 2*a*b + b^2)*log(sin(d*x + c) - 1))/d
```

Fricas [A] time = 1.60878, size = 186, normalized size = 2.38

$$\frac{b^2\cos(dx+c)^2-4\,ab\sin(dx+c)-\left(a^2-2\,ab+b^2\right)\log(\sin(dx+c)+1)-\left(a^2+2\,ab+b^2\right)\log(-\sin(dx+c)+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin(
d*x + c) + 1) - (a^2 + 2*a*b + b^2)*log(-sin(d*x + c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\sin(c+dx))^2\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c),x)

[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x), x)
```

Giac [B] time = 5.72047, size = 10604, normalized size = 135.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")

[Out] -1/4*(4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 +
 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c
)^2*tan(c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)
^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(
d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(t
an(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2
 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2
- b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a*b*log(2*(tan(1/
2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*
tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan
(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*t
an(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)
^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(
1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
+ 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(
tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^
2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*
log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2 + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*
```

x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c
) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)
/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x
)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 2
*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(
d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d
*x)^2*tan(c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*
x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2
*c)*tan(c)^2 + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c)
 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(t
an(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a^2*
log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^
2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*c)^2*t
an(c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*ta
n(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^
2*tan(1/2*c)^2*tan(c)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c
)^2 + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*ta
n(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/
2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(
1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1
/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a^2*l
og(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^
2*tan(c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3
*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/
2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 4*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^2*tan(d*x)^2*tan(1
/2*d*x)^2*tan(c)^2 - b^2*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(c)^2 + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*
tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2
+ tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan

```
(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2 - 4*a*b*log(2*(tan(1/2*c)^2 +
 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*ta
n(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2 + 2*a^2*l
og(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2
*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2
+ 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/
2*d*x)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*b*log(2*(tan(1
/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2
*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)
^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + ta
n(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2 -
4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)
^2*tan(1/2*c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d
*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*t
an(d*x)^2*tan(1/2*c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1))*tan(d*x)^2*tan(1/2*c)^2 - 16*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2
 + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(
1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*
tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(
tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^
2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*b^2*log(4*(tan(
c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + 4*a*b*
log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*ta
n(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan
(c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2
*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)
^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*
```

```
tan(d*x)^2*tan(c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*t
an(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1
))*tan(d*x)^2*tan(c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 -
2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c)
+ 1))*tan(d*x)^2*tan(c)^2 + 16*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(c)^2 + 4*a*b
*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*t
an(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^
2*tan(c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 -
2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^
4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
 1))*tan(1/2*d*x)^2*tan(c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
an(c) + 1))*tan(1/2*d*x)^2*tan(c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^
4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan
(d*x)*tan(c) + 1))*tan(1/2*d*x)^2*tan(c)^2 + 16*a*b*tan(d*x)^2*tan(1/2*c)*t
an(c)^2 - 16*a*b*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 + 4*a*b*log(2*(tan(1/2*
c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
- 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^2*tan(c)^2 - 4*a*b
*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*t
an(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)
^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/
2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^2*
tan(c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*t
an(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*
c)^2*tan(c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x
)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan
(1/2*c)^2*tan(c)^2 - 16*a*b*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + b^2*tan(d*
x)^2*tan(1/2*d*x)^2 + b^2*tan(d*x)^2*tan(1/2*c)^2 - b^2*tan(1/2*d*x)^2*tan(
1/2*c)^2 + 4*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(c) + 4*b^2*tan(d*x)*tan(1/2*c)
^2*tan(c) - b^2*tan(d*x)^2*tan(c)^2 + b^2*tan(1/2*d*x)^2*tan(c)^2 + b^2*tan
(1/2*c)^2*tan(c)^2 + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1
/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*
x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1))*tan(d*x)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*ta
n(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 +
tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1
/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1))*tan(d*x)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c
)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*t
```

an(c) + 1))*tan(d*x)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(d*x)^2 + 16*a*b*tan(d*x)^2*tan(1/2*d*x) + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*d*x)^2 + 16*a*b*tan(d*x)^2*tan(1/2*c) - 16*a*b*tan(1/2*d*x)^2*tan(1/2*c) + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(1/2*c)^2 - 16*a*b*tan(1/2*d*x)*tan(1/2*c)^2 + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(c)^2 - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(c)^2 + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(c)^2 + 2*b^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1))*tan(c)^2 + 16*a*b*tan(1/2*d*x)*tan(c)^2 + 16*a*b*tan(1/2*c)*tan(c)^2 + b^2*tan(d*x)^2 - b^2*tan(1/2*d*x)^2 - b^2*tan(1/2*c)^2 + 4*b^2*tan(d*x)*tan(c) + b^2*tan(c)^2 + 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)

```
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1)) - 4*a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 -
2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^
4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) +
 1)) + 2*a^2*log(4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c
) + tan(d*x)^2*tan(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 2*b^2*log(
4*(tan(c)^2 + 1)/(tan(d*x)^4*tan(c)^2 - 2*tan(d*x)^3*tan(c) + tan(d*x)^2*ta
n(c)^2 + tan(d*x)^2 - 2*tan(d*x)*tan(c) + 1)) + 16*a*b*tan(1/2*d*x) + 16*a*
b*tan(1/2*c) - b^2)/(d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*
tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)
^2 + d*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan
(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2 + d*tan(d*x)^2*tan(1/2*c)^2 + d*tan(1/2
*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)^2*tan(c)^2 + d*tan(1/2*d*x)^2*tan(c)^2 +
d*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2
+ d*tan(c)^2 + d)
```

3.152 $\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=46

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(
2*d)
```

Rubi [A] time = 0.038808, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 43}

$$\frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(
2*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2}{x}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(2a+\frac{a^2}{x}+x\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{a^2\log(\sin(c+dx))}{d} + \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d}$$

Mathematica [A] time = 0.0234396, size = 46, normalized size = 1.

$$\frac{a^2\log(\sin(c+dx))}{d} + \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(
2*d)
```

Maple [A] time = 0.023, size = 45, normalized size = 1.

$$\frac{a^2\ln(\sin(dx+c))}{d} + 2\frac{ab\sin(dx+c)}{d} + \frac{(\sin(dx+c))^2 b^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c))^2,x)
```

```
[Out] a^2*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d
```

Maxima [A] time = 1.89331, size = 54, normalized size = 1.17

$$\frac{b^2\sin(dx+c)^2 + 2a^2\log(\sin(dx+c)) + 4ab\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(sin(d*x + c)) + 4*a*b*sin(d*x + c))/d
```

Fricas [A] time = 1.57997, size = 108, normalized size = 2.35

$$-\frac{b^2\cos\left(dx+c\right)^2-2\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\,ab\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c)) - 4*a*b*sin(d*x + c)
)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+dx\right)\right)^2\cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x), x)
```

Giac [A] time = 2.32155, size = 55, normalized size = 1.2

$$\frac{b^2\sin\left(dx+c\right)^2+2\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+4\,ab\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c))) + 4*a*b*sin(d*x + c)
)/d
```

3.153 $\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=84

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}$$

```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((a^2 - b^2)*Log[Sin
[c + d*x]])/d - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0730727, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}-\frac{b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - ((a^2 - b^2)*Log[Sin
[c + d*x]])/d - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)
```


Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2(b^2-x^2)}{x^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a+\frac{a^2b^2}{x^3}+\frac{2ab^2}{x^2}+\frac{-a^2+b^2}{x}-x\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{2ab\csc(c+dx)}{d}-\frac{a^2\csc^2(c+dx)}{2d}-\frac{(a^2-b^2)\log(\sin(c+dx))}{d}-\frac{2ab\sin(c+}{d}$$

Mathematica [A] time = 0.23557, size = 70, normalized size = 0.83

$$-\frac{2(a^2-b^2)\log(\sin(c+dx))+a^2\csc^2(c+dx)+4ab\sin(c+dx)+4ab\csc(c+dx)+b^2\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(4*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*Log[Sin[c + d*x]]
 + 4*a*b*Sin[c + d*x] + b^2*Sin[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.053, size = 120, normalized size = 1.4

$$-\frac{a^2(\cot(dx+c))^2}{2d}-\frac{a^2\ln(\sin(dx+c))}{d}-2\frac{ab(\cos(dx+c))^4}{d\sin(dx+c)}-2\frac{ab(\cos(dx+c))^2\sin(dx+c)}{d}-4\frac{ab\sin(dx+c)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2*cot(d*x+c)^2-a^2*ln(sin(d*x+c))/d-2/d*a*b/sin(d*x+c)*cos(d*x+c)^
4-2/d*a*b*cos(d*x+c)^2*sin(d*x+c)-4*a*b*sin(d*x+c)/d+1/2/d*b^2*cos(d*x+c)^2
+1/d*b^2*ln(sin(d*x+c))
```

Maxima [A] time = 1.56796, size = 93, normalized size = 1.11

$$-\frac{b^2\sin(dx+c)^2+4ab\sin(dx+c)+2(a^2-b^2)\log(\sin(dx+c))+\frac{4ab\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + 2*(a^2 - b^2)*log(sin(d*x +
 c)) + (4*a*b*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.5208, size = 271, normalized size = 3.23

$$\frac{2\,b^2\cos\left(dx+c\right)^4-3\,b^2\cos\left(dx+c\right)^2+2\,a^2+b^2-4\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-8\left(ab\cos\left(a\right.\right.}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*cos(d*x + c)^4 - 3*b^2*cos(d*x + c)^2 + 2*a^2 + b^2 - 4*((a^2 -
b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(1/2*sin(d*x + c)) - 8*(a*b*cos(d*x + c
)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+dx\right)\right)^2\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**3, x)
```

Giac [A] time = 2.31265, size = 134, normalized size = 1.6

$$-\frac{b^2\sin\left(dx+c\right)^2+4\,ab\sin\left(dx+c\right)+2\left(a^2-b^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{3\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2-4\,ab\sin(dx+c)-a^2}{\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + 2*(a^2 - b^2)*log(abs(sin(d
*x + c))) - (3*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x +
c) - a^2)/sin(d*x + c)^2)/d
```

3.154 $\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\csc(}{d}$$

```
[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[
c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c +
 d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.103542, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 948}

$$\frac{(2a^2-b^2)\csc^2(c+dx)}{2d}+\frac{(a^2-2b^2)\log(\sin(c+dx))}{d}-\frac{a^2\csc^4(c+dx)}{4d}+\frac{2ab\sin(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}+\frac{4ab\csc(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*a*b*Csc[c + d*x])/d + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2*d) - (2*a*b*Csc[
c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + ((a^2 - 2*b^2)*Log[Sin[c +
 d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/(2*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(b^2-x^2\right)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(2a + \frac{a^2b^4}{x^5} + \frac{2ab^4}{x^4} + \frac{-2a^2b^2+b^4}{x^3} - \frac{4ab^2}{x^2} + \frac{a^2-2b^2}{x} + x\right) dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{4ab\csc(c+dx)}{d} + \frac{\left(2a^2-b^2\right)\csc^2(c+dx)}{2d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{a^2\csc^4(c+dx)}{4d}$$

Mathematica [A] time = 0.756517, size = 107, normalized size = 0.85

$$\frac{6\left(2a^2-b^2\right)\csc^2(c+dx)+6\left(2\left(a^2-2b^2\right)\log(\sin(c+dx))+4ab\sin(c+dx)+b^2\sin^2(c+dx)\right)-3a^2\csc^4(c+dx)-8a}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (48*a*b*Csc[c + d*x] + 6*(2*a^2 - b^2)*Csc[c + d*x]^2 - 8*a*b*Csc[c + d*x]^
3 - 3*a^2*Csc[c + d*x]^4 + 6*(2*(a^2 - 2*b^2)*Log[Sin[c + d*x]] + 4*a*b*Sin
[c + d*x] + b^2*Sin[c + d*x]^2))/(12*d)
```

Maple [A] time = 0.047, size = 220, normalized size = 1.8

$$-\frac{a^2(\cot(dx+c))^4}{4d} + \frac{a^2(\cot(dx+c))^2}{2d} + \frac{a^2\ln(\sin(dx+c))}{d} - \frac{2ab(\cos(dx+c))^6}{3d(\sin(dx+c))^3} + 2\frac{ab(\cos(dx+c))^6}{d\sin(dx+c)} + \frac{16ab\sin(}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/4/d*a^2*cot(d*x+c)^4+1/2/d*a^2*cot(d*x+c)^2+a^2*ln(sin(d*x+c))/d-2/3/d*a
*b/sin(d*x+c)^3*cos(d*x+c)^6+2/d*a*b/sin(d*x+c)*cos(d*x+c)^6+16/3*a*b*sin(d
*x+c)/d+2/d*a*b*sin(d*x+c)*cos(d*x+c)^4+8/3/d*a*b*cos(d*x+c)^2*sin(d*x+c)-1
/2/d*b^2/sin(d*x+c)^2*cos(d*x+c)^6-1/2/d*b^2*cos(d*x+c)^4-1/d*b^2*cos(d*x+c
)^2-2/d*b^2*ln(sin(d*x+c))
```

Maxima [A] time = 1.40308, size = 142, normalized size = 1.13

$$\frac{6\,b^2\sin\left(dx+c\right)^2+24\,ab\sin\left(dx+c\right)+12\left(a^2-2\,b^2\right)\log\left(\sin\left(dx+c\right)\right)+\frac{48\,ab\sin(dx+c)^3-8\,ab\sin(dx+c)+6\left(2\,a^2-b^2\right)\sin(dx+c)^2-3\,a^2}{\sin(dx+c)^4}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(sin
(d*x + c)) + (48*a*b*sin(d*x + c)^3 - 8*a*b*sin(d*x + c) + 6*(2*a^2 - b^2)*
sin(d*x + c)^2 - 3*a^2)/sin(d*x + c)^4)/d
```

Fricas [A] time = 1.62927, size = 437, normalized size = 3.47

$$-\frac{6\,b^2\cos\left(dx+c\right)^6-15\,b^2\cos\left(dx+c\right)^4+6\left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2-9\,a^2+3\,b^2-12\left(\left(a^2-2\,b^2\right)\cos\left(dx+c\right)^4-2\left(a^2-\right.\right.}{12\left(d\cos\left(dx+c\right)^4-2\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*cos(d*x + c)^6 - 15*b^2*cos(d*x + c)^4 + 6*(2*a^2 + b^2)*cos(d
*x + c)^2 - 9*a^2 + 3*b^2 - 12*((a^2 - 2*b^2)*cos(d*x + c)^4 - 2*(a^2 - 2*b
^2)*cos(d*x + c)^2 + a^2 - 2*b^2)*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d*x
+ c)^4 - 12*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2
*d*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.22304, size = 186, normalized size = 1.48

$$\frac{6\,b^{2}\sin\left(dx+c\right)^{2}+24\,ab\sin\left(dx+c\right)+12\left(a^{2}-2\,b^{2}\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{25\,a^{2}\sin(dx+c)^{4}-50\,b^{2}\sin(dx+c)^{4}-48\,ab\sin(dx+c)^{3}-12\,a^{2}}{\sin(dx+c)^{4}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(abs
(sin(d*x + c))) - (25*a^2*sin(d*x + c)^4 - 50*b^2*sin(d*x + c)^4 - 48*a*b*s
in(d*x + c)^3 - 12*a^2*sin(d*x + c)^2 + 6*b^2*sin(d*x + c)^2 + 8*a*b*sin(d*
x + c) + 3*a^2)/sin(d*x + c)^4)/d
```

3.155 $\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$

Optimal. Leaf size=149

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x-\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{4ab\sec(c+dx)}{d}+\frac{5b^2\tan^3(c+dx)}{6d}-\frac{5b}{}$$

```
[Out] a^2*x + (5*b^2*x)/2 - (2*a*b*Cos[c + d*x])/d - (4*a*b*Sec[c + d*x])/d + (2*
a*b*Sec[c + d*x]^3)/(3*d) - (a^2*Tan[c + d*x])/d - (5*b^2*Tan[c + d*x])/(2*
d) + (a^2*Tan[c + d*x]^3)/(3*d) + (5*b^2*Tan[c + d*x]^3)/(6*d) - (b^2*Sin[c
 + d*x]^2*Tan[c + d*x]^3)/(2*d)
```

Rubi [A] time = 0.161982, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x-\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{4ab\sec(c+dx)}{d}+\frac{5b^2\tan^3(c+dx)}{6d}-\frac{5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```



```
[Out] a^2*x + (5*b^2*x)/2 - (2*a*b*Cos[c + d*x])/d - (4*a*b*Sec[c + d*x])/d + (2*
a*b*Sec[c + d*x]^3)/(3*d) - (a^2*Tan[c + d*x])/d - (5*b^2*Tan[c + d*x])/(2*
d) + (a^2*Tan[c + d*x]^3)/(3*d) + (5*b^2*Tan[c + d*x]^3)/(6*d) - (b^2*Sin[c
 + d*x]^2*Tan[c + d*x]^3)/(2*d)
```


Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = \int \left(a^2\tan^4(c+dx)+2ab\sin(c+dx)\tan^4(c+dx)+b^2\sin^2(c+dx)\tan^4(c+dx\right.$$
$$= a^2\int \tan^4(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan^4(c+dx)\,dx+b^2\int \sin^2(c+dx)$$
$$= \frac{a^2\tan^3(c+dx)}{3d}-a^2\int \tan^2(c+dx)\,dx-\frac{(2ab)\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx,x,\cos(c+\right.}{d}$$
$$= -\frac{a^2\tan(c+dx)}{d}+\frac{a^2\tan^3(c+dx)}{3d}-\frac{b^2\sin^2(c+dx)\tan^3(c+dx)}{2d}+a^2\int 1\,dx-$$
$$= a^2x-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+$$
$$= a^2x-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}-$$
$$= a^2x+\frac{5b^2x}{2}-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a^2\tan(c}{d}$$

Mathematica [A] time = 0.686576, size = 176, normalized size = 1.18

$$-\frac{\sec^3(c+dx)\left(-36\left(2a^2+5b^2\right)(c+dx)\cos(c+dx)+32a^2\sin(3(c+dx))-24a^2c\cos(3(c+dx))-24a^2dx\cos(3(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

```
[Out] -(Sec[c + d*x]^3*(200*a*b - 36*(2*a^2 + 5*b^2)*(c + d*x)*Cos[c + d*x] + 288
*a*b*Cos[2*(c + d*x)] - 24*a^2*c*Cos[3*(c + d*x)] - 60*b^2*c*Cos[3*(c + d*x
)] - 24*a^2*d*x*Cos[3*(c + d*x)] - 60*b^2*d*x*Cos[3*(c + d*x)] + 24*a*b*Cos
[4*(c + d*x)] + 30*b^2*Sin[c + d*x] + 32*a^2*Sin[3*(c + d*x)] + 65*b^2*Sin[
3*(c + d*x)] + 3*b^2*Sin[5*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.044, size = 185, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(\frac{(\tan(dx+c))^3}{3}-\tan(dx+c)+dx+c\right)+2ab\left(1/3\,\frac{(\sin(dx+c))^6}{(\cos(dx+c))^3}-\frac{(\sin(dx+c))^6}{\cos(dx+c)}-\left(8/3+(\sin(dx+c))^4+4/3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a*b*(1/3*sin(d*x+c)^6/cos(d*
x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+
c))+b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(sin
(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

Maxima [A] time = 2.50033, size = 161, normalized size = 1.08

$$\frac{2\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^{2}+\left(2\,\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\,\tan\left(dx+c\right)\right)b^{2}-}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2 + (2*tan(d*x + c
)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x + c)
)*b^2 - 4*a*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

Fricas [A] time = 1.46423, size = 281, normalized size = 1.89

$$\frac{3\left(2\,a^{2}+5\,b^{2}\right)dx\cos\left(dx+c\right)^{3}-12\,ab\cos\left(dx+c\right)^{4}-24\,ab\cos\left(dx+c\right)^{2}+4\,ab-\left(3\,b^{2}\cos\left(dx+c\right)^{4}+2\left(4\,a^{2}+7\,b^{2}\right)\cos\right.}{6\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(2*a^2 + 5*b^2)*d*x*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)^4 - 24*a*b*
cos(d*x + c)^2 + 4*a*b - (3*b^2*cos(d*x + c)^4 + 2*(4*a^2 + 7*b^2)*cos(d*x
+ c)^2 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

3.156 $\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

Optimal. Leaf size=94

$$\frac{a^2\tan(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3b^2x}{2}$$

```
[Out] -(a^2*x) - (3*b^2*x)/2 + (2*a*b*Cos[c + d*x])/d + (2*a*b*Sec[c + d*x])/d +
(a^2*Tan[c + d*x])/d + (3*b^2*Tan[c + d*x])/(2*d) - (b^2*Sin[c + d*x]^2*Tan
[c + d*x])/(2*d)
```

Rubi [A] time = 0.122315, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203}

$$\frac{a^2\tan(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] -(a^2*x) - (3*b^2*x)/2 + (2*a*b*Cos[c + d*x])/d + (2*a*b*Sec[c + d*x])/d +
(a^2*Tan[c + d*x])/d + (3*b^2*Tan[c + d*x])/(2*d) - (b^2*Sin[c + d*x]^2*Tan
[c + d*x])/(2*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a + b\sin(c+dx))^2 \tan^2(c+dx)\,dx = \int \left(a^2\tan^2(c+dx) + 2ab\sin(c+dx)\tan^2(c+dx) + b^2\sin^2(c+dx)\tan^2(c+
$$

$$= a^2\int \tan^2(c+dx)\,dx + (2ab)\int \sin(c+dx)\tan^2(c+dx)\,dx + b^2\int \sin^2(c+d
$$

$$= \frac{a^2\tan(c+dx)}{d} - a^2\int 1\,dx - \frac{(2ab)\,\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{d} + \frac{b^2\,\text{Sub}}{}
$$

$$= -a^2x + \frac{a^2\tan(c+dx)}{d} - \frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d} - \frac{(2ab)\,\text{Subst}\left(\int\left(-1+\right.\right.}{d}
$$

$$= -a^2x + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\sec(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{3b^2\tan(c+dx)}{2d}
$$

$$= -a^2x - \frac{3b^2x}{2} + \frac{2ab\cos(c+dx)}{d} + \frac{2ab\sec(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d} + \frac{3b^2\tan}{2}
$$

Mathematica [A] time = 0.457161, size = 77, normalized size = 0.82

$$\frac{-4\left(2a^2+3b^2\right)(c+dx) + \left(8a^2+9b^2\right)\tan(c+dx) + b\sec(c+dx)(8a\cos(2(c+dx)) + 24a + b\sin(3(c+dx)))}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

```
[Out] (-4*(2*a^2 + 3*b^2)*(c + d*x) + b*Sec[c + d*x]*(24*a + 8*a*Cos[2*(c + d*x)]
 + b*Sin[3*(c + d*x)]) + (8*a^2 + 9*b^2)*Tan[c + d*x])/(8*d)
```

Maple [A] time = 0.039, size = 116, normalized size = 1.2

$$\frac{1}{d}\left(a^2(\tan(dx+c) - dx - c) + 2ab\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)} + \left(2 + (\sin(dx+c))^2\right)\cos(dx+c)\right) + b^2\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)} + \left((\sin(d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)
*cos(d*x+c))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))*cos
```

```
(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A] time = 2.52019, size = 112, normalized size = 1.19

$$-\frac{2\,(dx+c-\tan{(dx+c)})a^2+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan{(dx+c)}\right)b^2-4\,ab\left(\frac{1}{\cos(dx+c)}+\cos{(dx+c)}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(d*x + c - tan(d*x + c))*a^2 + (3*d*x + 3*c - tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 2*tan(d*x + c))*b^2 - 4*a*b*(1/cos(d*x + c) + cos(d*x + c)))
/d
```

Fricas [A] time = 1.3702, size = 190, normalized size = 2.02

$$-\frac{\left(2\,a^2+3\,b^2\right)dx\cos{(dx+c)}-4\,ab\cos{(dx+c)}^2-4\,ab-\left(b^2\cos{(dx+c)}^2+2\,a^2+2\,b^2\right)\sin{(dx+c)}}{2\,d\cos{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((2*a^2 + 3*b^2)*d*x*cos(d*x + c) - 4*a*b*cos(d*x + c)^2 - 4*a*b - (b^
2*cos(d*x + c)^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sin{(c+dx)})^2\tan^2{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**2, x)
```

Giac [B] time = 30.6123, size = 10355, normalized size = 110.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*b^2*d*x
*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*d*x*tan(d*x)^3*tan
(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1
/2*c)^4*tan(c) - 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2
- 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 8*a^2*d*x*tan
(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/
2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 2*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*
c)^4*tan(c)^3 + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - 8
*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*tan(d*x)^3*tan
(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 3*b^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2
*c)^4*tan(c)^2 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*
b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - 2*a^2*d*x*tan(d*x)^2*
tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c
)^4 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*
tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 2*a^2*d*x*tan(d*x)*tan(1/2*
d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4
*tan(c) - 8*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 8*a^2*d*x*t
an(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(
1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 2*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*ta
n(c)^2 - 3*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 8*a*b*tan(d*x)^2*
tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*
tan(c)^3 - 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*d*x*tan(d*x
)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)
^3*tan(1/2*c)*tan(c)^3 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan
(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*a^2*d*
x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(1
/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*
c)^3*tan(c)^3 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 3*b^2*d*x*tan(
d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*a*b*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*t
an(c)^3 + 2*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4 + 2*b^2*tan(d*x)^3*t
an(1/2*d*x)^4*tan(1/2*c)^4 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*t
an(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 12*b^2*tan(
```

d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*tan(d*x)*tan(1/2*d*x)^4
*tan(1/2*c)^4*tan(c)^2 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c
)^3 - 12*b^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 2*a^2*tan(1/
2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3
 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 12*b^2*d*x*tan(d*x)^2
*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*b^
2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4 + 8*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2
*c)^4 - 2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 3*b^2*d*x*tan(d*x)^3*t
an(1/2*d*x)^4*tan(c) - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c
) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 8*a^2*d*x*tan(
d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x
)*tan(1/2*c)^3*tan(c) - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(
c) - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 32*a*b*tan(d*
x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4
*tan(c) - 3*b^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c) - 8*a*b*tan(d*x)*tan(1/2
*d*x)^4*tan(1/2*c)^4*tan(c) + 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2
+ 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*d*x*tan(d*x)^2*tan(1
/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/
2*c)*tan(c)^2 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 1
2*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 8*a^2*d*x*tan(1/2
*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(
c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*d*x*t
an(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^4*tan(c)^
2 + 8*a*b*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 2*a^2*d*x*tan(d*x)*tan(1/2
*d*x)^4*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 8*a*b*tan(d
*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c
)*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 8*a^2
*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(
1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c
)*tan(c)^3 - 96*a*b*tan(d*x)^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 8*a^2
*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(
1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 32*a*b*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^
3*tan(c)^3 + 32*a*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 2*a^2*d
*x*tan(d*x)*tan(1/2*c)^4*tan(c)^3 - 3*b^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c)^
3 - 8*a*b*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^
3*tan(1/2*c)^3 - 8*b^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*a^2*tan(d
*x)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 3*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^
4 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 2*a^2*tan(1/2*d*x
)^4*tan(1/2*c)^4*tan(c) + 3*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 2*a^2*
tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)
^2 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 12*b^2*tan(d*x)^
3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/
2*c)^3*tan(c)^2 - 12*b^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 8*
a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 2*a^2*tan(d*x)^3*tan(1/
2*c)^4*tan(c)^2 - 3*b^2*tan(d*x)^3*tan(1/2*c)^4*tan(c)^2 - 2*a^2*tan(d*x)^2

*tan(1/2*d*x)^4*tan(c)^3 - 3*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 12*b^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 12*b^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*a^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 8*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 2*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c)^3 - 3*b^2*tan(d*x)^2*tan(1/2*c)^4*tan(c)^3 + 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 8*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3 + 12*b^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)^3 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*a^2*d*x*tan(d*x)^2*tan(1/2*c)^4 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^4 + 8*a*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - 2*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(c) - 8*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(c) - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 32*a*b*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 96*a*b*tan(d*x)^3*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 32*a*b*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 32*a*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 3*b^2*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 8*a*b*tan(d*x)^3*tan(1/2*c)^4*tan(c) + 2*a^2*d*x*tan(1/2*d*x)^4*tan(c)^2 + 3*b^2*d*x*tan(1/2*d*x)^4*tan(c)^2 + 8*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 8*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 8*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 32*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 32*a*b*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*d*x*tan(1/2*c)^4*tan(c)^2 + 3*b^2*d*x*tan(1/2*c)^4*tan(c)^2 + 8*a*b*tan(d*x)^2*tan(1/2*c)^4*tan(c)^2 + 2*a^2*d*x*tan(d*x)^3*tan(c)^3 + 3*b^2*d*x*tan(d*x)^3*tan(c)^3 - 8*a*b*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 12*b^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 + 32*a*b*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 32*a*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 96*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^3 - 32*a*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 8*a*b*tan(d*x)*tan(1/2*c)^4*tan(c)^3 - 2*a^2*tan(d*x)^3*tan(1/2*d*x)^4 - 2*b^2*tan(d*x)^3*tan(1/2*d*x)^4 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c) - 8*b^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3 - 8*b^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3 - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 12*b^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3 - 2*a^2*tan(d*x)^3*tan(1/2*c)^4 - 2*b^2*tan(d*x)^3*tan(1/2*c)^4 - 2*a^2

*tan(d*x)^2*tan(1/2*d*x)^4*tan(c) - 8*a^2*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2
*c)*tan(c) - 8*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 8*a^2*tan(
1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 12*b^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)
- 2*a^2*tan(d*x)^2*tan(1/2*c)^4*tan(c) - 2*a^2*tan(d*x)*tan(1/2*d*x)^4*tan(
c)^2 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 - 12*b^2*tan(d*x)^
3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*
c)*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 - 2*a^2*tan
(d*x)*tan(1/2*c)^4*tan(c)^2 - 2*a^2*tan(1/2*d*x)^4*tan(c)^3 - 2*b^2*tan(1/2
*d*x)^4*tan(c)^3 - 8*a^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 12*b
^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 8*a^2*tan(1/2*d*x)^3*tan(1
/2*c)*tan(c)^3 - 8*b^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 8*a^2*tan(1/2*d
*x)*tan(1/2*c)^3*tan(c)^3 - 8*b^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 2*a^
2*tan(1/2*c)^4*tan(c)^3 - 2*b^2*tan(1/2*c)^4*tan(c)^3 + 2*a^2*d*x*tan(1/2*d
*x)^4 + 3*b^2*d*x*tan(1/2*d*x)^4 + 8*a*b*tan(d*x)^2*tan(1/2*d*x)^4 + 8*a^2*
d*x*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 12*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)
*tan(1/2*c) + 8*a^2*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 12*b^2*d*x*tan(1/2*d*x)
^3*tan(1/2*c) + 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c) + 96*a*b*tan(d*
x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3 + 12
*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)^3 + 32*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2
*c)^3 - 32*a*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + 2*a^2*d*x*tan(1/2*c)^4 + 3*b^2
*d*x*tan(1/2*c)^4 + 8*a*b*tan(d*x)^2*tan(1/2*c)^4 + 2*a^2*d*x*tan(d*x)^3*ta
n(c) + 3*b^2*d*x*tan(d*x)^3*tan(c) - 8*a*b*tan(d*x)*tan(1/2*d*x)^4*tan(c) -
 8*a^2*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 12*b^2*d*x*tan(d*x)*ta
n(1/2*d*x)*tan(1/2*c)*tan(c) + 32*a*b*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*ta
n(c) - 32*a*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 96*a*b*tan(d*x)*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 32*a*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)
^3*tan(c) - 8*a*b*tan(d*x)*tan(1/2*c)^4*tan(c) - 2*a^2*d*x*tan(d*x)^2*tan(c
)^2 - 3*b^2*d*x*tan(d*x)^2*tan(c)^2 + 8*a*b*tan(1/2*d*x)^4*tan(c)^2 + 8*a^2
*d*x*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 12*b^2*d*x*tan(1/2*d*x)*tan(1/2*c)*
tan(c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 32*a*b*tan(
1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^
2 + 32*a*b*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + 8*a*b*tan(1/2*c)^4*tan(c)^2
 + 2*a^2*d*x*tan(d*x)*tan(c)^3 + 3*b^2*d*x*tan(d*x)*tan(c)^3 - 8*a*b*tan(d*
x)^3*tan(c)^3 + 32*a*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 2*a^2*ta
n(d*x)*tan(1/2*d*x)^4 - 3*b^2*tan(d*x)*tan(1/2*d*x)^4 - 8*a^2*tan(d*x)^3*ta
n(1/2*d*x)*tan(1/2*c) - 8*b^2*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c) - 8*a^2*ta
n(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 12*b^2*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c
) - 8*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3 - 12*b^2*tan(d*x)*tan(1/2*d*x)
*tan(1/2*c)^3 - 2*a^2*tan(d*x)*tan(1/2*c)^4 - 3*b^2*tan(d*x)*tan(1/2*c)^4 -
 2*a^2*tan(1/2*d*x)^4*tan(c) - 3*b^2*tan(1/2*d*x)^4*tan(c) - 8*a^2*tan(d*x)
^2*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 8*a^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)
- 12*b^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 8*a^2*tan(1/2*d*x)*tan(1/2*c)^3
*tan(c) - 12*b^2*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - 2*a^2*tan(1/2*c)^4*tan(
c) - 3*b^2*tan(1/2*c)^4*tan(c) + 2*a^2*tan(d*x)^3*tan(c)^2 + 3*b^2*tan(d*x)
^3*tan(c)^2 - 8*a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 2*a^2*tan(d

*x)^2*tan(c)^3 + 3*b^2*tan(d*x)^2*tan(c)^3 - 8*a^2*tan(1/2*d*x)*tan(1/2*c)*
tan(c)^3 - 8*b^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 2*a^2*d*x*tan(d*x)^2 -
3*b^2*d*x*tan(d*x)^2 + 8*a*b*tan(1/2*d*x)^4 + 8*a^2*d*x*tan(1/2*d*x)*tan(1/
2*c) + 12*b^2*d*x*tan(1/2*d*x)*tan(1/2*c) - 32*a*b*tan(d*x)^2*tan(1/2*d*x)*
tan(1/2*c) + 32*a*b*tan(1/2*d*x)^3*tan(1/2*c) + 96*a*b*tan(1/2*d*x)^2*tan(1
/2*c)^2 + 32*a*b*tan(1/2*d*x)*tan(1/2*c)^3 + 8*a*b*tan(1/2*c)^4 + 2*a^2*d*x
*tan(d*x)*tan(c) + 3*b^2*d*x*tan(d*x)*tan(c) - 8*a*b*tan(d*x)^3*tan(c) + 32
*a*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 2*a^2*d*x*tan(c)^2 - 3*b^2*d
*x*tan(c)^2 + 8*a*b*tan(d*x)^2*tan(c)^2 - 32*a*b*tan(1/2*d*x)*tan(1/2*c)*ta
n(c)^2 - 8*a*b*tan(d*x)*tan(c)^3 + 2*a^2*tan(d*x)^3 + 2*b^2*tan(d*x)^3 - 8*
a^2*tan(d*x)*tan(1/2*d*x)*tan(1/2*c) - 12*b^2*tan(d*x)*tan(1/2*d*x)*tan(1/2
*c) + 2*a^2*tan(d*x)^2*tan(c) - 8*a^2*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 12*b
^2*tan(1/2*d*x)*tan(1/2*c)*tan(c) + 2*a^2*tan(d*x)*tan(c)^2 + 2*a^2*tan(c)^
3 + 2*b^2*tan(c)^3 - 2*a^2*d*x - 3*b^2*d*x + 8*a*b*tan(d*x)^2 - 32*a*b*tan(
1/2*d*x)*tan(1/2*c) - 8*a*b*tan(d*x)*tan(c) + 8*a*b*tan(c)^2 + 2*a^2*tan(d*
x) + 3*b^2*tan(d*x) + 2*a^2*tan(c) + 3*b^2*tan(c) + 8*a*b)/(d*tan(d*x)^3*ta
n(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + d*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)
^4*tan(c) - d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 4*d*tan(d*x
)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + d*tan(d*x)*tan(1/2*d*x)^4*tan(1/
2*c)^4*tan(c)^3 - d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 4*d*tan(d*x)^3
*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + d*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^
4*tan(c) + 4*d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - d*tan(1/2*
d*x)^4*tan(1/2*c)^4*tan(c)^2 - d*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)^3 - 4*d*t
an(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)^3*tan(1/2*d*x)*
tan(1/2*c)^3*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 -
d*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 + 4*d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2
*c)^3 - d*tan(1/2*d*x)^4*tan(1/2*c)^4 - d*tan(d*x)^3*tan(1/2*d*x)^4*tan(c)
- 4*d*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)^3*tan(1/2*
d*x)*tan(1/2*c)^3*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)
- d*tan(d*x)^3*tan(1/2*c)^4*tan(c) + d*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 +
4*d*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^2 + 4*d*tan(d*x)^2*tan(1/2
*d*x)*tan(1/2*c)^3*tan(c)^2 + 4*d*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + d*
tan(d*x)^2*tan(1/2*c)^4*tan(c)^2 - d*tan(d*x)*tan(1/2*d*x)^4*tan(c)^3 - 4*d
*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)^3*
tan(1/2*c)*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - d*t
an(d*x)*tan(1/2*c)^4*tan(c)^3 + d*tan(d*x)^2*tan(1/2*d*x)^4 + 4*d*tan(d*x)^
2*tan(1/2*d*x)^3*tan(1/2*c) + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^3 + 4*
d*tan(1/2*d*x)^3*tan(1/2*c)^3 + d*tan(d*x)^2*tan(1/2*c)^4 - d*tan(d*x)*tan(
1/2*d*x)^4*tan(c) - 4*d*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)*tan(c) - 4*d*tan
(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*
c)^3*tan(c) - d*tan(d*x)*tan(1/2*c)^4*tan(c) + d*tan(1/2*d*x)^4*tan(c)^2 +
4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + 4*d*tan(1/2*d*x)^3*tan(1/
2*c)*tan(c)^2 + 4*d*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^2 + d*tan(1/2*c)^4*tan
(c)^2 + d*tan(d*x)^3*tan(c)^3 - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c)
^3 + d*tan(1/2*d*x)^4 + 4*d*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c) + 4*d*tan(1/

```
2*d*x)^3*tan(1/2*c) + 4*d*tan(1/2*d*x)*tan(1/2*c)^3 + d*tan(1/2*c)^4 + d*ta
n(d*x)^3*tan(c) - 4*d*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*tan(c) - d*tan(d*x)^
2*tan(c)^2 + 4*d*tan(1/2*d*x)*tan(1/2*c)*tan(c)^2 + d*tan(d*x)*tan(c)^3 - d
*tan(d*x)^2 + 4*d*tan(1/2*d*x)*tan(1/2*c) + d*tan(d*x)*tan(c) - d*tan(c)^2
- d)
```

3.157 $\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=78

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

```
[Out] -(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x
])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0855624, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473}

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*x
])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)+2ab\cos(c+dx)\cot(c+dx)+a^2\cot^2(c+dx)\right)dx$$

$$= a^2\int \cot^2(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot(c+dx)\,dx+b^2\int \cos^2(c+dx)\,d$$

$$= -\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}-a^2\int 1\,dx+\frac{1}{2}b^2\int 1\,dx-\frac{(2ab)\,\mathrm{S}}{}$$

$$= -a^2x+\frac{b^2x}{2}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}-$$

$$= -a^2x+\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2}{}$$

Mathematica [A] time = 0.416328, size = 116, normalized size = 1.49

$$\frac{2a^2\tan\left(\frac{1}{2}(c+dx)\right)-2a^2\cot\left(\frac{1}{2}(c+dx)\right)-4a^2c-4a^2dx+8ab\cos(c+dx)+8ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-8ab\log\left(\cos\left(\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*b*Cos[c + d*x] - 2*a^2*Co
t[(c + d*x)/2] - 8*a*b*Log[Cos[(c + d*x)/2]] + 8*a*b*Log[Sin[(c + d*x)/2]]
+ b^2*Sin[2*(c + d*x)] + 2*a^2*Tan[(c + d*x)/2])/(4*d)
```

Maple [A] time = 0.039, size = 102, normalized size = 1.3

$$-a^2x-\frac{a^2\cot(dx+c)}{d}-\frac{a^2c}{d}+2\,\frac{ab\cos(dx+c)}{d}+2\,\frac{ab\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{b^2\cos(dx+c)\sin(dx+c)}{2\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -a^2*x-a^2*cot(d*x+c)/d-1/d*a^2*c+2*a*b*cos(d*x+c)/d+2/d*a*b*ln(csc(d*x+c)-
cot(d*x+c))+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d+1/2*b^2*x+1/2/d*b^2*c
```

Maxima [A] time = 2.59493, size = 107, normalized size = 1.37

$$-\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-(2\,dx+2\,c+\sin(2\,dx+2\,c))b^2-4\,ab(2\,\cos(dx+c)-\log(\cos(dx+c)+1)+\log(\cos(dx}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(d*x + c + 1/tan(d*x + c))*a^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*b
^2 - 4*a*b*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1))
)/d
```

Fricas [A] time = 1.64996, size = 308, normalized size = 3.95

$$-\frac{b^2\cos(dx+c)^3+2\,ab\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,ab\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+\left(2\,a^2-b^2\right)\cos}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*cos(d*x + c)^3 + 2*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) -
 2*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 - b^2)*cos(d*x +
c) + ((2*a^2 - b^2)*d*x - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sin(c+dx))^2\cot^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**2, x)
```

Giac [A] time = 2.23738, size = 200, normalized size = 2.56

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\left(2\,a^2-b^2\right)(dx+c)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx\right.\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*a
^2 - b^2)*(d*x + c) - (4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*
```

```
c) - 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.158 $\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=133

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-3$$

```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
 d*x]^3)/(3*d)
```

Rubi [A] time = 0.149396, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 2591, 288, 321, 203, 2592, 206, 3473, 8}

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-3$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x]
)/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*x]
^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[c +
 d*x]^3)/(3*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^2(c+dx)+2ab\cos(c+dx)\cot^3(c+dx)+a^2\cot^4(c+dx\right.$$

$$= a^2\int \cot^4(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^3(c+dx)\,dx+b^2\int \cos^2(c+dx)$$

$$= -\frac{a^2\cot^3(c+dx)}{3d}-a^2\int \cot^2(c+dx)\,dx-\frac{(2ab)\,\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx,x,\cos(c+\right.}{d}$$

$$= \frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}-\frac{a^2}{}$$

$$= a^2x-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot(}{2d}$$

$$= a^2x-\frac{3b^2x}{2}+\frac{3ab\tanh^{-1}(\cos(c+dx))}{d}-\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d}-\frac{3b}{}$$

Mathematica [B] time = 6.23321, size = 293, normalized size = 2.2

$$\frac{(2a^2-3b^2)(c+dx)}{2d}+\frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)}{6d}+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right.}{6d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - 3*b^2)*(c + d*x))/(2*d) - (2*a*b*Cos[c + d*x])/d + ((4*a^2*Cos[(c
 + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (a*b*Csc[(c
+ d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (3*
a*b*Log[Cos[(c + d*x)/2]])/d - (3*a*b*Log[Sin[(c + d*x)/2]])/d + (a*b*Sec[(
c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 3*b^2*S
in[(c + d*x)/2]))/(6*d) - (b^2*Sin[2*(c + d*x)])/(4*d) + (a^2*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A] time = 0.049, size = 199, normalized size = 1.5

$$-\frac{a^2(\cot(dx+c))^3}{3d}+\frac{a^2\cot(dx+c)}{d}+a^2x+\frac{a^2c}{d}-\frac{ab(\cos(dx+c))^5}{d(\sin(dx+c))^2}-\frac{ab(\cos(dx+c))^3}{d}-3\frac{ab\cos(dx+c)}{d}-3\frac{ab\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3*a^2*cot(d*x+c)^3/d+a^2*cot(d*x+c)/d+a^2*x+1/d*a^2*c-1/d*a*b/sin(d*x+c)
^2*cos(d*x+c)^5-1/d*a*b*cos(d*x+c)^3-3*a*b*cos(d*x+c)/d-3/d*a*b*ln(csc(d*x+
c)-cot(d*x+c))-1/d*b^2/sin(d*x+c)*cos(d*x+c)^5-1/d*b^2*sin(d*x+c)*cos(d*x+c
)^3-3/2*b^2*cos(d*x+c)*sin(d*x+c)/d-3/2*b^2*x-3/2/d*b^2*c
```

Maxima [A] time = 2.45315, size = 186, normalized size = 1.4

$$\frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b^2+3\,ab\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*(3*d*x
 + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^2 + 3*a*
b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c
) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.82669, size = 539, normalized size = 4.05

$$\frac{3\,b^2\cos\left(dx+c\right)^5+4\left(2\,a^2-3\,b^2\right)\cos\left(dx+c\right)^3+9\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\left(ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*cos(d*x + c)^5 + 4*(2*a^2 - 3*b^2)*cos(d*x + c)^3 + 9*(a*b*cos(d
*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a*b*cos(d*x
+ c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(2*a^2 - 3*b^2)
*cos(d*x + c) + 3*((2*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)^
3 - (2*a^2 - 3*b^2)*d*x + 6*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c
```

```
)^2 - d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sin(c + dx))^2 \cot^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**4, x)
```

Giac [A] time = 2.05877, size = 325, normalized size = 2.44

$$a^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 6\,ab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 72\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right|\right) - 15\,a^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 12\,b^2\tan\left(\frac{1}{2}\,dx +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 72*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2
*d*x + 1/2*c) + 12*(2*a^2 - 3*b^2)*(d*x + c) + 24*(b^2*tan(1/2*d*x + 1/2*c)
^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(
1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*tan(1/
2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x + 1/2*
c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.159 $\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=202

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\cos}{}$$

```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos
[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a
*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2
*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*C
os[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.169485, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 2591, 288, 302, 203, 2592, 321, 206, 3473, 8}

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Cos
[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5*a
*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*b^2
*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a*b*C
os[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
```

```
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx = \int \left(b^2\cos^2(c+dx)\cot^4(c+dx)+2ab\cos(c+dx)\cot^5(c+dx)+a^2\cot^6(c+d\ldots\right.$$

$$= a^2\int \cot^6(c+dx)\,dx+(2ab)\int \cos(c+dx)\cot^5(c+dx)\,dx+b^2\int \cos^2(c+d\ldots$$

$$= -\frac{a^2\cot^5(c+dx)}{5d}-a^2\int \cot^4(c+dx)\,dx-\frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3}\,dx,x,\cos(c\ldots\right.}{d}$$

$$= \frac{a^2\cot^3(c+dx)}{3d}+\frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}-\ldots$$

$$= -\frac{a^2\cot(c+dx)}{d}+\frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d}+\frac{a^2\cot^3(c+dx)}{3d}+\frac{b^2\cos^2(c\ldots}{}$$

$$= -a^2x+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c+dx)}{d}+\frac{5b^2\cot(c+dx)}{2d}+\frac{5ab\cos(c+dx\ldots}{4d\ldots}$$

$$= -a^2x+\frac{5b^2x}{2}-\frac{15ab\tanh^{-1}(\cos(c+dx))}{4d}+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c+dx)}{d}\ldots$$

Mathematica [A] time = 1.09302, size = 351, normalized size = 1.74

$$\frac{\left(560b^2-368a^2\right)\cot\left(\frac{1}{2}(c+dx)\right)+368a^2\tan\left(\frac{1}{2}(c+dx)\right)+96a^2\sin^6\left(\frac{1}{2}(c+dx)\right)\csc^5(c+dx)-328a^2\sin^4\left(\frac{1}{2}(c+dx)\right)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-480*a^2*c + 1200*b^2*c - 480*a^2*d*x + 1200*b^2*d*x + 960*a*b*Cos[c + d*x
] + (-368*a^2 + 560*b^2)*Cot[(c + d*x)/2] + 270*a*b*Csc[(c + d*x)/2]^2 - 15
*a*b*Csc[(c + d*x)/2]^4 - 1800*a*b*Log[Cos[(c + d*x)/2]] + 1800*a*b*Log[Sin
```

```
[(c + d*x)/2]] - 270*a*b*Sec[(c + d*x)/2]^2 + 15*a*b*Sec[(c + d*x)/2]^4 - 3
28*a^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 160*b^2*Csc[c + d*x]^3*Sin[(c +
d*x)/2]^4 + 96*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + (41*a^2*Csc[(c + d*x
)/2]^4*Sin[c + d*x])/2 - 10*b^2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - (3*a^2*Cs
c[(c + d*x)/2]^6*Sin[c + d*x])/2 + 120*b^2*Sin[2*(c + d*x)] + 368*a^2*Tan[(
c + d*x)/2] - 560*b^2*Tan[(c + d*x)/2])/(480*d)
```

Maple [A] time = 0.052, size = 302, normalized size = 1.5

$$-\frac{a^2(\cot(dx+c))^5}{5d}+\frac{a^2(\cot(dx+c))^3}{3d}-\frac{a^2\cot(dx+c)}{d}-a^2x-\frac{a^2c}{d}-\frac{ab(\cos(dx+c))^7}{2d(\sin(dx+c))^4}+\frac{3ab(\cos(dx+c))^7}{4d(\sin(dx+c))^2}+\frac{3ab(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/5*a^2*cot(d*x+c)^5/d+1/3*a^2*cot(d*x+c)^3/d-a^2*cot(d*x+c)/d-a^2*x-1/d*a
^2*c-1/2/d*a*b/sin(d*x+c)^4*cos(d*x+c)^7+3/4/d*a*b/sin(d*x+c)^2*cos(d*x+c)^
7+3/4/d*a*b*cos(d*x+c)^5+5/4/d*a*b*cos(d*x+c)^3+15/4*a*b*cos(d*x+c)/d+15/4/
d*a*b*ln(csc(d*x+c)-cot(d*x+c))-1/3/d*b^2/sin(d*x+c)^3*cos(d*x+c)^7+4/3/d*b
^2/sin(d*x+c)*cos(d*x+c)^7+4/3/d*b^2*sin(d*x+c)*cos(d*x+c)^5+5/3/d*b^2*sin(
d*x+c)*cos(d*x+c)^3+5/2*b^2*cos(d*x+c)*sin(d*x+c)/d+5/2*b^2*x+5/2/d*b^2*c
```

Maxima [A] time = 1.67638, size = 247, normalized size = 1.22

$$-\frac{8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-20\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)b^2+15\,ab\left(\frac{2\left(9\,\cos(dx+c)^3-7\right.}{\cos(dx+c)^4-2\,\mathrm{co}}\right.}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d
*x + c)^5)*a^2 - 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)^2
 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*b^2 + 15*a*b*(2*(9*cos(d*x + c)^3
- 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
 + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.63935, size = 786, normalized size = 3.89

$$-\frac{60\,b^2\cos\left(dx+c\right)^7+92\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^5-140\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^3+225\left(ab\cos\left(dx+c\right)^4-2\,ab\cos\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/120*(60*b^2*cos(d*x + c)^7 + 92*(2*a^2 - 5*b^2)*cos(d*x + c)^5 - 140*(2*
a^2 - 5*b^2)*cos(d*x + c)^3 + 225*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c)^
2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(a*b*cos(d*x + c)^4
 - 2*a*b*cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
60*(2*a^2 - 5*b^2)*cos(d*x + c) + 30*(2*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^4
- 8*a*b*cos(d*x + c)^5 - 4*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^2 + 25*a*b*cos(
d*x + c)^3 + 2*(2*a^2 - 5*b^2)*d*x - 15*a*b*cos(d*x + c))*sin(d*x + c))/((d
*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.77852, size = 455, normalized size = 2.25

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,ab\tan\left(\frac{1}{2}\,dx+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^
2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*a*b*tan(1/2*
d*x + 1/2*c)^2 + 1800*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 330*a^2*tan(1/2*
d*x + 1/2*c) - 540*b^2*tan(1/2*d*x + 1/2*c) - 240*(2*a^2 - 5*b^2)*(d*x + c)
 - 480*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan
(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (4110*a*b*tan(1
/2*d*x + 1/2*c)^5 + 330*a^2*tan(1/2*d*x + 1/2*c)^4 - 540*b^2*tan(1/2*d*x +
1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 35*a^2*tan(1/2*d*x + 1/2*c)^2 +
 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a^2)/tan(1
/2*d*x + 1/2*c)^5)/d
```

3.160 $\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$

Optimal. Leaf size=150

$$\frac{b\left(6a^2+5b^2\right)\sin(c+dx)}{2d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-5b)(a-b)^2\log(\sin(c+}{4d}$$

```
[Out] ((a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 5*b)*(a - b)^
2*Log[1 + Sin[c + d*x]])/(4*d) + (b*(6*a^2 + 5*b^2)*Sin[c + d*x])/(2*d) + (
3*a*b^2*Sin[c + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d) + (Sec[c + d*x]^
2*(a + b*Sin[c + d*x])^3)/(2*d)
```

Rubi [A] time = 0.241513, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2721, 1645, 1629, 633, 31}

$$\frac{b\left(6a^2+5b^2\right)\sin(c+dx)}{2d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-5b)(a-b)^2\log(\sin(c+}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] ((a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]])/(4*d) + ((2*a - 5*b)*(a - b)^
2*Log[1 + Sin[c + d*x]])/(4*d) + (b*(6*a^2 + 5*b^2)*Sin[c + d*x])/(2*d) + (
3*a*b^2*Sin[c + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d) + (Sec[c + d*x]^
2*(a + b*Sin[c + d*x])^3)/(2*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1645

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[Pq, a + c*x^2, x], f = Coeff[PolynomialRemai
nder[Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + c*x^2,
 x], x, 1]}, Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p
 + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p +
```

```
 1)*ExpandToSum[2*a*c*(p + 1)*(d + e*x)*Q - a*e*g*m + c*d*f*(2*p + 3) + c*e
*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && Rati
onalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x^3(a+x)^3}{\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^3}{2d} + \frac{\text{Subst}\left(\int \frac{(a+x)^2\left(-3b^4-2ab^2x-2b^2x^2\right)}{b^2-x^2}\,dx, x, b\sin(c\right.}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a+b\sin(c+dx))^3}{2d} + \frac{\text{Subst}\left(\int \left(6a^2b^2+5b^4+6ab^2x+2b^2x^2-\frac{9a}{}\right.\right.}{2b^2d}$$

$$= \frac{b\left(6a^2+5b^2\right)\sin(c+dx)}{2d} + \frac{3ab^2\sin^2(c+dx)}{2d} + \frac{b^3\sin^3(c+dx)}{3d} + \frac{\sec^2(c+dx)(}{}$$

$$= \frac{b\left(6a^2+5b^2\right)\sin(c+dx)}{2d} + \frac{3ab^2\sin^2(c+dx)}{2d} + \frac{b^3\sin^3(c+dx)}{3d} + \frac{\sec^2(c+dx)(}{}$$

$$= \frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d} + \frac{(2a-5b)(a-b)^2\log(1+\sin(c+dx))}{4d} +$$

Mathematica [A] time = 0.263131, size = 141, normalized size = 0.94

$$\frac{12b\left(3a^2+2b^2\right)\sin(c+dx)+18ab^2\sin^2(c+dx)+\frac{3(a-b)^3}{\sin(c+dx)+1}-\frac{3(a+b)^3}{\sin(c+dx)-1}+3(2a-5b)(a-b)^2\log(\sin(c+dx)+1)+3}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

```
[Out] (3*(a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]] + 3*(2*a - 5*b)*(a - b)^2*Lo
g[1 + Sin[c + d*x]] - (3*(a + b)^3)/(-1 + Sin[c + d*x]) + 12*b*(3*a^2 + 2*b
^2)*Sin[c + d*x] + 18*a*b^2*Sin[c + d*x]^2 + 4*b^3*Sin[c + d*x]^3 + (3*(a -
 b)^3)/(1 + Sin[c + d*x]))/(12*d)
```

Maple [B] time = 0.056, size = 279, normalized size = 1.9

$$\frac{a^3(\tan(dx+c))^2}{2d}+\frac{a^3\ln(\cos(dx+c))}{d}+\frac{3a^2b(\sin(dx+c))^5}{2d(\cos(dx+c))^2}+\frac{3a^2b(\sin(dx+c))^3}{2d}+\frac{9a^2b\sin(dx+c)}{2d}-\frac{9a^2b\ln(\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x)
```

```
[Out] 1/2/d*a^3*tan(d*x+c)^2+1/d*a^3*ln(cos(d*x+c))+3/2/d*a^2*b*sin(d*x+c)^5/cos(
d*x+c)^2+3/2/d*a^2*b*sin(d*x+c)^3+9/2*a^2*b*sin(d*x+c)/d-9/2/d*a^2*b*ln(sec
(d*x+c)+tan(d*x+c))+3/2/d*a*b^2*sin(d*x+c)^6/cos(d*x+c)^2+3/2/d*a*b^2*sin(d
*x+c)^4+3*a*b^2*sin(d*x+c)^2/d+6/d*a*b^2*ln(cos(d*x+c))+1/2/d*b^3*sin(d*x+c
)^7/cos(d*x+c)^2+1/2/d*b^3*sin(d*x+c)^5+5/6*b^3*sin(d*x+c)^3/d+5/2/d*b^3*si
n(d*x+c)-5/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.54646, size = 219, normalized size = 1.46

$$\frac{4b^3\sin(dx+c)^3+18ab^2\sin(dx+c)^2+3\left(2a^3-9a^2b+12ab^2-5b^3\right)\log(\sin(dx+c)+1)+3\left(2a^3+9a^2b+12ab^2\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 3*(2*a^3 - 9*a^2*b +
 12*a*b^2 - 5*b^3)*log(sin(d*x + c) + 1) + 3*(2*a^3 + 9*a^2*b + 12*a*b^2 +
5*b^3)*log(sin(d*x + c) - 1) + 12*(3*a^2*b + 2*b^3)*sin(d*x + c) - 6*(a^3 +
 3*a*b^2 + (3*a^2*b + b^3)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 1.72138, size = 470, normalized size = 3.13

$$-\frac{18\,ab^2\cos\left(dx+c\right)^4-9\,ab^2\cos\left(dx+c\right)^2-3\left(2\,a^3-9\,a^2b+12\,ab^2-5\,b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(18*a*b^2*cos(d*x + c)^4 - 9*a*b^2*cos(d*x + c)^2 - 3*(2*a^3 - 9*a^2*
b + 12*a*b^2 - 5*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^3 + 9*a
^2*b + 12*a*b^2 + 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 6*a^3 - 18
*a*b^2 + 2*(2*b^3*cos(d*x + c)^4 - 9*a^2*b - 3*b^3 - 2*(9*a^2*b + 7*b^3)*co
s(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.161 $\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx$

Optimal. Leaf size=105

$$-\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\sin^3(c+dx)}{3d}$$

```
[Out] -((a + b)^3*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (b*(3*a^2 + b^2)*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d*x]^2)/(2*d)
 - (b^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.113594, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2721, 801, 633, 31}

$$-\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{(a-b)^3\log(\sin(c+dx)+1)}{2d}-\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```


```
[Out] -((a + b)^3*Log[1 - Sin[c + d*x]])/(2*d) - ((a - b)^3*Log[1 + Sin[c + d*x]]
)/(2*d) - (b*(3*a^2 + b^2)*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d*x]^2)/(2*d)
 - (b^3*Sin[c + d*x]^3)/(3*d)
```


Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+x)^3}{b^2-x^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^2-b^2-3ax-x^2+\frac{3a^2b^2+b^4+a\left(a^2+3b^2\right)x}{b^2-x^2}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{b^3\sin^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{3a^2}{}\right.}{}$$
$$= -\frac{b\left(3a^2+b^2\right)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{b^3\sin^3(c+dx)}{3d}+\frac{(a-b)^3\,\text{Sub}}{}$$
$$= -\frac{(a+b)^3\log(1-\sin(c+dx))}{2d}-\frac{(a-b)^3\log(1+\sin(c+dx))}{2d}-\frac{b\left(3a^2+b^2\right)\sin}{d}$$

Mathematica [A] time = 0.189961, size = 90, normalized size = 0.86

$$-\frac{6b\left(3a^2+b^2\right)\sin(c+dx)+9ab^2\sin^2(c+dx)+3\left((a-b)^3\log(\sin(c+dx)+1)+(a+b)^3\log(1-\sin(c+dx))\right)+2b^3\text{s}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

```
[Out] -(3*((a + b)^3*Log[1 - Sin[c + d*x]] + (a - b)^3*Log[1 + Sin[c + d*x]]) + 6
*b*(3*a^2 + b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2*b^3*Sin[c + d*x]
^3)/(6*d)
```

Maple [A] time = 0.046, size = 139, normalized size = 1.3

$$-\frac{a^3\ln(\cos(dx+c))}{d}-3\,\frac{a^2b\sin(dx+c)}{d}+3\,\frac{a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{3\,(\sin(dx+c))^2\,ab^2}{2\,d}-3\,\frac{ab^2\ln(\cos(\ldots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c),x)
```

```
[Out] -1/d*a^3*ln(cos(d*x+c))-3*a^2*b*sin(d*x+c)/d+3/d*a^2*b*ln(sec(d*x+c)+tan(d*
x+c))-3/2*a*b^2*sin(d*x+c)^2/d-3/d*a*b^2*ln(cos(d*x+c))-1/3*b^3*sin(d*x+c)^
3/d-1/d*b^3*sin(d*x+c)+1/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.70671, size = 153, normalized size = 1.46

$$-\frac{2\,b^3\sin(dx+c)^3+9\,ab^2\sin(dx+c)^2+3\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log(\sin(dx+c)+1)+3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\mathrm{lo}\ldots}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*log(sin(d*x + c) + 1) + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(
sin(d*x + c) - 1) + 6*(3*a^2*b + b^3)*sin(d*x + c))/d
```

Fricas [A] time = 1.60646, size = 277, normalized size = 2.64

$$\frac{9\,ab^2\cos(dx+c)^2-3\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log(\sin(dx+c)+1)-3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log(-\sin(dx+c)+1)\ldots}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(9*a*b^2*cos(d*x + c)^2 - 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(sin(d*x
 + c) + 1) - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(-sin(d*x + c) + 1) + 2*(
b^3*cos(d*x + c)^2 - 9*a^2*b - 4*b^3)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c),x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.162 $\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{3a^2b\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x
]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0449595, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 43}

$$\frac{3a^2b\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x
]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3}{x}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(3a^2+\frac{a^3}{x}+3ax+x^2\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0252867, size = 67, normalized size = 1.

$$\frac{3a^2b\sin(c+dx)}{d}+\frac{a^3\log(\sin(c+dx))}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x
]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.027, size = 64, normalized size = 1.

$$\frac{a^3\ln(\sin(dx+c))}{d}+3\frac{a^2b\sin(dx+c)}{d}+\frac{3\,(\sin(dx+c))^2\,ab^2}{2\,d}+\frac{(\sin(dx+c))^3\,b^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c))^3,x)
```

```
[Out] a^3*ln(sin(d*x+c))/d+3*a^2*b*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*
sin(d*x+c)^3/d
```

Maxima [A] time = 1.56722, size = 77, normalized size = 1.15

$$\frac{2\,b^3\sin(dx+c)^3+9\,ab^2\sin(dx+c)^2+6\,a^3\log(\sin(dx+c))+18\,a^2b\sin(dx+c)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c)
) + 18*a^2*b*sin(d*x + c))/d
```

Fricas [A] time = 1.49174, size = 158, normalized size = 2.36

$$-\frac{9\,ab^2\cos\left(dx+c\right)^2-6\,a^3\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+2\left(b^3\cos\left(dx+c\right)^2-9\,a^2b-b^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a*b^2*cos(d*x + c)^2 - 6*a^3*log(1/2*sin(d*x + c)) + 2*(b^3*cos(d*x
 + c)^2 - 9*a^2*b - b^3)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+dx\right)\right)^3\cot\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x), x)
```

Giac [A] time = 1.99966, size = 78, normalized size = 1.16

$$\frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+6\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+18\,a^2b\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x
+ c))) + 18*a^2*b*sin(d*x + c))/d
```

3.163 $\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=116

$$-\frac{b(3a^2-b^2)\sin(c+dx)}{d}-\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)}{2d}-b^3\,\mathrm{s}$$

```
[Out] (-3*a^2*b*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*L
og[Sin[c + d*x]])/d - (b*(3*a^2 - b^2)*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d
*x]^2)/(2*d) - (b^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0943571, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{b(3a^2-b^2)\sin(c+dx)}{d}-\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}-\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)}{2d}-b^3\,\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*L
og[Sin[c + d*x]])/d - (b*(3*a^2 - b^2)*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d
*x]^2)/(2*d) - (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3(b^2-x^2)}{x^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^2\left(1-\frac{b^2}{3a^2}\right)+\frac{a^3b^2}{x^3}+\frac{3a^2b^2}{x^2}+\frac{-a^3+3ab^2}{x}-3ax-x^2\right)dx, x, b\sin(c+\right.}{d}$$
$$= -\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a\left(a^2-3b^2\right)\log(\sin(c+dx))}{d}-\frac{b\left(3a^2-\right.}{}$$

Mathematica [A] time = 0.320514, size = 97, normalized size = 0.84

$$-\frac{-6b\left(b^2-3a^2\right)\sin(c+dx)+6a\left(a^2-3b^2\right)\log(\sin(c+dx))+18a^2b\csc(c+dx)+3a^3\csc^2(c+dx)+9ab^2\sin^2(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(18*a^2*b*Csc[c + d*x] + 3*a^3*Csc[c + d*x]^2 + 6*a*(a^2 - 3*b^2)*Log[Sin[
c + d*x]] - 6*b*(-3*a^2 + b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2*b^
3*Sin[c + d*x]^3)/(6*d)
```

Maple [A] time = 0.065, size = 165, normalized size = 1.4

$$-\frac{a^3(\cot(dx+c))^2}{2d}-\frac{a^3\ln(\sin(dx+c))}{d}-3\frac{a^2b(\cos(dx+c))^4}{d\sin(dx+c)}-3\frac{a^2b\sin(dx+c)(\cos(dx+c))^2}{d}-6\frac{a^2b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3*cot(d*x+c)^2-a^3*ln(sin(d*x+c))/d-3/d*a^2*b/sin(d*x+c)*cos(d*x+c
)^4-3/d*a^2*b*sin(d*x+c)*cos(d*x+c)^2-6*a^2*b*sin(d*x+c)/d+3/2/d*a*b^2*cos(
d*x+c)^2+3/d*a*b^2*ln(sin(d*x+c))+1/3/d*b^3*cos(d*x+c)^2*sin(d*x+c)+2/3/d*b
^3*sin(d*x+c)
```

Maxima [A] time = 1.89972, size = 132, normalized size = 1.14

$$-\frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+6\left(a^3-3\,ab^2\right)\log\left(\sin\left(dx+c\right)\right)+6\left(3\,a^2b-b^3\right)\sin\left(dx+c\right)+\frac{3\left(6\,a^2b\sin(dx+c)+a^3\right)}{\sin(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*(a^3 - 3*a*b^2)*log
(sin(d*x + c)) + 6*(3*a^2*b - b^3)*sin(d*x + c) + 3*(6*a^2*b*sin(d*x + c) +
 a^3)/sin(d*x + c)^2)/d
```

Fricas [A] time = 1.56308, size = 358, normalized size = 3.09

$$\frac{18\,ab^2\cos\left(dx+c\right)^4-27\,ab^2\cos\left(dx+c\right)^2+6\,a^3+9\,ab^2+12\left(a^3-3\,ab^2-\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)}{12\left(d\cos\left(dx+c\right)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*cos(d*x + c)^4 - 27*a*b^2*cos(d*x + c)^2 + 6*a^3 + 9*a*b^2 +
 12*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))
+ 4*(b^3*cos(d*x + c)^4 + 18*a^2*b - 2*b^3 - (9*a^2*b - b^3)*cos(d*x + c)^2
)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(c+dx\right)\right)^3\cot^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**3, x)
```

Giac [A] time = 2.0221, size = 177, normalized size = 1.53

$$-\frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+18\,a^2b\sin\left(dx+c\right)-6\,b^3\sin\left(dx+c\right)+6\left(a^3-3\,ab^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{3(3}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 18*a^2*b*sin(d*x + c)
 - 6*b^3*sin(d*x + c) + 6*(a^3 - 3*a*b^2)*log(abs(sin(d*x + c))) - 3*(3*a^3
*sin(d*x + c)^2 - 9*a*b^2*sin(d*x + c)^2 - 6*a^2*b*sin(d*x + c) - a^3)/sin(
d*x + c)^2)/d
```

3.164 $\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=165

$$\frac{b(3a^2-2b^2)\sin(c+dx)}{d}+\frac{a(2a^2-3b^2)\csc^2(c+dx)}{2d}+\frac{b(6a^2-b^2)\csc(c+dx)}{d}+\frac{a(a^2-6b^2)\log(\sin(c+dx))}{d}-\frac{a^2b\,\mathrm{cs}}{}$$

```
[Out] (b*(6*a^2 - b^2)*Csc[c + d*x])/d + (a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2*d)
 - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Csc[c + d*x]^4)/(4*d) + (a*(a^2 - 6*b^2)
*Log[Sin[c + d*x]])/d + (b*(3*a^2 - 2*b^2)*Sin[c + d*x])/d + (3*a*b^2*Sin[c
 + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.14103, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 948}

$$\frac{b(3a^2-2b^2)\sin(c+dx)}{d}+\frac{a(2a^2-3b^2)\csc^2(c+dx)}{2d}+\frac{b(6a^2-b^2)\csc(c+dx)}{d}+\frac{a(a^2-6b^2)\log(\sin(c+dx))}{d}-\frac{a^2b\,\mathrm{cs}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (b*(6*a^2 - b^2)*Csc[c + d*x])/d + (a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2*d)
 - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Csc[c + d*x]^4)/(4*d) + (a*(a^2 - 6*b^2)
*Log[Sin[c + d*x]])/d + (b*(3*a^2 - 2*b^2)*Sin[c + d*x])/d + (3*a*b^2*Sin[c
 + d*x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
```

```
& EqQ[d, 0]))
```

Rubi steps

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\text{Subst}\left(\int \frac{(a+x)^3\left(b^2-x^2\right)^2}{x^5}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)+\frac{a^3b^4}{x^5}+\frac{3a^2b^4}{x^4}+\frac{-2a^3b^2+3ab^4}{x^3}+\frac{-6a^2b^2+b^4}{x^2}+\frac{a^3-6ab^2}{x}+3a\right.}{d}$$

$$= \frac{b\left(6a^2-b^2\right)\csc(c+dx)}{d}+\frac{a\left(2a^2-3b^2\right)\csc^2(c+dx)}{2d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{a^3}{}$$

Mathematica [A] time = 1.16383, size = 144, normalized size = 0.87

$$\frac{6a\left(2a^2-3b^2\right)\csc^2(c+dx)-12b\left(b^2-6a^2\right)\csc(c+dx)+2\left(6b\left(3a^2-2b^2\right)\sin(c+dx)+6a\left(a^2-6b^2\right)\log(\sin(c+dx))\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-12*b*(-6*a^2 + b^2)*Csc[c + d*x] + 6*a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2 - 1
2*a^2*b*Csc[c + d*x]^3 - 3*a^3*Csc[c + d*x]^4 + 2*(6*a*(a^2 - 6*b^2)*Log[Si
n[c + d*x]] + 6*b*(3*a^2 - 2*b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2
*b^3*Sin[c + d*x]^3))/(12*d)
```

Maple [B] time = 0.062, size = 316, normalized size = 1.9

$$-\frac{a^3(\cot(dx+c))^4}{4d}+\frac{a^3(\cot(dx+c))^2}{2d}+\frac{a^3\ln(\sin(dx+c))}{d}-\frac{a^2b(\cos(dx+c))^6}{d(\sin(dx+c))^3}+3\frac{a^2b(\cos(dx+c))^6}{d\sin(dx+c)}+8\frac{a^2b\sin(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/4/d*a^3*cot(d*x+c)^4+1/2/d*a^3*cot(d*x+c)^2+a^3*ln(sin(d*x+c))/d-1/d*a^2
*b/sin(d*x+c)^3*cos(d*x+c)^6+3/d*a^2*b/sin(d*x+c)*cos(d*x+c)^6+8*a^2*b*sin(
```

```
d*x+c)/d+3/d*a^2*b*sin(d*x+c)*cos(d*x+c)^4+4/d*a^2*b*sin(d*x+c)*cos(d*x+c)^
2-3/2/d*a*b^2/sin(d*x+c)^2*cos(d*x+c)^6-3/2/d*a*b^2*cos(d*x+c)^4-3/d*a*b^2*
cos(d*x+c)^2-6/d*a*b^2*ln(sin(d*x+c))-1/d*b^3/sin(d*x+c)*cos(d*x+c)^6-8/3/d
*b^3*sin(d*x+c)-1/d*b^3*sin(d*x+c)*cos(d*x+c)^4-4/3/d*b^3*cos(d*x+c)^2*sin(
d*x+c)
```

<u>Maxima</u> [A] time = 1.35412, size = 192, normalized size = 1.16

$$\frac{4\,b^3\sin\left(dx+c\right)^3+18\,ab^2\sin\left(dx+c\right)^2+12\left(a^3-6\,ab^2\right)\log\left(\sin\left(dx+c\right)\right)+12\left(3\,a^2b-2\,b^3\right)\sin\left(dx+c\right)-\frac{3\left(4\,a^2b\sin(dx+c)\right.}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 12*(a^3 - 6*a*b^2)*l
og(sin(d*x + c)) + 12*(3*a^2*b - 2*b^3)*sin(d*x + c) - 3*(4*a^2*b*sin(d*x +
 c) - 4*(6*a^2*b - b^3)*sin(d*x + c)^3 + a^3 - 2*(2*a^3 - 3*a*b^2)*sin(d*x
+ c)^2)/sin(d*x + c)^4)/d
```

<u>Fricas</u> [A] time = 1.60026, size = 536, normalized size = 3.25

$$-\frac{18\,ab^2\cos\left(dx+c\right)^6-45\,ab^2\cos\left(dx+c\right)^4-9\,a^3+9\,ab^2+6\left(2\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^2-12\left(\left(a^3-6\,ab^2\right)\cos\left(dx+c\right)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(18*a*b^2*cos(d*x + c)^6 - 45*a*b^2*cos(d*x + c)^4 - 9*a^3 + 9*a*b^2
+ 6*(2*a^3 + 3*a*b^2)*cos(d*x + c)^2 - 12*((a^3 - 6*a*b^2)*cos(d*x + c)^4 +
 a^3 - 6*a*b^2 - 2*(a^3 - 6*a*b^2)*cos(d*x + c)^2)*log(1/2*sin(d*x + c)) +
4*(b^3*cos(d*x + c)^6 - 3*(3*a^2*b - b^3)*cos(d*x + c)^4 - 24*a^2*b + 8*b^3
 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4 - 2*d
*cos(d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.1406, size = 250, normalized size = 1.52

$$\frac{4\,b^3\sin\left(dx+c\right)^3+18\,ab^2\sin\left(dx+c\right)^2+36\,a^2b\sin\left(dx+c\right)-24\,b^3\sin\left(dx+c\right)+12\left(a^3-6\,ab^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\ldots}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 36*a^2*b*sin(d*x + c
) - 24*b^3*sin(d*x + c) + 12*(a^3 - 6*a*b^2)*log(abs(sin(d*x + c))) - (25*a
^3*sin(d*x + c)^4 - 150*a*b^2*sin(d*x + c)^4 - 72*a^2*b*sin(d*x + c)^3 + 12
*b^3*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 18*a*b^2*sin(d*x + c)^2 + 12*
a^2*b*sin(d*x + c) + 3*a^3)/sin(d*x + c)^4)/d
```

3.165 $\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$

Optimal. Leaf size=220

$$-\frac{3a^2b\cos(c+dx)}{d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{6a^2b\sec(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x+\frac{5ab^2\tan^3(c+dx)}{2d}$$

```
[Out] a^3*x + (15*a*b^2*x)/2 - (3*a^2*b*Cos[c + d*x])/d - (3*b^3*Cos[c + d*x])/d
+ (b^3*Cos[c + d*x]^3)/(3*d) - (6*a^2*b*Sec[c + d*x])/d - (3*b^3*Sec[c + d*
x])/d + (a^2*b*Sec[c + d*x]^3)/d + (b^3*Sec[c + d*x]^3)/(3*d) - (a^3*Tan[c
+ d*x])/d - (15*a*b^2*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (5
*a*b^2*Tan[c + d*x]^3)/(2*d) - (3*a*b^2*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d
)
```

Rubi [A] time = 0.224299, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 3473, 8, 2590, 270, 2591, 288, 302, 203}

$$-\frac{3a^2b\cos(c+dx)}{d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{6a^2b\sec(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x+\frac{5ab^2\tan^3(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```


```
[Out] a^3*x + (15*a*b^2*x)/2 - (3*a^2*b*Cos[c + d*x])/d - (3*b^3*Cos[c + d*x])/d
+ (b^3*Cos[c + d*x]^3)/(3*d) - (6*a^2*b*Sec[c + d*x])/d - (3*b^3*Sec[c + d*
x])/d + (a^2*b*Sec[c + d*x]^3)/d + (b^3*Sec[c + d*x]^3)/(3*d) - (a^3*Tan[c
+ d*x])/d - (15*a*b^2*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d) + (5
*a*b^2*Tan[c + d*x]^3)/(2*d) - (3*a*b^2*Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*d
)
```


Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx = \int \left(a^3\tan^4(c+dx)+3a^2b\sin(c+dx)\tan^4(c+dx)+3ab^2\sin^2(c+dx)\tan^4(c+$$

$$= a^3\int \tan^4(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan^4(c+dx)\,dx+\left(3ab^2\right)\int \sin^2(c$$

$$= \frac{a^3\tan^3(c+dx)}{3d}-a^3\int \tan^2(c+dx)\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4}\,dx,x,\cos(c+\right.}{d}$$

$$= -\frac{a^3\tan(c+dx)}{d}+\frac{a^3\tan^3(c+dx)}{3d}-\frac{3ab^2\sin^2(c+dx)\tan^3(c+dx)}{2d}+a^3\int 1\,d$$

$$= a^3x-\frac{3a^2b\cos(c+dx)}{d}-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}-\frac{6a^2b\sec(c+dx)}{d}$$

$$= a^3x-\frac{3a^2b\cos(c+dx)}{d}-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}-\frac{6a^2b\sec(c+dx)}{d}$$

$$= a^3x+\frac{15}{2}ab^2x-\frac{3a^2b\cos(c+dx)}{d}-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}-\frac{6a^2b\,\text{se}}{}$$

Mathematica [A] time = 0.678412, size = 226, normalized size = 1.03

$$\frac{\sec^3(c+dx)\left(36a\left(2a^2+15b^2\right)(c+dx)\cos(c+dx)-3\left(144a^2b+91b^3\right)\cos(2(c+dx))-36a^2b\cos(4(c+dx))-300a^2b-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

```
[Out] (Sec[c + d*x]^3*(-300*a^2*b - 210*b^3 + 36*a*(2*a^2 + 15*b^2)*(c + d*x)*Cos
[c + d*x] - 3*(144*a^2*b + 91*b^3)*Cos[2*(c + d*x)] + 24*a^3*c*Cos[3*(c + d
*x)] + 180*a*b^2*c*Cos[3*(c + d*x)] + 24*a^3*d*x*Cos[3*(c + d*x)] + 180*a*b
^2*d*x*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 30*b^3*Cos[4*(c + d*x
)] + b^3*Cos[6*(c + d*x)] - 90*a*b^2*Sin[c + d*x] - 32*a^3*Sin[3*(c + d*x)]
 - 195*a*b^2*Sin[3*(c + d*x)] - 9*a*b^2*Sin[5*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.058, size = 268, normalized size = 1.2

$$\frac{1}{d}\left(a^3\left(\frac{(\tan(dx+c))^3}{3}-\tan(dx+c)+dx+c\right)+3a^2b\left(1/3\frac{(\sin(dx+c))^6}{(\cos(dx+c))^3}-\frac{(\sin(dx+c))^6}{\cos(dx+c)}-\left(8/3+(\sin(dx+c))^4+4/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x)
```

```
[Out] 1/d*(a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+3*a^2*b*(1/3*sin(d*x+c)^6/cos(
d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*
x+c))+3*a*b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/
3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c)
+b^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(16/5+s
in(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 1.82353, size = 225, normalized size = 1.02

$$\frac{2\left(\tan\left(dx+c\right)^{3}+3\,dx+3\,c-3\,\tan\left(dx+c\right)\right)a^{3}+3\left(2\,\tan\left(dx+c\right)^{3}+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^{2}+1}-12\,\tan\left(dx+c\right)\right)ab}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3 + 3*(2*tan(d*x +
 c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x +
c))*a*b^2 + 2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*c
os(d*x + c))*b^3 - 6*a^2*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d
*x + c)))/d
```

Fricas [A] time = 1.59267, size = 369, normalized size = 1.68

$$\frac{2\,b^{3}\cos\left(dx+c\right)^{6}+3\left(2\,a^{3}+15\,ab^{2}\right)dx\cos\left(dx+c\right)^{3}-18\left(a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{4}+6\,a^{2}b+2\,b^{3}-18\left(2\,a^{2}b+b^{3}\right)\cos\left(d\right.}{6\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*cos(d*x + c)^6 + 3*(2*a^3 + 15*a*b^2)*d*x*cos(d*x + c)^3 - 18*(a
^2*b + b^3)*cos(d*x + c)^4 + 6*a^2*b + 2*b^3 - 18*(2*a^2*b + b^3)*cos(d*x +
 c)^2 - (9*a*b^2*cos(d*x + c)^4 - 2*a^3 - 6*a*b^2 + 2*(4*a^3 + 21*a*b^2)*co
```

```
s(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**4,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")

[Out] Timed out
```

3.166 $\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

Optimal. Leaf size=146

$$\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-$$

```
[Out] -(a^3*x) - (9*a*b^2*x)/2 + (3*a^2*b*Cos[c + d*x])/d + (2*b^3*Cos[c + d*x])/
d - (b^3*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Sec[c + d*x])/d + (b^3*Sec[c + d*
x])/d + (a^3*Tan[c + d*x])/d + (9*a*b^2*Tan[c + d*x])/(2*d) - (3*a*b^2*Sin[
c + d*x]^2*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.169452, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 3473, 8, 2590, 14, 2591, 288, 321, 203, 270}

$$\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```


```
[Out] -(a^3*x) - (9*a*b^2*x)/2 + (3*a^2*b*Cos[c + d*x])/d + (2*b^3*Cos[c + d*x])/
d - (b^3*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Sec[c + d*x])/d + (b^3*Sec[c + d*
x])/d + (a^3*Tan[c + d*x])/d + (9*a*b^2*Tan[c + d*x])/(2*d) - (3*a*b^2*Sin[
c + d*x]^2*Tan[c + d*x])/(2*d)
```


Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \int \left(a^3\tan^2(c+dx)+3a^2b\sin(c+dx)\tan^2(c+dx)+3ab^2\sin^2(c+dx)\tan^2(c\right.$$

$$= a^3\int \tan^2(c+dx)\,dx+\left(3a^2b\right)\int \sin(c+dx)\tan^2(c+dx)\,dx+\left(3ab^2\right)\int \sin^2($$

$$= \frac{a^3\tan(c+dx)}{d}-a^3\int 1\,dx-\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx,x,\cos(c+dx)\right)}{d}+\frac{\left(3ab^2\right.}{}$$

$$= -a^3x+\frac{a^3\tan(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int \left(-1\right.\right.}{}$$

$$= -a^3x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2b\sec(c+a}{d}$$

$$= -a^3x-\frac{9}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2b}{}$$

Mathematica [A] time = 0.729084, size = 113, normalized size = 0.77

$$\frac{3a\left(\left(8a^2+27b^2\right)\tan(c+dx)-4\left(2a^2+9b^2\right)(c+dx)\right)+b\sec(c+dx)\left(4\left(9a^2+5b^2\right)\cos(2(c+dx))+108a^2+9ab\sin(3\right.}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

```
[Out] (b*Sec[c + d*x]*(108*a^2 + 45*b^2 + 4*(9*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b^
2*Cos[4*(c + d*x)] + 9*a*b*Sin[3*(c + d*x)]) + 3*a*(-4*(2*a^2 + 9*b^2)*(c +
 d*x) + (8*a^2 + 27*b^2)*Tan[c + d*x]))/(24*d)
```

Maple [A] time = 0.047, size = 169, normalized size = 1.2

$$\frac{1}{d}\left(a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{(\sin(dx+c))^4}{\cos(dx+c)}+\left(2+(\sin(dx+c))^2\right)\cos(dx+c)\right)+3ab^2\left(\frac{(\sin(dx+c))^5}{\cos(dx+c)}+\left((\mathrm{si}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x)
```

```
[Out] 1/d*(a^3*(tan(d*x+c)-d*x-c)+3*a^2*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^
2)*cos(d*x+c))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c
))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x+c)^4
+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A] time = 3.01256, size = 161, normalized size = 1.1

$$-\frac{6\,(dx+c-\tan{(dx+c)})a^3+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan{(dx+c)}\right)ab^2+2\left(\cos{(dx+c)}^3-\frac{3}{\cos(dx+c)}-6\,\cos{(dx}\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(d*x + c - tan(d*x + c))*a^3 + 9*(3*d*x + 3*c - tan(d*x + c)/(tan(d
*x + c)^2 + 1) - 2*tan(d*x + c))*a*b^2 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c)
 - 6*cos(d*x + c))*b^3 - 18*a^2*b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

Fricas [A] time = 1.48544, size = 271, normalized size = 1.86

$$-\frac{2\,b^3\cos{(dx+c)}^4+3\left(2\,a^3+9\,ab^2\right)dx\cos{(dx+c)}-18\,a^2b-6\,b^3-6\left(3\,a^2b+2\,b^3\right)\cos{(dx+c)}^2-3\left(3\,ab^2\cos{(dx+c)}\right.}{6\,d\cos{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*cos(d*x + c)^4 + 3*(2*a^3 + 9*a*b^2)*d*x*cos(d*x + c) - 18*a^2*
b - 6*b^3 - 6*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 3*(3*a*b^2*cos(d*x + c)^2
+ 2*a^3 + 6*a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sin(c + dx))^3 \tan^2(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.167 $\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=102

$$\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3\cos}{}$$

```
[Out] -(a^3*x) + (3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + (3*a^2*b*Cos
[c + d*x])/d - (b^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.111823, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2722, 2635, 8, 2592, 321, 206, 3473, 2565, 30}

$$\frac{3a^2b\cos(c+dx)}{d}-\frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+a^3(-x)+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(a^3*x) + (3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + (3*a^2*b*Cos
[c + d*x])/d - (b^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a*b^2
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(3ab^2\cos^2(c+dx) + 3a^2b\cos(c+dx)\cot(c+dx) + a^3\cot^2(c+dx) + b^3\cos^2\right.$$
$$= a^3\int \cot^2(c+dx)\,dx + \left(3a^2b\right)\int \cos(c+dx)\cot(c+dx)\,dx + \left(3ab^2\right)\int \cos^2(c+$$
$$= -\frac{a^3\cot(c+dx)}{d} + \frac{3ab^2\cos(c+dx)\sin(c+dx)}{2d} - a^3\int 1\,dx + \frac{1}{2}\left(3ab^2\right)\int 1\,dx -$$
$$= -a^3x + \frac{3}{2}ab^2x + \frac{3a^2b\cos(c+dx)}{d} - \frac{b^3\cos^3(c+dx)}{3d} - \frac{a^3\cot(c+dx)}{d} + \frac{3ab^2\cos}{}$$
$$= -a^3x + \frac{3}{2}ab^2x - \frac{3a^2b\tanh^{-1}(\cos(c+dx))}{d} + \frac{3a^2b\cos(c+dx)}{d} - \frac{b^3\cos^3(c+dx)}{3d}$$

Mathematica [A] time = 1.29539, size = 143, normalized size = 1.4

$$\frac{\left(36a^2b-3b^3\right)\cos(c+dx) + 6a\left(a^2\tan\left(\frac{1}{2}(c+dx)\right) - 2a^2c - 2a^2dx + 6ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 6ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right.\right.\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((36*a^2*b - 3*b^3)*Cos[c + d*x] - b^3*Cos[3*(c + d*x)] - 6*a^3*Cot[(c + d*
x)/2] + 9*a*b^2*Sin[2*(c + d*x)] + 6*a*(-2*a^2*c + 3*b^2*c - 2*a^2*d*x + 3*
b^2*d*x - 6*a*b*Log[Cos[(c + d*x)/2]] + 6*a*b*Log[Sin[(c + d*x)/2]] + a^2*T
an[(c + d*x)/2]))/(12*d)
```

Maple [A] time = 0.052, size = 125, normalized size = 1.2

$$-a^3x - \frac{a^3\cot(dx+c)}{d} - \frac{a^3c}{d} + 3\,\frac{a^2b\ln(\csc(dx+c)-\cot(dx+c))}{d} + 3\,\frac{a^2b\cos(dx+c)}{d} + \frac{3\,ab^2\cos(dx+c)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -a^3*x-a^3*cot(d*x+c)/d-1/d*a^3*c+3/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))+3*a^2
*b*cos(d*x+c)/d+3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d+3/2*a*b^2*x+3/2/d*a*b^2*c
-1/3*b^3*cos(d*x+c)^3/d
```

Maxima [A] time = 1.51545, size = 128, normalized size = 1.25

$$-\frac{4\,b^3\cos\left(dx+c\right)^3+12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)ab^2-18\,a^2b(2\,\cos\left(dx+c\right)-\log\left(\cos\left(d\right.\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b^3*cos(d*x + c)^3 + 12*(d*x + c + 1/tan(d*x + c))*a^3 - 9*(2*d*x
+ 2*c + sin(2*d*x + 2*c))*a*b^2 - 18*a^2*b*(2*cos(d*x + c) - log(cos(d*x +
c) + 1) + log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.6047, size = 370, normalized size = 3.63

$$-\frac{9\,ab^2\cos\left(dx+c\right)^3+9\,a^2b\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,a^2b\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+3\left(2\,a^3\right.}{6\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(9*a*b^2*cos(d*x + c)^3 + 9*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 9*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(2*a^3 - 3*a*b
^2)*cos(d*x + c) + (2*b^3*cos(d*x + c)^3 - 18*a^2*b*cos(d*x + c) + 3*(2*a^3
 - 3*a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sin\left(c+dx\right)\right)^3\cot^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**2, x)
```

Giac [B] time = 2.62788, size = 269, normalized size = 2.64

$$\frac{18\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\left(2\,a^3-3\,ab^2\right)(dx+c)-\frac{3\left(6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(9\,ab^2\tan\left(\frac{1}{2}\,dx+\cdots\right.\right.}{\cdots}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(18*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c) -
 3*(2*a^3 - 3*a*b^2)*(d*x + c) - 3*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/tan
(1/2*d*x + 1/2*c) - 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*
x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)
^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 2*b^3)/(tan(1/2*d*x + 1/2*c)
^2 + 1)^3)/d
```

3.168 $\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+a$$

```
[Out] a^3*x - (9*a*b^2*x)/2 + (9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTan
h[Cos[c + d*x]])/d - (9*a^2*b*Cos[c + d*x])/(2*d) + (b^3*Cos[c + d*x])/d +
(b^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/
(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (3*a^2*b*Cos[c + d*x]
*Cot[c + d*x]^2)/(2*d) - (a^3*Cot[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.183918, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 2592, 302, 206, 2591, 288, 321, 203, 3473, 8}

$$-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+a$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] a^3*x - (9*a*b^2*x)/2 + (9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTan
h[Cos[c + d*x]])/d - (9*a^2*b*Cos[c + d*x])/(2*d) + (b^3*Cos[c + d*x])/d +
(b^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/
(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (3*a^2*b*Cos[c + d*x]
*Cot[c + d*x]^2)/(2*d) - (a^3*Cot[c + d*x]^3)/(3*d)
```


Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\cos^3(c+dx)\cot(c+dx)+3ab^2\cos^2(c+dx)\cot^2(c+dx)+3a^2b\cos(c+\right.$$

$$= a^3\int \cot^4(c+dx)\,dx+\left(3a^2b\right)\int \cos(c+dx)\cot^3(c+dx)\,dx+\left(3ab^2\right)\int \cos^2($$

$$= -\frac{a^3\cot^3(c+dx)}{3d}-a^3\int \cot^2(c+dx)\,dx-\frac{\left(3a^2b\right)\mathrm{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^2}\,dx,x,\cos(c\right.}{d}$$

$$= \frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx}{2d}$$

$$= a^3x-\frac{9a^2b\cos(c+dx)}{2d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}-$$

$$= a^3x-\frac{9}{2}ab^2x+\frac{9a^2b\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b^3\tanh^{-1}(\cos(c+dx))}{d}-\frac{9a^2b\cos}{2}$$

Mathematica [A] time = 6.23632, size = 355, normalized size = 1.83

$$\frac{a\left(2a^2-9b^2\right)(c+dx)}{2d}+\frac{b\left(5b^2-12a^2\right)\cos(c+dx)}{4d}+\frac{\left(2b^3-9a^2b\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{\left(9a^2b-2b^3\right)\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 9*b^2)*(c + d*x))/(2*d) + (b*(-12*a^2 + 5*b^2)*Cos[c + d*x])/(4
*d) + (b^3*Cos[3*(c + d*x)])/(12*d) + ((4*a^3*Cos[(c + d*x)/2] - 9*a*b^2*Co
s[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (3*a^2*b*Csc[(c + d*x)/2]^2)/(8*d
) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + ((9*a^2*b - 2*b^3)*L
og[Cos[(c + d*x)/2]])/(2*d) + ((-9*a^2*b + 2*b^3)*Log[Sin[(c + d*x)/2]])/(2
*d) + (3*a^2*b*Sec[(c + d*x)/2]^2)/(8*d) + (Sec[(c + d*x)/2]*(-4*a^3*Sin[(c
```

```
 + d*x)/2] + 9*a*b^2*Sin[(c + d*x)/2]))/(6*d) - (3*a*b^2*Sin[2*(c + d*x)])/
(4*d) + (a^3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

Maple [A] time = 0.062, size = 264, normalized size = 1.4

$$-\frac{a^3(\cot(dx+c))^3}{3d}+\frac{a^3\cot(dx+c)}{d}+a^3x+\frac{a^3c}{d}-\frac{3a^2b(\cos(dx+c))^5}{2d(\sin(dx+c))^2}-\frac{3a^2b(\cos(dx+c))^3}{2d}-\frac{9a^2b\cos(dx+c)}{2d}-9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3*a^3*cot(d*x+c)^3/d+a^3*cot(d*x+c)/d+a^3*x+1/d*a^3*c-3/2/d*a^2*b/sin(d*
x+c)^2*cos(d*x+c)^5-3/2/d*a^2*b*cos(d*x+c)^3-9/2*a^2*b*cos(d*x+c)/d-9/2/d*a
^2*b*ln(csc(d*x+c)-cot(d*x+c))-3/d*a*b^2/sin(d*x+c)*cos(d*x+c)^5-3/d*a*b^2*
sin(d*x+c)*cos(d*x+c)^3-9/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d-9/2*a*b^2*x-9/2/d
*a*b^2*c+1/3*b^3*cos(d*x+c)^3/d+b^3*cos(d*x+c)/d+1/d*b^3*ln(csc(d*x+c)-cot(
d*x+c))
```

Maxima [A] time = 2.21021, size = 252, normalized size = 1.3

$$\frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3-18\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab^2+2\left(2\,\cos(dx+c)^3+6\,\cos(dx+c)-3\,\log(c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 18*(3*d
*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a*b^2 +
2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(cos(
d*x + c) - 1))*b^3 + 9*a^2*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d
*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.65166, size = 699, normalized size = 3.6

$$\frac{18\,ab^2\cos(dx+c)^5+8\left(2\,a^3-9\,ab^2\right)\cos(dx+c)^3-3\left(9\,a^2b-2\,b^3-\left(9\,a^2b-2\,b^3\right)\cos(dx+c)^2\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*cos(d*x + c)^5 + 8*(2*a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(9*a
^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(9*a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log
(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(2*a^3 - 9*a*b^2)*cos(d*x + c) +
 2*(2*b^3*cos(d*x + c)^5 + 3*(2*a^3 - 9*a*b^2)*d*x*cos(d*x + c)^2 - 2*(9*a^
2*b - 2*b^3)*cos(d*x + c)^3 - 3*(2*a^3 - 9*a*b^2)*d*x + 3*(9*a^2*b - 2*b^3)
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 2.6525, size = 568, normalized size = 2.93

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+108\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\left(2\,a^3-9\,ab^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 45*a
^3*tan(1/2*d*x + 1/2*c) + 108*a*b^2*tan(1/2*d*x + 1/2*c) + 36*(2*a^3 - 9*a*
b^2)*(d*x + c) - 36*(9*a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (198
*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 44*b^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^3*tan(
1/2*d*x + 1/2*c)^8 + 108*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 135*a^2*b*tan(1/2*d
*x + 1/2*c)^7 + 156*b^3*tan(1/2*d*x + 1/2*c)^7 + 132*a^3*tan(1/2*d*x + 1/2*
```

```
c)^6 - 324*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 351*a^2*b*tan(1/2*d*x + 1/2*c)^5
+ 156*b^3*tan(1/2*d*x + 1/2*c)^5 + 126*a^3*tan(1/2*d*x + 1/2*c)^4 - 540*a*b
^2*tan(1/2*d*x + 1/2*c)^4 - 315*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 148*b^3*tan(
1/2*d*x + 1/2*c)^3 + 36*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 - 27*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/(tan(1/2*d*x + 1/2*c)^3
 + tan(1/2*d*x + 1/2*c))^3)/d
```

3.169 $\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=291

$$\frac{45a^2b\cos(c+dx)}{8d}-\frac{3a^2b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{15a^2b\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{45a^2b\tanh^{-1}(\cos(c+dx))}{8d}-a^3$$

```
[Out] -(a^3*x) + (15*a*b^2*x)/2 - (45*a^2*b*ArcTanh[Cos[c + d*x]])/(8*d) + (5*b^3
*ArcTanh[Cos[c + d*x]])/(2*d) + (45*a^2*b*Cos[c + d*x])/(8*d) - (5*b^3*Cos[
c + d*x])/(2*d) - (5*b^3*Cos[c + d*x]^3)/(6*d) - (a^3*Cot[c + d*x])/d + (15
*a*b^2*Cot[c + d*x])/(2*d) + (15*a^2*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) -
 (b^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(2*d) + (a^3*Cot[c + d*x]^3)/(3*d) - (
5*a*b^2*Cot[c + d*x]^3)/(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*
d) - (3*a^2*b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a^3*Cot[c + d*x]^5)/(5*
d)
```

Rubi [A] time = 0.2309, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2722, 2592, 288, 302, 206, 2591, 203, 321, 3473, 8}

$$\frac{45a^2b\cos(c+dx)}{8d}-\frac{3a^2b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{15a^2b\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{45a^2b\tanh^{-1}(\cos(c+dx))}{8d}-a^3$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -(a^3*x) + (15*a*b^2*x)/2 - (45*a^2*b*ArcTanh[Cos[c + d*x]])/(8*d) + (5*b^3
*ArcTanh[Cos[c + d*x]])/(2*d) + (45*a^2*b*Cos[c + d*x])/(8*d) - (5*b^3*Cos[
c + d*x])/(2*d) - (5*b^3*Cos[c + d*x]^3)/(6*d) - (a^3*Cot[c + d*x])/d + (15
*a*b^2*Cot[c + d*x])/(2*d) + (15*a^2*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) -
 (b^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(2*d) + (a^3*Cot[c + d*x]^3)/(3*d) - (
5*a*b^2*Cot[c + d*x]^3)/(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*
d) - (3*a^2*b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a^3*Cot[c + d*x]^5)/(5*
d)
```


Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
```

```
&& IGtQ[m, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(b^3\cos^3(c+dx)\cot^3(c+dx)+3ab^2\cos^2(c+dx)\cot^4(c+dx)+3a^2b\cos(c$$

$$= a^3\int \cot^6(c+dx)\,dx + \left(3a^2b\right)\int \cos(c+dx)\cot^5(c+dx)\,dx + \left(3ab^2\right)\int \cos^2($$

$$= -\frac{a^3\cot^5(c+dx)}{5d} - a^3\int \cot^4(c+dx)\,dx - \frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx, x, \cos(c}{d}$$

$$= -\frac{b^3\cos^3(c+dx)\cot^2(c+dx)}{2d} + \frac{a^3\cot^3(c+dx)}{3d} + \frac{3ab^2\cos^2(c+dx)\cot^3(c+}{2d}$$

$$= -\frac{a^3\cot(c+dx)}{d} + \frac{15a^2b\cos(c+dx)\cot^2(c+dx)}{8d} - \frac{b^3\cos^3(c+dx)\cot^2(c+d}{2d}$$

$$= -a^3x + \frac{45a^2b\cos(c+dx)}{8d} - \frac{5b^3\cos(c+dx)}{2d} - \frac{5b^3\cos^3(c+dx)}{6d} - \frac{a^3\cot(c+d}{d}$$

$$= -a^3x + \frac{15}{2}ab^2x - \frac{45a^2b\tanh^{-1}(\cos(c+dx))}{8d} + \frac{5b^3\tanh^{-1}(\cos(c+dx))}{2d} + \frac{45}{}$$

Mathematica [A] time = 2.57765, size = 346, normalized size = 1.19

$$\frac{-600a\left(2a^2-15b^2\right)(c+dx)\csc^4(c+dx)+1200b\left(4b^2-9a^2\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\csc^5(c-}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-600*a*(2*a^2 - 15*b^2)*(c + d*x)*Csc[c + d*x]^4 + 1200*b*(-9*a^2 + 4*b^2)
*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 5*Cot[c + d*x]*Csc[c + d
*x]^4*(-80*a^3 + 285*a*b^2 + 12*b*(60*a^2 - 29*b^2)*Sin[c + d*x]) + Csc[c +
 d*x]^5*(5*(40*a^3 - 489*a*b^2)*Cos[3*(c + d*x)] + (-184*a^3 + 1065*a*b^2)*
Cos[5*(c + d*x)] + 5*(-9*a*b^2*Cos[7*(c + d*x)] + 60*a*(2*a^2 - 15*b^2)*(c
+ d*x)*Sin[3*(c + d*x)] - 306*a^2*b*Sin[4*(c + d*x)] + 122*b^3*Sin[4*(c + d
*x)] - 24*a^3*c*Sin[5*(c + d*x)] + 180*a*b^2*c*Sin[5*(c + d*x)] - 24*a^3*d*
x*Sin[5*(c + d*x)] + 180*a*b^2*d*x*Sin[5*(c + d*x)] + 36*a^2*b*Sin[6*(c + d
*x)] - 22*b^3*Sin[6*(c + d*x)] - b^3*Sin[8*(c + d*x)])))/(1920*d)
```

Maple [A] time = 0.063, size = 415, normalized size = 1.4

$$-\frac{a^3 (\cot(dx+c))^5}{5d} + \frac{a^3 (\cot(dx+c))^3}{3d} - \frac{a^3 \cot(dx+c)}{d} - a^3 x - \frac{a^3 c}{d} - \frac{3a^2 b (\cos(dx+c))^7}{4d (\sin(dx+c))^4} + \frac{9a^2 b (\cos(dx+c))^7}{8d (\sin(dx+c))^2} + \frac{9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/5*a^3*cot(d*x+c)^5/d+1/3*a^3*cot(d*x+c)^3/d-a^3*cot(d*x+c)/d-a^3*x-1/d*a
^3*c-3/4/d*a^2*b/sin(d*x+c)^4*cos(d*x+c)^7+9/8/d*a^2*b/sin(d*x+c)^2*cos(d*x
+c)^7+9/8/d*a^2*b*cos(d*x+c)^5+15/8/d*a^2*b*cos(d*x+c)^3+45/8*a^2*b*cos(d*x
+c)/d+45/8/d*a^2*b*ln(csc(d*x+c)-cot(d*x+c))-1/d*a*b^2/sin(d*x+c)^3*cos(d*x
+c)^7+4/d*a*b^2/sin(d*x+c)*cos(d*x+c)^7+4/d*a*b^2*sin(d*x+c)*cos(d*x+c)^5+5
/d*a*b^2*sin(d*x+c)*cos(d*x+c)^3+15/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d+15/2*a*
b^2*x+15/2/d*a*b^2*c-1/2/d*b^3/sin(d*x+c)^2*cos(d*x+c)^7-1/2/d*b^3*cos(d*x+
c)^5-5/6*b^3*cos(d*x+c)^3/d-5/2*b^3*cos(d*x+c)/d-5/2/d*b^3*ln(csc(d*x+c)-co
t(d*x+c))
```

Maxima [A] time = 2.64117, size = 340, normalized size = 1.17

$$-\frac{16\left(15\,dx + 15\,c + \frac{15\,\tan(dx+c)^4 - 5\,\tan(dx+c)^2 + 3}{\tan(dx+c)^5}\right)a^3 - 120\left(15\,dx + 15\,c + \frac{15\,\tan(dx+c)^4 + 10\,\tan(dx+c)^2 - 2}{\tan(dx+c)^5 + \tan(dx+c)^3}\right)ab^2 + 20\left(4\,\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a^3 - 120*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)
^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a*b^2 + 20*(4*cos(d*x + c)^3 - 6
*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x + c)
+ 1) + 15*log(cos(d*x + c) - 1))*b^3 + 45*a^2*b*(2*(9*cos(d*x + c)^3 - 7*co
s(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*
log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

Fricas [A] time = 1.83246, size = 1014, normalized size = 3.48

$$-\frac{360\,ab^2\cos(dx+c)^7+184\left(2\,a^3-15\,ab^2\right)\cos(dx+c)^5-280\left(2\,a^3-15\,ab^2\right)\cos(dx+c)^3+75\left(\left(9\,a^2b-4\,b^3\right)\cos(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(360*a*b^2*cos(d*x + c)^7 + 184*(2*a^3 - 15*a*b^2)*cos(d*x + c)^5 -
280*(2*a^3 - 15*a*b^2)*cos(d*x + c)^3 + 75*((9*a^2*b - 4*b^3)*cos(d*x + c)^
4 + 9*a^2*b - 4*b^3 - 2*(9*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) - 75*((9*a^2*b - 4*b^3)*cos(d*x + c)^4 + 9*a^2*b -
4*b^3 - 2*(9*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*si
n(d*x + c) + 120*(2*a^3 - 15*a*b^2)*cos(d*x + c) + 10*(8*b^3*cos(d*x + c)^7
 + 12*(2*a^3 - 15*a*b^2)*d*x*cos(d*x + c)^4 - 8*(9*a^2*b - 4*b^3)*cos(d*x +
 c)^5 - 24*(2*a^3 - 15*a*b^2)*d*x*cos(d*x + c)^2 + 25*(9*a^2*b - 4*b^3)*cos
(d*x + c)^3 + 12*(2*a^3 - 15*a*b^2)*d*x - 15*(9*a^2*b - 4*b^3)*cos(d*x + c)
)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.01324, size = 636, normalized size = 2.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 70*
a^3*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 720*a^2*b*t
an(1/2*d*x + 1/2*c)^2 + 120*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^3*tan(1/2*d*
x + 1/2*c) - 3240*a*b^2*tan(1/2*d*x + 1/2*c) - 480*(2*a^3 - 15*a*b^2)*(d*x
+ c) + 600*(9*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) - 320*(9*a*b^2*
tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 18*b^3*tan(1/2*d
*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 24*b^3*tan(1/2*d*x + 1/2*
c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 14*b^3)/(tan(1/2*d*x + 1/2
*c)^2 + 1)^3 - (12330*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 5480*b^3*tan(1/2*d*x +
 1/2*c)^5 + 660*a^3*tan(1/2*d*x + 1/2*c)^4 - 3240*a*b^2*tan(1/2*d*x + 1/2*c
)^4 - 720*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1/2*c)^3 - 7
0*a^3*tan(1/2*d*x + 1/2*c)^2 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^2 + 45*a^2*b*
tan(1/2*d*x + 1/2*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.170 $\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=204

$$\frac{a^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c+$$

```
[Out] -((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((8*a^2
 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a + b*
Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4
*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*Si
n[c + d*x]))/(8*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.363019, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 801}

$$\frac{a^5\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16d(a+b)^3}-\frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{16d(a-b)^3}+\frac{\sec^4(c+$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```


```
[Out] -((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^3*d) - ((8*a^2
 - 9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^3*d) + (a^5*Log[a + b*
Sin[c + d*x]])/((a^2 - b^2)^3*d) + (Sec[c + d*x]^4*(a - b*Sin[c + d*x]))/(4
*(a^2 - b^2)*d) - (Sec[c + d*x]^2*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*Si
n[c + d*x]))/(8*(a^2 - b^2)^2*d)
```


Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^6}{a^2-b^2}-\frac{b^4\left(4a^2-b^2\right)x}{a^2-b^2}-4b^2x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{4b^2d}$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2d} + \text{Su}\cdots$$

$$= \frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\left(a^2-b^2\right)d} - \frac{\sec^2(c+dx)\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx)\right)}{8\left(a^2-b^2\right)^2d} + \text{Su}\cdots$$

$$= -\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{16(a+b)^3d} - \frac{\left(8a^2-9ab+3b^2\right)\log(1+\sin(c+dx))}{16(a-b)^3d} + \frac{a^5\log(a+\cdots}{\left(a^2\cdots\right.}$$

Mathematica [A] time = 1.46175, size = 184, normalized size = 0.9

$$\frac{-\frac{\left(8a^2+9ab+3b^2\right)\log(1-\sin(c+dx))}{(a+b)^3} - \frac{\left(8a^2-9ab+3b^2\right)\log(\sin(c+dx)+1)}{(a-b)^3} + \frac{16a^5\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{7a+5b}{(a+b)^2(\sin(c+dx)-1)} + \frac{5b-7a}{(a-b)^2(\sin(c+dx)+1)} + \frac{\cdots}{(a+\cdots}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-(((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((8*a^2 - 9
*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^5*Log[a + b*Sin[c +
d*x]])/((a - b)^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (7*a + 5
*b)/((a + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2) + (-
7*a + 5*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

Maple [A] time = 0.066, size = 304, normalized size = 1.5

$$\frac{a^5 \ln(a + b \sin(dx + c))}{d (a + b)^3 (a - b)^3} + \frac{1}{2 d (8 a + 8 b) (\sin(dx + c) - 1)^2} + \frac{7 a}{16 d (a + b)^2 (\sin(dx + c) - 1)} + \frac{5 b}{16 d (a + b)^2 (\sin(dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a^5/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/d/(8*a+8*b)/(sin(d*x+c)-1)^2
+7/16/d/(a+b)^2/(sin(d*x+c)-1)*a+5/16/d/(a+b)^2/(sin(d*x+c)-1)*b-1/2/d/(a+b
)^3*ln(sin(d*x+c)-1)*a^2-9/16/d/(a+b)^3*ln(sin(d*x+c)-1)*a*b-3/16/d/(a+b)^3
*ln(sin(d*x+c)-1)*b^2+1/2/d/(8*a-8*b)/(1+sin(d*x+c))^2-7/16/d/(a-b)^2/(1+si
n(d*x+c))*a+5/16/d/(a-b)^2/(1+sin(d*x+c))*b-1/2/d/(a-b)^3*ln(1+sin(d*x+c))*
a^2+9/16/d/(a-b)^3*ln(1+sin(d*x+c))*a*b-3/16/d/(a-b)^3*ln(1+sin(d*x+c))*b^2
```

Maxima [A] time = 1.8256, size = 389, normalized size = 1.91

$$\frac{\frac{16 a^5 \log(b \sin(dx+c)+a)}{a^6-3 a^4 b^2+3 a^2 b^4-b^6} - \frac{(8 a^2-9 ab+3 b^2) \log(\sin(dx+c)+1)}{a^3-3 a^2 b+3 ab^2-b^3} - \frac{(8 a^2+9 ab+3 b^2) \log(\sin(dx+c)-1)}{a^3+3 a^2 b+3 ab^2+b^3} - \frac{2 ((9 a^2 b-5 b^3) \sin(dx+c)^3+6 a^3-2 ab^2-4 (2 a^3-ab^2}{(a^4-2 a^2 b^2+b^4) \sin(dx+c)^4+a^4-2 a^2 b^2+b^4-}}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
(8*a^2 - 9*a*b + 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) - (8*a^2 + 9*a*b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2
```

```
 + b^3) - 2*((9*a^2*b - 5*b^3)*sin(d*x + c)^3 + 6*a^3 - 2*a*b^2 - 4*(2*a^3
- a*b^2)*sin(d*x + c)^2 - (7*a^2*b - 3*b^3)*sin(d*x + c))/((a^4 - 2*a^2*b^2
 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*
sin(d*x + c)^2))/d
```

Fricas [A] time = 2.47436, size = 594, normalized size = 2.91

$$\frac{16\,a^5\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)-\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-\left(8\,a^5-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (8*a^5 + 15*a^4*b - 1
0*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^5 - 15*a^4*b
 + 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8*a^
3*b^2 + 4*a*b^4 - 8*(2*a^5 - 3*a^3*b^2 + a*b^4)*cos(d*x + c)^2 - 2*(2*a^4*b
 - 4*a^2*b^3 + 2*b^5 - (9*a^4*b - 14*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d
*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 5.11783, size = 463, normalized size = 2.27

$$\frac{\frac{16\,a^5b\log\left(\left|b\sin(dx+c)+a\right|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{\left(8\,a^2-9\,ab+3\,b^2\right)\log\left(\left|\sin(dx+c)+1\right|\right)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+9\,ab+3\,b^2\right)\log\left(\left|\sin(dx+c)-1\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(6\,a^5\sin(dx+c)^4-9\,a^4b\sin(dx+c)^3+14\,a^2b^3\sin(\right.}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/16*(16*a^5*b*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
- b^7) - (8*a^2 - 9*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*a^5*sin(d*x + c)^4 - 9*a^4*b*sin(d*x + c)
^3 + 14*a^2*b^3*sin(d*x + c)^3 - 5*b^5*sin(d*x + c)^3 - 4*a^5*sin(d*x + c)^
2 - 12*a^3*b^2*sin(d*x + c)^2 + 4*a*b^4*sin(d*x + c)^2 + 7*a^4*b*sin(d*x +
c) - 10*a^2*b^3*sin(d*x + c) + 3*b^5*sin(d*x + c) + 8*a^3*b^2 - 2*a*b^4)/((
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

3.171 $\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{a^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+dx)+}{4d(a-b)^2}$$

```
[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d
) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.190922, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 801}

$$-\frac{a^3\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2d\left(a^2-b^2\right)}+\frac{(2a+b)\log(1-\sin(c+dx))}{4d(a+b)^2}+\frac{(2a-b)\log(\sin(c+dx)+}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((2*a + b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((2*a - b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^2*d) - (a^3*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d
) + (Sec[c + d*x]^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2-b^2}-\frac{b^2\left(2a^2-b^2\right)x}{a^2-b^2}}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{2b^2d}$$

$$= \frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \left(-\frac{b^2(2a+b)}{2(a+b)^2(b-x)} - \frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)} + \frac{(2a-b)b^2}{2(a-b)^2(b+x)}\right) dx, \ldots\right.}{2b^2d}$$

$$= \frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{a^3\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} + \ldots$$

Mathematica [A] time = 0.484179, size = 117, normalized size = 0.93

$$\frac{-\frac{4a^3\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2} - \frac{1}{(a+b)(\sin(c+dx)-1)} + \frac{1}{(a-b)(\sin(c+dx)+1)} + \frac{(2a+b)\log(1-\sin(c+dx))}{(a+b)^2} + \frac{(2a-b)\log(\sin(c+dx)+1)}{(a-b)^2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (((2*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^2 + ((2*a - b)*Log[1 + Sin[c + d
*x]])/(a - b)^2 - (4*a^3*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) - 1
/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)
```

Maple [A] time = 0.059, size = 164, normalized size = 1.3

$$-\frac{a^{3} \ln \left(a+b \sin \left(d x+c\right)\right)}{d \left(a+b\right)^{2} \left(a-b\right)^{2}}-\frac{1}{d \left(4 a+4 b\right) \left(\sin \left(d x+c\right)-1\right)}+\frac{\ln \left(\sin \left(d x+c\right)-1\right) a}{2 d \left(a+b\right)^{2}}+\frac{\ln \left(\sin \left(d x+c\right)-1\right) b}{4 d \left(a+b\right)^{2}}+\frac{}{d \left(4 a-4 b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d*a^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-1/d/(4*a+4*b)/(sin(d*x+c)-1)+1/
2/d/(a+b)^2*ln(sin(d*x+c)-1)*a+1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*b+1/d/(4*a-4*
b)/(1+sin(d*x+c))+1/2/d/(a-b)^2*ln(1+sin(d*x+c))*a-1/4/d/(a-b)^2*ln(1+sin(d
*x+c))*b
```

Maxima [A] time = 1.53645, size = 192, normalized size = 1.52

$$-\frac{\frac{4 a^{3} \log\left(b \sin\left(d x+c\right)+a\right)}{a^{4}-2 a^{2} b^{2}+b^{4}}-\frac{\left(2 a-b\right) \log\left(\sin\left(d x+c\right)+1\right)}{a^{2}-2 a b+b^{2}}-\frac{\left(2 a+b\right) \log\left(\sin\left(d x+c\right)-1\right)}{a^{2}+2 a b+b^{2}}-\frac{2 \left(b \sin\left(d x+c\right)-a\right)}{\left(a^{2}-b^{2}\right) \sin\left(d x+c\right)^{2}-a^{2}+b^{2}}}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(4*a^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - (2*a - b)*log
(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (2*a + b)*log(sin(d*x + c) - 1)/(a
^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 - a^
2 + b^2))/d
```

Fricas [A] time = 1.96149, size = 367, normalized size = 2.91

$$-\frac{4 a^{3} \cos \left(d x+c\right)^{2} \log \left(b \sin \left(d x+c\right)+a\right)-\left(2 a^{3}+3 a^{2} b-b^{3}\right) \cos \left(d x+c\right)^{2} \log \left(\sin \left(d x+c\right)+1\right)-\left(2 a^{3}-3 a^{2} b+b^{3}\right) \mathrm{co}}{4 \left(a^{4}-2 a^{2} b^{2}+b^{4}\right) d \cos \left(d x+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (2*a^3 + 3*a^2*b - b^3
)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a^2*b + b^3)*cos(d*x +
c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x + c
))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.98003, size = 239, normalized size = 1.9

$$-\frac{\frac{4\,a^3b\log\left(|b\sin(dx+c)+a|\right)}{a^4b-2\,a^2b^3+b^5}-\frac{(2\,a-b)\log\left(|\sin(dx+c)+1|\right)}{a^2-2\,ab+b^2}-\frac{(2\,a+b)\log\left(|\sin(dx+c)-1|\right)}{a^2+2\,ab+b^2}+\frac{2\left(a^3\sin(dx+c)^2-a^2b\sin(dx+c)+b^3\sin(dx+c)-ab^2\right)}{\left(a^4-2\,a^2b^2+b^4\right)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(4*a^3*b*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - (2*a
 - b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (2*a + b)*log(abs(si
n(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(a^3*sin(d*x + c)^2 - a^2*b*sin(d*
x + c) + b^3*sin(d*x + c) - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2
 - 1)))/d
```

3.172 $\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{a\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
+ (a*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.0661064, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 801}

$$\frac{a\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)}-\frac{\log(1-\sin(c+dx))}{2d(a+b)}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)*d)
+ (a*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+x)\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(b-x)} + \frac{a}{(a-b)(a+b)(a+x)} - \frac{1}{2(a-b)(b+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} - \frac{\log(1+\sin(c+dx))}{2(a-b)d} + \frac{a\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.0863228, size = 87, normalized size = 1.18

$$\frac{a\log(a+b\sin(c+dx)) + (b-a)\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - (a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right)}{d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((-a + b)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - (a + b)*Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]] + a*Log[a + b*Sin[c + d*x]])/((a - b)*(a + b)*d)
```

Maple [A] time = 0.049, size = 76, normalized size = 1.

$$\frac{a\ln(a+b\sin(dx+c))}{d(a+b)(a-b)} - \frac{\ln(\sin(dx+c)-1)}{d(2a+2b)} - \frac{\ln(1+\sin(dx+c))}{d(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/d/(2*a+2*b)*ln(sin(d*x+c)-1)-1/d/(2*
a-2*b)*ln(1+sin(d*x+c))
```

Maxima [A] time = 1.55148, size = 88, normalized size = 1.19

$$\frac{\frac{2a\log(b\sin(dx+c)+a)}{a^2-b^2} - \frac{\log(\sin(dx+c)+1)}{a-b} - \frac{\log(\sin(dx+c)-1)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a - b
) - log(sin(d*x + c) - 1)/(a + b))/d
```

Fricas [A] time = 1.61071, size = 157, normalized size = 2.12

$$\frac{2\,a\log\left(b\sin\left(dx+c\right)+a\right)-\left(a+b\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a-b\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) - (a - b)*
log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.3001, size = 96, normalized size = 1.3

$$\frac{\frac{2\,ab\log(|b\sin(dx+c)+a|)}{a^2b-b^3}-\frac{\log(|\sin(dx+c)+1|)}{a-b}-\frac{\log(|\sin(dx+c)-1|)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a*b*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x + c
) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b))/d
```

3.173 $\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0397367, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2721, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sin(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sin(c+dx)\right)}{ad}$$
$$= \frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Mathematica [A] time = 0.0203458, size = 34, normalized size = 1.

$$\frac{\log(\sin(c+dx))}{ad} - \frac{\log(a+b\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

Maple [A] time = 0.026, size = 35, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{da} - \frac{\ln(a+b\sin(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] ln(sin(d*x+c))/a/d-1/d/a*ln(a+b*sin(d*x+c))
```

Maxima [A] time = 1.46505, size = 45, normalized size = 1.32

$$-\frac{\frac{\log(b\sin(dx+c)+a)}{a}-\frac{\log(\sin(dx+c))}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(log(b*sin(d*x + c) + a)/a - log(sin(d*x + c))/a)/d
```

Fricas [A] time = 1.56974, size = 80, normalized size = 2.35

$$-\frac{\log\left(b\sin\left(dx+c\right)+a\right)-\log\left(-\frac{1}{2}\sin\left(dx+c\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(log(b*sin(d*x + c) + a) - log(-1/2*sin(d*x + c)))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.6945, size = 47, normalized size = 1.38

$$-\frac{\frac{\log(|b\sin(dx+c)+a|)}{a}-\frac{\log(|\sin(dx+c)|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(log(abs(b*sin(d*x + c) + a))/a - log(abs(sin(d*x + c)))/a)/d
```

3.174 $\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c
+ d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.0888076, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d}+\frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}+\frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] (b*Csc[c + d*x])/(a^2*d) - Csc[c + d*x]^2/(2*a*d) - ((a^2 - b^2)*Log[Sin[c
+ d*x]])/(a^3*d) + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/(a^3*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{ax^3}-\frac{b^2}{a^2x^2}+\frac{-a^2+b^2}{a^3x}+\frac{a^2-b^2}{a^3(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{b\csc(c+dx)}{a^2d}-\frac{\csc^2(c+dx)}{2ad}-\frac{\left(a^2-b^2\right)\log(\sin(c+dx))}{a^3d}+\frac{\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^3d}$$

Mathematica [A] time = 0.1613, size = 65, normalized size = 0.77

$$-\frac{2\left(a^2-b^2\right)(\log(\sin(c+dx))-\log(a+b\sin(c+dx)))+a^2\csc^2(c+dx)-2ab\csc(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

```
[Out] -(-2*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*(Log[Sin[c + d*x
]] - Log[a + b*Sin[c + d*x]]))/(2*a^3*d)
```

Maple [A] time = 0.063, size = 106, normalized size = 1.3

$$\frac{\ln(a+b\sin(dx+c))}{da}-\frac{\ln(a+b\sin(dx+c))\,b^2}{da^3}-\frac{1}{2\,da\,(\sin(dx+c))^2}-\frac{\ln(\sin(dx+c))}{da}+\frac{\ln(\sin(dx+c))\,b^2}{da^3}+\frac{}{da^2\,\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/d/a*ln(a+b*sin(d*x+c))-1/d/a^3*ln(a+b*sin(d*x+c))*b^2-1/2/d/a/sin(d*x+c)^
2-ln(sin(d*x+c))/a/d+1/d/a^3*ln(sin(d*x+c))*b^2+1/d/a^2*b/sin(d*x+c)
```

Maxima [A] time = 1.61607, size = 104, normalized size = 1.24

$$\frac{\frac{2\left(a^2-b^2\right)\log(b\sin(dx+c)+a)}{a^3}-\frac{2\left(a^2-b^2\right)\log(\sin(dx+c))}{a^3}+\frac{2\,b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^2 - b^2)*log(b*sin(d*x + c) + a)/a^3 - 2*(a^2 - b^2)*log(sin(d*x
+ c))/a^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

Fricas [A] time = 1.5958, size = 271, normalized size = 3.23

$$-\frac{2\,ab\sin\left(dx+c\right)-a^{2}-2\left(\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}-a^{2}+b^{2}\right)\log\left(b\sin\left(dx+c\right)+a\right)+2\left(\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}-a^{2}+b^{2}}{2\left(a^{3}d\cos\left(dx+c\right)^{2}-a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) - a^2 - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)
*log(b*sin(d*x + c) + a) + 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(-
1/2*sin(d*x + c)))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot^{3}\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 2.0115, size = 154, normalized size = 1.83

$$-\frac{\frac{2\left(a^{2}-b^{2}\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{3}}-\frac{2\left(a^{2}b-b^{3}\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{3}b}-\frac{3\,a^{2}\sin\left(dx+c\right)^{2}-3\,b^{2}\sin\left(dx+c\right)^{2}+2\,ab\sin\left(dx+c\right)-a^{2}}{a^{3}\sin\left(dx+c\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^2*b - b^3)*log(abs(b*
sin(d*x + c) + a))/(a^3*b) - (3*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 +
 2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2))/d
```

3.175 $\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{\left(2a^2-b^2\right)\csc^2(c+dx)}{2a^3d}-\frac{b\left(2a^2-b^2\right)\csc(c+dx)}{a^4d}+\frac{\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^5d}-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^5d}+\frac{b\,c}{}$$

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 -
 b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]
)/(a^5*d)
```

Rubi [A] time = 0.138352, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{\left(2a^2-b^2\right)\csc^2(c+dx)}{2a^3d}-\frac{b\left(2a^2-b^2\right)\csc(c+dx)}{a^4d}+\frac{\left(a^2-b^2\right)^2\log(\sin(c+dx))}{a^5d}-\frac{\left(a^2-b^2\right)^2\log(a+b\sin(c+dx))}{a^5d}+\frac{b\,c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] -((b*(2*a^2 - b^2)*Csc[c + d*x])/(a^4*d)) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/
(2*a^3*d) + (b*Csc[c + d*x]^3)/(3*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a^2 -
 b^2)^2*Log[Sin[c + d*x]])/(a^5*d) - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]]
)/(a^5*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^5(a+x)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{ax^5}-\frac{b^4}{a^2x^4}+\frac{-2a^2b^2+b^4}{a^3x^3}+\frac{2a^2b^2-b^4}{a^4x^2}+\frac{\left(a^2-b^2\right)^2}{a^5x}-\frac{\left(a^2-b^2\right)^2}{a^5(a+x)}\right)dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{b\left(2a^2-b^2\right)\csc(c+dx)}{a^4d}+\frac{\left(2a^2-b^2\right)\csc^2(c+dx)}{2a^3d}+\frac{b\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4ad}+\frac{\left(a^2-\right.}{}$$

Mathematica [A] time = 3.84689, size = 115, normalized size = 0.78

$$\frac{6a^2\left(2a^2-b^2\right)\csc^2(c+dx)+12ab\left(b^2-2a^2\right)\csc(c+dx)+12\left(a^2-b^2\right)^2(\log(\sin(c+dx))-\log(a+b\sin(c+dx)))+4a}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

```
[Out] (12*a*b*(-2*a^2 + b^2)*Csc[c + d*x] + 6*a^2*(2*a^2 - b^2)*Csc[c + d*x]^2 +
4*a^3*b*Csc[c + d*x]^3 - 3*a^4*Csc[c + d*x]^4 + 12*(a^2 - b^2)^2*(Log[Sin[c
 + d*x]] - Log[a + b*Sin[c + d*x]]))/(12*a^5*d)
```

Maple [A] time = 0.063, size = 216, normalized size = 1.5

$$-\frac{\ln(a+b\sin(dx+c))}{da}+2\frac{\ln(a+b\sin(dx+c))b^2}{da^3}-\frac{\ln(a+b\sin(dx+c))b^4}{da^5}-\frac{1}{4da(\sin(dx+c))^4}+\frac{1}{da(\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c)),x)
```

```
[Out] -1/d/a*ln(a+b*sin(d*x+c))+2/d/a^3*ln(a+b*sin(d*x+c))*b^2-1/d/a^5*ln(a+b*sin
(d*x+c))*b^4-1/4/d/a/sin(d*x+c)^4+1/d/a/sin(d*x+c)^2-1/2/d/a^3/sin(d*x+c)^2
```

```
*b^2+ln(sin(d*x+c))/a/d-2/d/a^3*ln(sin(d*x+c))*b^2+1/d/a^5*ln(sin(d*x+c))*b
^4-2/d/a^2*b/sin(d*x+c)+1/d/a^4*b^3/sin(d*x+c)+1/3/d/a^2*b/sin(d*x+c)^3
```

Maxima [A] time = 1.46289, size = 188, normalized size = 1.27

$$-\frac{\frac{12\left(a^4-2a^2b^2+b^4\right)\log\left(b\sin(dx+c)+a\right)}{a^5}-\frac{12\left(a^4-2a^2b^2+b^4\right)\log\left(\sin(dx+c)\right)}{a^5}-\frac{4a^2b\sin(dx+c)-12\left(2a^2b-b^3\right)\sin(dx+c)^3-3a^3+6\left(2a^3-ab^2\right)\sin(dx+c)^2}{a^4\sin(dx+c)^4}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/a^5 - 12*(a^4 - 2
*a^2*b^2 + b^4)*log(sin(d*x + c))/a^5 - (4*a^2*b*sin(d*x + c) - 12*(2*a^2*b
 - b^3)*sin(d*x + c)^3 - 3*a^3 + 6*(2*a^3 - a*b^2)*sin(d*x + c)^2)/(a^4*sin
(d*x + c)^4))/d
```

Fricas [A] time = 1.663, size = 629, normalized size = 4.25

$$\frac{9a^4-6a^2b^2-6\left(2a^4-a^2b^2\right)\cos(dx+c)^2-12\left(\left(a^4-2a^2b^2+b^4\right)\cos(dx+c)^4+a^4-2a^2b^2+b^4-2\left(a^4-2a^2b^2+b^4\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(9*a^4 - 6*a^2*b^2 - 6*(2*a^4 - a^2*b^2)*cos(d*x + c)^2 - 12*((a^4 - 2
*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2
 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) + 12*((a^4 - 2*a^2*b^2 + b^
4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d
*x + c)^2)*log(-1/2*sin(d*x + c)) - 4*(5*a^3*b - 3*a*b^3 - 3*(2*a^3*b - a*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos(d*x +
 c)^2 + a^5*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.53765, size = 271, normalized size = 1.83

$$\frac{\frac{12\left(a^4-2a^2b^2+b^4\right)\log(|\sin(dx+c)|)}{a^5}-\frac{12\left(a^4b-2a^2b^3+b^5\right)\log(|b\sin(dx+c)+a|)}{a^5b}-\frac{25a^4\sin(dx+c)^4-50a^2b^2\sin(dx+c)^4+25b^4\sin(dx+c)^4+24a^3b\sin(dx+c)^3-}{a^5\sin}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(sin(d*x + c)))/a^5 - 12*(a^4*b - 2
*a^2*b^3 + b^5)*log(abs(b*sin(d*x + c) + a))/(a^5*b) - (25*a^4*sin(d*x + c)
^4 - 50*a^2*b^2*sin(d*x + c)^4 + 25*b^4*sin(d*x + c)^4 + 24*a^3*b*sin(d*x +
 c)^3 - 12*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 6*a^2*b^2*sin(d*x
 + c)^2 - 4*a^3*b*sin(d*x + c) + 3*a^4)/(a^5*sin(d*x + c)^4))/d
```

3.176 $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^3\tan(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2b\sec(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2
)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^
2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.239069, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2727, 2607, 30, 2606, 3767, 8, 2660, 618, 204}

$$\frac{2a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{a^3\tan(c+dx)}{d\left(a^2-b^2\right)^2}-\frac{b\sec^3(c+dx)}{3d\left(a^2-b^2\right)}+\frac{a^2b\sec(c+dx)}{d\left(a^2-b^2\right)^2}+\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2)
*d) + (a^2*b*Sec[c + d*x])/((a^2 - b^2)^2*d) + (b*Sec[c + d*x])/((a^2 - b^2
)*d) - (b*Sec[c + d*x]^3)/(3*(a^2 - b^2)*d) - (a^3*Tan[c + d*x])/((a^2 - b^
2)^2*d) + (a*Tan[c + d*x]^3)/(3*(a^2 - b^2)*d)
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan^3(c+dx)\,dx}{a^2-b^2}$$

$$= -\frac{a^3\int \sec^2(c+dx)\,dx}{\left(a^2-b^2\right)^2} + \frac{a^4\int \frac{1}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2} + \frac{\left(a^2b\right)\int \sec(c+dx)\tan(c+dx)\,dx}{\left(a^2-b^2\right)^2} + \frac{a\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{a^3\,\mathrm{Subst}\left(\int 1\,dx, x, -\tan(c+dx)\right)}{\left(a^2-b^2\right)^2 d} + \frac{\left(2a^4\right)\mathrm{S}}{}$$

$$= \frac{a^2b\sec(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^3\tan(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{a\tan^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{\left(4a^4\right)\mathrm{Su}}{}$$

$$= \frac{2a^4\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d} + \frac{a^2b\sec(c+dx)}{\left(a^2-b^2\right)^2 d} + \frac{b\sec(c+dx)}{\left(a^2-b^2\right)d} - \frac{b\sec^3(c+dx)}{3\left(a^2-b^2\right)d} - \frac{a^3\tan(c+dx}{\left(a^2-b^2\right)^2 d}$$

Mathematica [A] time = 1.41876, size = 195, normalized size = 1.1

$$\frac{\frac{48a^4\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} - \frac{\sec^3(c+dx)\left(3b\left(11a^2-5b^2\right)\cos(c+dx)+12b\left(b^2-2a^2\right)\cos(2(c+dx))+11a^2b\cos(3(c+dx))-16a^2b+8a^3\sin(3(c+dx))+6ab^2\sin(c+dx\right.}{(a-b)^2(a+b)^2}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] ((48*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
) - (Sec[c + d*x]^3*(-16*a^2*b + 4*b^3 + 3*b*(11*a^2 - 5*b^2)*Cos[c + d*x]
+ 12*b*(-2*a^2 + b^2)*Cos[2*(c + d*x)] + 11*a^2*b*Cos[3*(c + d*x)] - 5*b^3*
Cos[3*(c + d*x)] + 6*a*b^2*Sin[c + d*x] + 8*a^3*Sin[3*(c + d*x)] - 2*a*b^2*
Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

Maple [A] time = 0.067, size = 269, normalized size = 1.5

$$-\frac{32}{3\,d\,(32\,a+32\,b)}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-16\,\frac{1}{d\,(32\,a+32\,b)\,(\tan\,(1/2\,dx+c/2)-1)^{2}}+\frac{a}{d\,(a+b)^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] -32/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(32*a+32*b)-16/d/(32*a+32*b)/(tan(1/2*d*x+
1/2*c)-1)^2+1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a+1/2/d/(a+b)^2/(tan(1/2*d*x
+1/2*c)-1)*b+2/d*a^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-32/3/d/(tan(1/2*d*x+1/2*c)+1)^3/(32*a-32
*b)+16/d/(32*a-32*b)/(tan(1/2*d*x+1/2*c)+1)^2+1/d/(a-b)^2/(tan(1/2*d*x+1/2*
c)+1)*a-1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.63934, size = 1054, normalized size = 5.95

$$\left[-\frac{3\sqrt{-a^{2}+b^{2}}a^{4}\cos\left(dx+c\right)^{3}\log\left(\frac{\left(2\,a^{2}-b^{2}\right)\cos(dx+c)^{2}-2\,ab\sin(dx+c)-a^{2}-b^{2}+2\,(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^{2}+b^{2}}}{b^{2}\cos(dx+c)^{2}-2\,ab\sin(dx+c)-a^{2}-b^{2}}\right)+2\,a^{4}b-4}{6\left(a^{6}-3\,a^{4}b^{2}+3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(-a^2 + b^2)*a^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*c
os(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(2*a^4*b - 3*a^2*b^3 + b^5)*co
s(d*x + c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x +
 c)^3), -1/3*(3*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2
- b^2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*(2*a^4*b
 - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*
a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 3.30637, size = 325, normalized size = 1.84

$$\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{\left(a^4-2a^2b^2+b^4\right)\sqrt{a^2-b^2}}+\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12a^2b\,\mathrm{t}\cdots}{\left(a^4-2a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\cdots}\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*a^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2))
+ (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*a^3*t
an(1/2*d*x + 1/2*c)^3 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1/2*d
*x + 1/2*c)^2 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) -
```

```
 5*a^2*b + 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))
/d
```

3.177 $\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.0988934, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2727, 3767, 8, 2606, 2660, 618, 204}

$$-\frac{2a^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(3/2
)*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)
```

Rule 2727

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[(b*g)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[(a^2*g^2)/(a^2 - b^2), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\int \sec^2(c+dx)\,dx}{a^2-b^2} - \frac{a^2\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2-b^2} - \frac{b\int \sec(c+dx)\tan(c+dx)\,dx}{a^2-b^2}$$

$$= -\frac{a\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{(a^2-b^2)\,d} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)\,d} - \frac{b\,\text{Sub}}{}$$

$$= -\frac{b\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d} + \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{-4(a^2-b^2)-x^2}\,dx, x, 2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2-b^2)\,d}$$

$$= -\frac{2a^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{b\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d}$$

Mathematica [A] time = 0.202612, size = 152, normalized size = 1.58

$$\frac{\sqrt{a^2-b^2}(a\sin(c+dx)+b\cos(c+dx)-b)-2a^2\cos(c+dx)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a-b)(a+b)\sqrt{a^2-b^2}\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sqr
t[a^2 - b^2]*(-b + b*Cos[c + d*x] + a*Sin[c + d*x]))/((a - b)*(a + b)*Sqrt[
a^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]))
```

Maple [A] time = 0.053, size = 117, normalized size = 1.2

$$-2\,\frac{a^2}{d\,(a-b)\,(a+b)\,\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)+2\,b}{\sqrt{a^2-b^2}}\right)-8\,\frac{1}{d\,(8\,a-8\,b)\,(\tan(1/2\,dx+c/2)+1)}-8\,\frac{}{d\,(8\,a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] -2/d*a^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b
)/(a^2-b^2)^(1/2))-8/d/(8*a-8*b)/(tan(1/2*d*x+1/2*c)+1)-8/d/(8*a+8*b)/(tan(
1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67045, size = 684, normalized size = 7.12

$$\left[\frac{\sqrt{-a^2+b^2}a^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)-2a^2b+2b^3 -}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a^2 + b^2)*a^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*
x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 -
b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
 b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) + b)/(s
qrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*si
n(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 2.05295, size = 144, normalized size = 1.5

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{\left(a^2-b^2\right)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b}{\left(a^2-b^2\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*
c) + b)/sqrt(a^2 - b^2)))*a^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c) -
 b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.178 $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] (-2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*
d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rubi [A] time = 0.242757, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^2d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2*
d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
```

```
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx$$

$$= -\frac{\cot(c+dx)}{ad} + \frac{\int \frac{\csc(c+dx)(-b-a\sin(c+dx))}{a+b\sin(c+dx)}\,dx}{a}$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{b\int \csc(c+dx)\,dx}{a^2} + \frac{\left(-a^2+b^2\right)\int \frac{1}{a+b\sin(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} - \frac{\left(2\left(a^2-b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right.\right.}{a^2d}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad} + \frac{\left(4\left(a^2-b^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\right.}{a^2d}$$

$$= -\frac{2\sqrt{a^2-b^2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d} + \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A] time = 0.235631, size = 108, normalized size = 1.35

$$\frac{-4\sqrt{a^2-b^2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) - 2b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2b\log\left(\cos\left(\frac{1}{2}(\right.\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - a*Co
t[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]] + a*
Tan[(c + d*x)/2])/(2*a^2*d)
```

Maple [B] time = 0.06, size = 155, normalized size = 1.9

$$\frac{1}{2\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) - 2\,\frac{1}{d\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)+2\,b}{\sqrt{a^2-b^2}}\right) + 2\,\frac{b^2}{da^2\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan(1/2\,dx}{\sqrt{a^2-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/a*tan(1/2*d*x+1/2*c)-2/d/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+
1/2*c)+2*b)/(a^2-b^2)^(1/2))+2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/2/d/a/tan(1/2*d*x+1/2*c)-1/d/a^2*b
*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.74227, size = 801, normalized size = 10.01

$$\left[\frac{b \log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right) - b \log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right) + \sqrt{-a^2+b^2}\log\left(\frac{\left(2a^2-b^2\right)\cos(dx+c)^2-2ab\,s}{\cdots}\right.\cdots}{2a^2d\sin\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c)), 1/2*(b*log(1/2*
cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d
*x + c)))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.86967, size = 174, normalized size = 2.17

$$-\frac{\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}+\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{a^2}-\frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a + 4*(
pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) +
b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/a^2 - (2*b*tan(1/2*d*x + 1/2*c) - a)/(
a^2*tan(1/2*d*x + 1/2*c)))/d
```

3.179 $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=154

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{b\cot(c+dx)\csc(}{2a^2d}$$

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2
)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rubi [A] time = 0.425707, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2725, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d}+\frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d}-\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d}+\frac{b\cot(c+dx)\csc(}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^4
*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(2*a^4*d) + ((4*a^2 - 3*b^2
)*Cot[c + d*x])/(3*a^3*d) + (b*Cot[c + d*x]*Csc[c + d*x])/(2*a^2*d) - (Cot[
c + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

Rule 2725

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m*Simp[8*a^2 -
 b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*m*(m - 2))*Sin[e +
 f*x]^2, x])/Sin[e + f*x]^2, x], x] - Simp[(b*(m - 2)*Cos[e + f*x]*(a + b*S
in[e + f*x])^(m + 1))/(6*a^2*f*Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f, m
}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \frac{\int \frac{\csc^2(c+dx)\left(2\left(4a^2-3b^2\right)-ab\sin(c+dx)-3\left(2a^2-b^2\right)}{a+b\sin(c+dx)}}{6a^2}$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} - \int \frac{\csc(c+dx)\left(-3\right.}{}$$

$$= \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad} + \frac{\left(b\left(3a^2-2b^2\right.\right.}{}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{c}{}$$

$$= -\frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{c}{}$$

$$= \frac{2\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b\left(3a^2-2b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^4d} + \frac{\left(4a^2-3b^2\right)\cot(c}{3a^3d}$$

Mathematica [B] time = 6.11882, size = 350, normalized size = 2.27

$$\frac{\left(3a^2b-2b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\left(2b^3-3a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{\csc\left(\frac{1}{2}(c+dx)\right)\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\right.}{6a^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[(
c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((4*a^2*Cos[(c + d*x)/2] - 3*b^2*
Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(8*a
^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-3*a^2*b + 2*b^3
)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((3*a^2*b - 2*b^3)*Log[Sin[(c + d*x)/2
]])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*(-4*a^
2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c + d*x)/2]
```

```
^2*Tan[(c + d*x)/2])/(24*a*d)
```

Maple [B] time = 0.069, size = 348, normalized size = 2.3

$$\frac{1}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{b}{8\,da^{2}}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{5}{8\,da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{b^{2}}{2\,da^{3}}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{1}{d\sqrt{a^{2}-b^{2}}}\arctan\left(1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c)),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3-1/8/d/a^2*b*tan(1/2*d*x+1/2*c)^2-5/8/d/a*tan(
1/2*d*x+1/2*c)+1/2/d/a^3*b^2*tan(1/2*d*x+1/2*c)+2/d/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-4/d/a^2/(a^2-b^2)^(1/2)*a
rctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+2/d/a^4/(a^2-b^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-1/24/
d/a/tan(1/2*d*x+1/2*c)^3+5/8/d/a/tan(1/2*d*x+1/2*c)-1/2/d/a^3/tan(1/2*d*x+1
/2*c)*b^2+1/8/d/a^2*b/tan(1/2*d*x+1/2*c)^2+3/2/d/a^2*b*ln(tan(1/2*d*x+1/2*c
))-1/d/a^4*b^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.92888, size = 1507, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 - 3*a*b^2)*cos(d*x + c
)^3 + 6*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a^2 + b^2)*log(((2*a
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2
*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*
b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*
a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 12*(a^3 - a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 -
a^4*d)*sin(d*x + c)), -1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 -
 3*a*b^2)*cos(d*x + c)^3 + 12*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt
(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*si
n(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2
*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*
cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 12*(a^3 - a*b^2
)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 1.91054, size = 369, normalized size = 2.4

$$\frac{\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-15a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}+\frac{12\left(3a^2b-2b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}+\frac{48\left(a^4-2a^2b^2+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\right.\right.}{}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*t
an(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(3*a^2*b - 2*b^
```

```
3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*(pi*floo
r(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt
(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (66*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 44
*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*tan(
1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/2*d*x +
 1/2*c)^3))/d
```

3.180 $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

Optimal. Leaf size=307

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{\left(-35a^2b^2+23a^4+15b^4\right)\cot(c+dx)}{15a^5d}+\frac{b\left(-20a^2b^2+15a^4+8b^4\right)\tanh^{-1}(\cos(c}{8a^6d}$$

```
[Out] (-2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) -
((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(b*d) + ((8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x])
/(8*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*b^2*d) - ((15*a^4 - 22*a^
2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^3*b^2*d) + (b*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```

Rubi [A] time = 1.10601, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d}-\frac{\left(-35a^2b^2+23a^4+15b^4\right)\cot(c+dx)}{15a^5d}+\frac{b\left(-20a^2b^2+15a^4+8b^4\right)\tanh^{-1}(\cos(c}{8a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```


```
[Out] (-2*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
6*d) + (b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(8*a^6*d) -
((23*a^4 - 35*a^2*b^2 + 15*b^4)*Cot[c + d*x])/(15*a^5*d) - (Cot[c + d*x]*Cs
c[c + d*x])/(b*d) + ((8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[c + d*x]*Csc[c + d*x])
/(8*a^4*b*d) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*b^2*d) - ((15*a^4 - 22*a^
2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^3*b^2*d) + (b*Cot[c + d*
x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d)
```


Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
```

```
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
 - 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
 - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+}{}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} - \frac{\left(15a^4-22a^2b^2+10b^4\right)\cot(c+dx)\,\mathrm{cs}}{30a^3b^2d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{8a^4bd} + \frac{a\cot(c+dx)\csc^2(c}{2b^2d}$$

$$= -\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(c+}{8a^4bd}$$

$$= -\frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{\left(8a^4-9a^2b^2+4b^4\right)\cot(c+}{8a^4bd}$$

$$= \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c+}{}$$

$$= \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{15a^5d} - \frac{\cot(c+}{}$$

$$= -\frac{2\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b\left(15a^4-20a^2b^2+8b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^6d} - \frac{\left(23a^4\right.}{}$$

Mathematica [A] time = 1.38587, size = 504, normalized size = 1.64

$$-1920\left(a^2-b^2\right)^{5/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)-1120a^3b^2\tan\left(\frac{1}{2}(c+dx)\right)-32\left(-35a^3b^2+23a^5+15ab^4\right)\cot\left(\frac{1}{2}(c+dx)\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

```
[Out] (-1920*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] -
 32*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 270*a^4*b*Csc[(c +
d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4 +
1800*a^4*b*Log[Cos[(c + d*x)/2]] - 2400*a^2*b^3*Log[Cos[(c + d*x)/2]] + 960
*b^5*Log[Cos[(c + d*x)/2]] - 1800*a^4*b*Log[Sin[(c + d*x)/2]] + 2400*a^2*b^
3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 270*a^4*b*Sec[(c
+ d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x)/2]^4
- 656*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d*x]^3*Si
n[(c + d*x)/2]^4 + 41*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3*b^2*Csc[
(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*x] + 736*a
^5*Tan[(c + d*x)/2] - 1120*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*Tan[(c + d*
x)/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

Maple [B] time = 0.073, size = 629, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c)),x)
```

```
[Out] -15/8/d/a^2*b*ln(tan(1/2*d*x+1/2*c))+5/2/d/a^4*b^3*ln(tan(1/2*d*x+1/2*c))+1
/4/d/a^2*b*tan(1/2*d*x+1/2*c)^2-9/8/d/a^3*b^2*tan(1/2*d*x+1/2*c)+9/8/d/a^3/
tan(1/2*d*x+1/2*c)*b^2-1/4/d/a^2*b/tan(1/2*d*x+1/2*c)^2+6/d/a^2/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/160/d/a
/tan(1/2*d*x+1/2*c)^5+1/160/d/a*tan(1/2*d*x+1/2*c)^5-7/96/d/a*tan(1/2*d*x+1
/2*c)^3+7/96/d/a/tan(1/2*d*x+1/2*c)^3-1/2/d/a^5/tan(1/2*d*x+1/2*c)*b^4-1/64
/d/a^2*b*tan(1/2*d*x+1/2*c)^4+1/24/d/a^3*b^2*tan(1/2*d*x+1/2*c)^3-1/8/d/a^4
*tan(1/2*d*x+1/2*c)^2*b^3+1/2/d/a^5*b^4*tan(1/2*d*x+1/2*c)+1/64/d/a^2*b/tan
(1/2*d*x+1/2*c)^4+1/8/d/a^4*b^3/tan(1/2*d*x+1/2*c)^2-1/d/a^6*b^5*ln(tan(1/2
*d*x+1/2*c))-1/24/d/a^3/tan(1/2*d*x+1/2*c)^3*b^2-2/d/(a^2-b^2)^(1/2)*arctan
```

```
(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+11/16/d/a*tan(1/2*d*x+1/
2*c)-11/16/d/a/tan(1/2*d*x+1/2*c)+2/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6-6/d/a^4/(a^2-b^2)^(1/2)*arctan(
1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.83409, size = 2566, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/240*(16*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 1
3*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 120*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x
+ c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*
sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2
))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 15
*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x +
 c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 2
0*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d
*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2
 + a*b^4)*cos(d*x + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*
b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d
*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(16*(23*a^5 - 35*a^3*b^2 + 1
5*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 13*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3
- 240*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(
a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x +
```

```
 c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 15*(15*a^4*b - 20*a
^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4
*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*
x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5
)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(-1
/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x
 + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)^3 - (7*a^4*b - 4*a^2*b^3)*co
s(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c)^4 - 2*a^6*d*cos(d*x + c)^2 +
 a^6*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x)), x)
```

Giac [A] time = 2.21212, size = 662, normalized size = 2.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 70
*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^3*b
*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^4*tan(1/
2*d*x + 1/2*c) - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*x +
1/2*c))/a^5 - 120*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x + 1/2
*c)))/a^6 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d*x + c
)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/
(sqrt(a^2 - b^2)*a^6) + (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 5480*a^2*b^3*t
an(1/2*d*x + 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(1/2*d
*x + 1/2*c)^4 + 1080*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*tan(1/2*d*x
 + 1/2*c)^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan(1/2*d*x +
```

```
1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^
2 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x + 1/2*c)^5))/d
```

3.181 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=242

$$-\frac{a^5}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a^4\left(a^2+5b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}+\frac{\sec^4(c+dx)\left(a^2-2ab\sin(c+dx)+b^2\right)}{4d\left(a^2-b^2\right)^2}-\frac{\text{sec}}{}$$

```
[Out] -(a*(4*a + b)*Log[1 - Sin[c + d*x]])/(8*(a + b)^4*d) - (a*(4*a - b)*Log[1 +
 Sin[c + d*x]])/(8*(a - b)^4*d) + (a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]
])/((a^2 - b^2)^4*d) - a^5/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[c
+ d*x]^4*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(4*(a^2 - b^2)^2*d) - (Sec[c + d
*x]^2*(2*(2*a^4 + 3*a^2*b^2 - b^4) - a*b*(9*a^2 - b^2)*Sin[c + d*x]))/(4*(a
^2 - b^2)^3*d)
```

Rubi [A] time = 0.632621, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 1629}

$$-\frac{a^5}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a^4\left(a^2+5b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}+\frac{\sec^4(c+dx)\left(a^2-2ab\sin(c+dx)+b^2\right)}{4d\left(a^2-b^2\right)^2}-\frac{\text{sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] -(a*(4*a + b)*Log[1 - Sin[c + d*x]])/(8*(a + b)^4*d) - (a*(4*a - b)*Log[1 +
 Sin[c + d*x]])/(8*(a - b)^4*d) + (a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]
])/((a^2 - b^2)^4*d) - a^5/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[c
+ d*x]^4*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(4*(a^2 - b^2)^2*d) - (Sec[c + d
*x]^2*(2*(2*a^4 + 3*a^2*b^2 - b^4) - a*b*(9*a^2 - b^2)*Sin[c + d*x]))/(4*(a
^2 - b^2)^3*d)
```


Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^2\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{4\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^6}{\left(a^2-b^2\right)^2}-\frac{4a^4b^4x}{\left(a^2-b^2\right)^2}-\frac{6ab^6x^2}{\left(a^2-b^2\right)^2}-4b^2x^3}{(a+x)^2\left(b^2-x^2\right)^2}\,dx, x, b\sin\right.}{4b^2d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{4\left(a^2-b^2\right)^2 d} - \frac{\sec^2(c+dx)\left(2\left(2a^4+3a^2b^2-b^4\right)-ab\left(9a^2-b^2\right)\right.}{4\left(a^2-b^2\right)^3 d}$$

$$= \frac{\sec^4(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{4\left(a^2-b^2\right)^2 d} - \frac{\sec^2(c+dx)\left(2\left(2a^4+3a^2b^2-b^4\right)-ab\left(9a^2-b^2\right)\right.}{4\left(a^2-b^2\right)^3 d}$$

$$= -\frac{a(4a+b)\log(1-\sin(c+dx))}{8(a+b)^4d} - \frac{a(4a-b)\log(1+\sin(c+dx))}{8(a-b)^4d} + \frac{a^4\left(a^2+5b^2\right)\log(a+b\,\text{si}}{\left(a^2-b^2\right)^4 d}$$

Mathematica [A] time = 6.25186, size = 240, normalized size = 0.99

$$-\frac{a^5}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}+\frac{a^4\left(a^2+5b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}-\frac{7a+3b}{16d(a+b)^3(1-\sin(c+dx))}-\frac{7a-}{16d(a-b)^3(\sin}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*(4*a + b)*Log[1 - Sin[c + d*x]])/(8*(a + b)^4*d) - (a*(4*a - b)*Log[1 +
 Sin[c + d*x]])/(8*(a - b)^4*d) + (a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]
])/((a^2 - b^2)^4*d) + 1/(16*(a + b)^2*d*(1 - Sin[c + d*x])^2) - (7*a + 3*b
)/(16*(a + b)^3*d*(1 - Sin[c + d*x])) + 1/(16*(a - b)^2*d*(1 + Sin[c + d*x]
)^2) - (7*a - 3*b)/(16*(a - b)^3*d*(1 + Sin[c + d*x])) - a^5/((a^2 - b^2)^3
*d*(a + b*Sin[c + d*x]))
```

Maple [A] time = 0.093, size = 318, normalized size = 1.3

$$-\frac{a^5}{d(a+b)^3(a-b)^3(a+b\sin(dx+c))}+\frac{a^6\ln(a+b\sin(dx+c))}{d(a+b)^4(a-b)^4}+5\frac{a^4\ln(a+b\sin(dx+c))b^2}{d(a+b)^4(a-b)^4}+\frac{1}{16d(a+b)^2(\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))+1/d*a^6/(a+b)^4/(a-b)^4*ln(a+b*si
n(d*x+c))+5/d*a^4/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*b^2+1/16/d/(a+b)^2/(si
n(d*x+c)-1)^2+3/16/d/(a+b)^3/(sin(d*x+c)-1)*b+7/16/d/(a+b)^3/(sin(d*x+c)-1)
*a-1/2/d*a^2/(a+b)^4*ln(sin(d*x+c)-1)-1/8/d*a/(a+b)^4*ln(sin(d*x+c)-1)*b+1/
16/d/(a-b)^2/(1+sin(d*x+c))^2+3/16/d/(a-b)^3/(1+sin(d*x+c))*b-7/16/d/(a-b)^
3/(1+sin(d*x+c))*a-1/2/d*a^2/(a-b)^4*ln(1+sin(d*x+c))+1/8/d*a/(a-b)^4*ln(1+
sin(d*x+c))*b
```

Maxima [B] time = 1.8279, size = 682, normalized size = 2.82

$$\frac{\frac{8\left(a^6+5a^4b^2\right)\log(b\sin(dx+c)+a)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{\left(4a^2-ab\right)\log(\sin(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{\left(4a^2+ab\right)\log(\sin(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{2\left(7a^5+6a^3b^2-ab^4+\left(4a^5\right.\right.}{a^7-3a^5b^2+3a^3b^4-ab^6+\left(a^6b-3a^4b^3+3a^2b^5-b^7\right)\sin(dx+}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(a^6 + 5*a^4*b^2)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*b
^4 - 4*a^2*b^6 + b^8) - (4*a^2 - a*b)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*b
+ 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + a*b)*log(sin(d*x + c) - 1)/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(7*a^5 + 6*a^3*b^2 - a*b^4 + (4*a^5
 + 9*a^3*b^2 - a*b^4)*sin(d*x + c)^4 + (5*a^4*b - 7*a^2*b^3 + 2*b^5)*sin(d*
x + c)^3 - (12*a^5 + 13*a^3*b^2 - a*b^4)*sin(d*x + c)^2 - (4*a^4*b - 5*a^2*
b^3 + b^5)*sin(d*x + c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^5 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
a*b^6)*sin(d*x + c)^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c
)^3 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^2 + (a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)))/d
```

Fricas [B] time = 3.38575, size = 1231, normalized size = 5.09

$$\frac{2\,a^{7}-6\,a^{5}b^{2}+6\,a^{3}b^{4}-2\,ab^{6}-2\left(4\,a^{7}+5\,a^{5}b^{2}-10\,a^{3}b^{4}+ab^{6}\right)\cos\left(dx+c\right)^{4}-2\left(4\,a^{7}-9\,a^{5}b^{2}+6\,a^{3}b^{4}-ab^{6}\right)\cos\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 2*(4*a^7 + 5*a^5*b^2 - 10*a^
3*b^4 + a*b^6)*cos(d*x + c)^4 - 2*(4*a^7 - 9*a^5*b^2 + 6*a^3*b^4 - a*b^6)*c
os(d*x + c)^2 + 8*((a^6*b + 5*a^4*b^3)*cos(d*x + c)^4*sin(d*x + c) + (a^7 +
 5*a^5*b^2)*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) - ((4*a^6*b + 15*a^5*b^
2 + 20*a^4*b^3 + 10*a^3*b^4 - a*b^6)*cos(d*x + c)^4*sin(d*x + c) + (4*a^7 +
 15*a^6*b + 20*a^5*b^2 + 10*a^4*b^3 - a^2*b^5)*cos(d*x + c)^4)*log(sin(d*x
+ c) + 1) - ((4*a^6*b - 15*a^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d
*x + c)^4*sin(d*x + c) + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*
b^5)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*
b^5 - b^7 - (5*a^6*b - 12*a^4*b^3 + 9*a^2*b^5 - 2*b^7)*cos(d*x + c)^2)*sin(
d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)
^4*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d
*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

Giac [B] time = 4.67247, size = 667, normalized size = 2.76

$$\frac{\frac{8\left(a^6b+5a^4b^3\right)\log(|b\sin(dx+c)+a|)}{a^8b-4a^6b^3+6a^4b^5-4a^2b^7+b^9}-\frac{\left(4a^2-ab\right)\log(|\sin(dx+c)+1|)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}-\frac{\left(4a^2+ab\right)\log(|\sin(dx+c)-1|)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{8\left(a^6b\sin(dx+c)+5a^4b^3\sin(dx+c)+2a^7+4a^5b^2\right)}{\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)(b\sin(dx+c)+a)}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*(a^6*b + 5*a^4*b^3)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3
+ 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (4*a^2 - a*b)*log(abs(sin(d*x + c) + 1))/(
a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + a*b)*log(abs(sin(d*x
+ c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 8*(a^6*b*sin(d*x +
 c) + 5*a^4*b^3*sin(d*x + c) + 2*a^7 + 4*a^5*b^2)/((a^8 - 4*a^6*b^2 + 6*a^4
*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c) + a)) + 2*(3*a^6*sin(d*x + c)^4 + 1
5*a^4*b^2*sin(d*x + c)^4 - 9*a^5*b*sin(d*x + c)^3 + 10*a^3*b^3*sin(d*x + c)
^3 - a*b^5*sin(d*x + c)^3 - 2*a^6*sin(d*x + c)^2 - 28*a^4*b^2*sin(d*x + c)^
2 - 8*a^2*b^4*sin(d*x + c)^2 + 2*b^6*sin(d*x + c)^2 + 7*a^5*b*sin(d*x + c)
- 6*a^3*b^3*sin(d*x + c) - a*b^5*sin(d*x + c) + 12*a^4*b^2 + 7*a^2*b^4 - b^
6)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)^2)
)/d
```

3.182 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{a^3}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{a^2\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^2(c+dx)\left(a^2-2ab\sin(c+dx)+b^2\right)}{2d\left(a^2-b^2\right)^2}+\frac{a\log(}{}$$

```
[Out] (a*Log[1 - Sin[c + d*x]])/(2*(a + b)^3*d) + (a*Log[1 + Sin[c + d*x]])/(2*(a
 - b)^3*d) - (a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d)
+ a^3/((a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(a^2 + b^2 -
 2*a*b*Sin[c + d*x]))/(2*(a^2 - b^2)^2*d)
```

Rubi [A] time = 0.311696, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 1629}

$$\frac{a^3}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{a^2\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}+\frac{\sec^2(c+dx)\left(a^2-2ab\sin(c+dx)+b^2\right)}{2d\left(a^2-b^2\right)^2}+\frac{a\log(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (a*Log[1 - Sin[c + d*x]])/(2*(a + b)^3*d) + (a*Log[1 + Sin[c + d*x]])/(2*(a
 - b)^3*d) - (a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d)
+ a^3/((a^2 - b^2)^2*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x]^2*(a^2 + b^2 -
 2*a*b*Sin[c + d*x]))/(2*(a^2 - b^2)^2*d)
```


Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^2\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^4}{\left(a^2-b^2\right)^2}-\frac{2a^2b^2x}{a^2-b^2}-\frac{2ab^4x^2}{\left(a^2-b^2\right)^2}}{(a+x)^2\left(b^2-x^2\right)}\,dx, x, b\sin(c+d\right.}{2b^2d}$$

$$= \frac{\sec^2(c+dx)\left(a^2+b^2-2ab\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int\left(-\frac{ab^2}{(a+b)^3(b-x)}-\frac{2a^3b^2}{(a-b)^2(a+b)^2(a+x)^2}-\frac{2a^2}{(a-b)}\right.\right.}{2b^2d}$$

$$= \frac{a\log(1-\sin(c+dx))}{2(a+b)^3d} + \frac{a\log(1+\sin(c+dx))}{2(a-b)^3d} - \frac{a^2\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^3 d} + \frac{}{(a}$$

Mathematica [A] time = 0.719396, size = 145, normalized size = 0.9

$$\frac{\frac{4a^3}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))}-\frac{4a^2\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^3}-\frac{1}{(a+b)^2(\sin(c+dx)-1)}+\frac{1}{(a-b)^2(\sin(c+dx)+1)}+\frac{2a\log(1-\sin(c+dx))}{(a+b)^3}+\frac{2a\log(\sin(c+dx)}{(a-b)^3}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a*Log[1 - Sin[c + d*x]])/(a + b)^3 + (2*a*Log[1 + Sin[c + d*x]])/(a - b
)^3 - (4*a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^3 - 1/((a +
```

```
 b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)^2*(1 + Sin[c + d*x])) + (4*a^3)/((a
^2 - b^2)^2*(a + b*Sin[c + d*x])))/(4*d)
```

Maple [A] time = 0.087, size = 182, normalized size = 1.1

$$\frac{a^3}{d\left(a+b\right)^2\left(a-b\right)^2\left(a+b\sin\left(dx+c\right)\right)}-\frac{a^4\ln\left(a+b\sin\left(dx+c\right)\right)}{d\left(a+b\right)^3\left(a-b\right)^3}-3\,\frac{a^2\ln\left(a+b\sin\left(dx+c\right)\right)b^2}{d\left(a+b\right)^3\left(a-b\right)^3}-\frac{1}{4\,d\left(a+b\right)^2\left(\sin\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))-1/d*a^4/(a+b)^3/(a-b)^3*ln(a+b*sin
(d*x+c))-3/d*a^2/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*b^2-1/4/d/(a+b)^2/(sin(
d*x+c)-1)+1/2/d*a/(a+b)^3*ln(sin(d*x+c)-1)+1/4/d/(a-b)^2/(1+sin(d*x+c))+1/2
*a*ln(1+sin(d*x+c))/(a-b)^3/d
```

Maxima [A] time = 1.75269, size = 370, normalized size = 2.3

$$-\frac{\frac{2\left(a^4+3\,a^2b^2\right)\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{a\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{a\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{3\,a^3+ab^2-2\left(a^3+ab^2\right)\sin(dx+c)^2-\left(a^2b-b^3\right)\sin(d}{a^5-2\,a^3b^2+ab^4-\left(a^4b-2\,a^2b^3+b^5\right)\sin(dx+c)^3-\left(a^5-2\,a^3b^2+ab^4\right)\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^4 + 3*a^2*b^2)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6) - a*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - a*lo
g(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3 + a*b^2 - 2*(a
^3 + a*b^2)*sin(d*x + c)^2 - (a^2*b - b^3)*sin(d*x + c))/(a^5 - 2*a^3*b^2 +
 a*b^4 - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^2 + a*b^
4)*sin(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

Fricas [B] time = 2.3941, size = 869, normalized size = 5.4

$$\frac{a^5-2\,a^3b^2+ab^4+2\left(a^5-ab^4\right)\cos\left(dx+c\right)^2-2\left(\left(a^4b+3\,a^2b^3\right)\cos\left(dx+c\right)^2\sin\left(dx+c\right)+\left(a^5+3\,a^3b^2\right)\cos\left(dx+c\right)^2\right)\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^5 - 2*a^3*b^2 + a*b^4 + 2*(a^5 - a*b^4)*cos(d*x + c)^2 - 2*((a^4*b +
 3*a^2*b^3)*cos(d*x + c)^2*sin(d*x + c) + (a^5 + 3*a^3*b^2)*cos(d*x + c)^2)
*log(b*sin(d*x + c) + a) + ((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x
 + c)^2*sin(d*x + c) + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(d*x + c)^2
)*log(sin(d*x + c) + 1) + ((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*cos(d*x
+ c)^2*sin(d*x + c) + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x + c)^2)
*log(-sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((a^6*b -
 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^2*sin(d*x + c) + (a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.84256, size = 335, normalized size = 2.08

$$-\frac{\frac{2\left(a^4b+3\,a^2b^3\right)\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7}-\frac{a\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{a\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,a^3\sin(dx+c)^2+2\,ab^2\sin(dx+c)^2+a^2b\sin(dx+c)-b^3\sin(dx+c)-3\,a^3}{\left(a^4-2\,a^2b^2+b^4\right)\left(b\sin(dx+c)^3+a\sin(dx+c)^2-b\sin(dx+c)-a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^4*b + 3*a^2*b^3)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3
 + 3*a^2*b^5 - b^7) - a*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2
 - b^3) - a*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (2
*a^3*sin(d*x + c)^2 + 2*a*b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3*sin
```

```
(d*x + c) - 3*a^3 - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)^3 + a*s
in(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

3.183 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{a}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{\left(a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^2*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)^2
*d) + ((a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - a/((a^2 - b
^2)*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.0956814, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 801}

$$-\frac{a}{d\left(a^2-b^2\right)(a+b\sin(c+dx))}+\frac{\left(a^2+b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^2}-\frac{\log(1-\sin(c+dx))}{2d(a+b)^2}-\frac{\log(\sin(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^2*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)^2
*d) + ((a^2 + b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^2*d) - a/((a^2 - b
^2)*d*(a + b*Sin[c + d*x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+x)^2\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)^2(b-x)} + \frac{a}{(a-b)(a+b)(a+x)^2} + \frac{a^2+b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{1}{2(a-b)^2(b+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} - \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{\left(a^2+b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2 d} - \frac{}{\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.286521, size = 162, normalized size = 1.49

$$-\frac{a\left(-2\left(\left(a^2+b^2\right)\log(a+b\sin(c+dx))-a^2+b^2\right)+(a-b)^2\log(1-\sin(c+dx))+(a+b)^2\log(\sin(c+dx)+1)\right)+b\sin}{2d(a-b)^2(a+b)^2(a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(a*((a - b)^2*Log[1 - Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]] - 2*
(-a^2 + b^2 + (a^2 + b^2)*Log[a + b*Sin[c + d*x]])) + b*((a - b)^2*Log[1 -
Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]] - 2*(a^2 + b^2)*Log[a + b*S
in[c + d*x]])*Sin[c + d*x])/(2*(a - b)^2*(a + b)^2*d*(a + b*Sin[c + d*x]))
```

Maple [A] time = 0.078, size = 132, normalized size = 1.2

$$-\frac{a}{d\,(a+b)\,(a-b)\,(a+b\sin(dx+c))} + \frac{\ln(a+b\sin(dx+c))\,a^2}{d\,(a+b)^2\,(a-b)^2} + \frac{\ln(a+b\sin(dx+c))\,b^2}{d\,(a+b)^2\,(a-b)^2} - \frac{\ln(\sin(dx+c)-1)}{2\,d\,(a+b)^2} - \frac{\ln(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*a/(a+b)/(a-b)/(a+b*sin(d*x+c))+1/d/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*
a^2+1/d/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))*b^2-1/2/d/(a+b)^2*ln(sin(d*x+c)-
1)-1/2*ln(1+sin(d*x+c))/(a-b)^2/d
```

Maxima [A] time = 1.7641, size = 167, normalized size = 1.53

$$\frac{\frac{2\,(a^2+b^2)\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{2\,a}{a^3-ab^2+(a^2b-b^3)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^2 + b^2)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - 2*a/(a
^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^2 - 2*a
*b + b^2) - log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Fricas [A] time = 1.69463, size = 463, normalized size = 4.25

$$-\frac{2\,a^3-2\,ab^2-2\,(a^3+ab^2+(a^2b+b^3)\sin(dx+c))\log(b\sin(dx+c)+a)+(a^3+2\,a^2b+ab^2+(a^2b+2\,ab^2+b^3)\sin}{2\,((a^4b-2\,a^2b^3+b^5)d\sin(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3 - 2*a*b^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*sin(d*x + c))*log(b*
sin(d*x + c) + a) + (a^3 + 2*a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*sin(d*
x + c))*log(sin(d*x + c) + 1) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^2 +
 b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*d*si
n(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.28426, size = 211, normalized size = 1.94

$$\frac{\frac{2\left(a^{2}b+b^{3}\right)\log(|b\sin(dx+c)+a|)}{a^{4}b-2\,a^{2}b^{3}+b^{5}}-\frac{\log(|\sin(dx+c)+1|)}{a^{2}-2\,ab+b^{2}}-\frac{\log(|\sin(dx+c)-1|)}{a^{2}+2\,ab+b^{2}}-\frac{2\left(a^{2}b\sin(dx+c)+b^{3}\sin(dx+c)+2\,a^{3}\right)}{\left(a^{4}-2\,a^{2}b^{2}+b^{4}\right)(b\sin(dx+c)+a)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^2*b + b^3)*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5)
 - log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - log(abs(sin(d*x + c) -
1))/(a^2 + 2*a*b + b^2) - 2*(a^2*b*sin(d*x + c) + b^3*sin(d*x + c) + 2*a^3)
/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

3.184 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=53

$$-\frac{\log(a+b\sin(c+dx))}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}+\frac{1}{ad(a+b\sin(c+dx))}$$

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[a + b*Sin[c + d*x]]/(a^2*d) + 1/(a*d*(a + b
*Sin[c + d*x]))
```

Rubi [A] time = 0.0526635, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 44}

$$-\frac{\log(a+b\sin(c+dx))}{a^2d}+\frac{\log(\sin(c+dx))}{a^2d}+\frac{1}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^2*d) - Log[a + b*Sin[c + d*x]]/(a^2*d) + 1/(a*d*(a + b
*Sin[c + d*x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2x} - \frac{1}{a(a+x)^2} - \frac{1}{a^2(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^2d} - \frac{\log(a+b\sin(c+dx))}{a^2d} + \frac{1}{ad(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.0760863, size = 42, normalized size = 0.79

$$\frac{\frac{a}{a+b\sin(c+dx)} - \log(a+b\sin(c+dx)) + \log(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]] + a/(a + b*Sin[c + d*x]))/(a^2
*d)
```

Maple [A] time = 0.038, size = 54, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{a^2d} - \frac{\ln(a+b\sin(dx+c))}{a^2d} + \frac{1}{da(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c))^2,x)
```

```
[Out] ln(sin(d*x+c))/a^2/d-ln(a+b*sin(d*x+c))/a^2/d+1/a/d/(a+b*sin(d*x+c))
```

Maxima [A] time = 1.47039, size = 63, normalized size = 1.19

$$\frac{\frac{1}{ab\sin(dx+c)+a^2} - \frac{\log(b\sin(dx+c)+a)}{a^2} + \frac{\log(\sin(dx+c))}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a*b*sin(d*x + c) + a^2) - log(b*sin(d*x + c) + a)/a^2 + log(sin(d*x + c
))/a^2)/d
```

Fricas [A] time = 1.63667, size = 176, normalized size = 3.32

$$-\frac{(b\sin(dx+c)+a)\log(b\sin(dx+c)+a)-(b\sin(dx+c)+a)\log\left(-\frac{1}{2}\sin(dx+c)\right)-a}{a^2bd\sin(dx+c)+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((b*sin(d*x + c) + a)*log(b*sin(d*x + c) + a) - (b*sin(d*x + c) + a)*log(-
1/2*sin(d*x + c)) - a)/(a^2*b*d*sin(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.49888, size = 69, normalized size = 1.3

$$\frac{b\left(\frac{\log\left(\left|-\frac{a}{b\sin(dx+c)+a}+1\right|\right)}{a^2b}+\frac{1}{(b\sin(dx+c)+a)ab}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] b*(log(abs(-a/(b*sin(d*x + c) + a) + 1))/(a^2*b) + 1/((b*sin(d*x + c) + a)*
a*b))/d
```

3.185 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=114

$$-\frac{a^2-b^2}{a^3d(a+b\sin(c+dx))}-\frac{(a^2-3b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^2-3b^2)\log(a+b\sin(c+dx))}{a^4d}+\frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}$$

```
[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((a^2 - 3*b^2)*Log[
Sin[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^4*d) -
(a^2 - b^2)/(a^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.114796, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{a^2-b^2}{a^3d(a+b\sin(c+dx))}-\frac{(a^2-3b^2)\log(\sin(c+dx))}{a^4d}+\frac{(a^2-3b^2)\log(a+b\sin(c+dx))}{a^4d}+\frac{2b\csc(c+dx)}{a^3d}-\frac{\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b*Csc[c + d*x])/(a^3*d) - Csc[c + d*x]^2/(2*a^2*d) - ((a^2 - 3*b^2)*Log[
Sin[c + d*x]])/(a^4*d) + ((a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^4*d) -
(a^2 - b^2)/(a^3*d*(a + b*Sin[c + d*x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^2x^3} - \frac{2b^2}{a^3x^2} + \frac{-a^2+3b^2}{a^4x} + \frac{a^2-b^2}{a^3(a+x)^2} + \frac{a^2-3b^2}{a^4(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{2b\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^2d} - \frac{\left(a^2-3b^2\right)\log(\sin(c+dx))}{a^4d} + \frac{\left(a^2-3b^2\right)\log(a+b\sin(c+}{a^4d}$$

Mathematica [A] time = 0.607729, size = 96, normalized size = 0.84

$$-\frac{2\left(a^2-3b^2\right)\log(\sin(c+dx)) - 2\left(a^2-3b^2\right)\log(a+b\sin(c+dx)) + a^2\csc^2(c+dx) + \frac{2a(a-b)(a+b)}{a+b\sin(c+dx)} - 4ab\csc(c+dx)}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(-4*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - 3*b^2)*Log[Sin[c + d*
x]] - 2*(a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]] + (2*a*(a - b)*(a + b))/(a +
b*Sin[c + d*x]))/(2*a^4*d)
```

Maple [A] time = 0.089, size = 150, normalized size = 1.3

$$\frac{\ln(a+b\sin(dx+c))}{a^2d} - 3\frac{\ln(a+b\sin(dx+c))\,b^2}{da^4} - \frac{1}{da\,(a+b\sin(dx+c))} + \frac{b^2}{da^3\,(a+b\sin(dx+c))} - \frac{1}{2\,a^2d\,(\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x)
```

```
[Out] ln(a+b*sin(d*x+c))/a^2/d-3/d/a^4*ln(a+b*sin(d*x+c))*b^2-1/a/d/(a+b*sin(d*x+
c))+1/d/a^3/(a+b*sin(d*x+c))*b^2-1/2/d/a^2/sin(d*x+c)^2-ln(sin(d*x+c))/a^2/
d+3/d/a^4*ln(sin(d*x+c))*b^2+2/d/a^3*b/sin(d*x+c)
```

Maxima [A] time = 1.36351, size = 157, normalized size = 1.38

$$\frac{\frac{3\,ab\sin(dx+c)-2\left(a^2-3\,b^2\right)\sin(dx+c)^2-a^2}{a^3b\sin(dx+c)^3+a^4\sin(dx+c)^2}+\frac{2\left(a^2-3\,b^2\right)\log(b\sin(dx+c)+a)}{a^4}-\frac{2\left(a^2-3\,b^2\right)\log(\sin(dx+c))}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*((3*a*b*sin(d*x + c) - 2*(a^2 - 3*b^2)*sin(d*x + c)^2 - a^2)/(a^3*b*sin
(d*x + c)^3 + a^4*sin(d*x + c)^2) + 2*(a^2 - 3*b^2)*log(b*sin(d*x + c) + a)
/a^4 - 2*(a^2 - 3*b^2)*log(sin(d*x + c))/a^4)/d
```

Fricas [B] time = 1.61861, size = 599, normalized size = 5.25

$$-\frac{3\,a^2b\sin\left(dx+c\right)-3\,a^3+6\,ab^2+2\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2+2\left(a^3-3\,ab^2-\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2+\left(a^2b-3\,b^3\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^2*b*sin(d*x + c) - 3*a^3 + 6*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x +
c)^2 + 2*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - 3*b^3 -
 (a^2*b - 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) - 2*
(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - 3*b^3 - (a^2*b -
 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x + c)))/(a^5*d*cos(d*
x + c)^2 - a^5*d + (a^4*b*d*cos(d*x + c)^2 - a^4*b*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.97623, size = 223, normalized size = 1.96

$$-\frac{\frac{2\left(a^2-3\,b^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^4}-\frac{2\left(a^2b-3\,b^3\right)\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^4b}+\frac{2\left(a^2b\sin\left(dx+c\right)-3\,b^3\sin\left(dx+c\right)+2\,a^3-4\,ab^2\right)}{\left(b\sin\left(dx+c\right)+a\right)a^4}-\frac{3\,a^2\sin\left(dx+c\right)^2-9\,b^2\sin\left(dx+c\right)^2+4\,ab\sin\left(dx+c\right)-a^2}{a^4\sin\left(dx+c\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - 3*b^2)*log(abs(sin(d*x + c)))/a^4 - 2*(a^2*b - 3*b^3)*log(ab
s(b*sin(d*x + c) + a))/(a^4*b) + 2*(a^2*b*sin(d*x + c) - 3*b^3*sin(d*x + c)
 + 2*a^3 - 4*a*b^2)/((b*sin(d*x + c) + a)*a^4) - (3*a^2*sin(d*x + c)^2 - 9*
b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) - a^2)/(a^4*sin(d*x + c)^2))/d
```

3.186 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$\frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}-\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d}+\frac{\left(-6a^2b^2+a^4+5b^4\right)\log(\sin(c+dx))}{a^6d}-$$

```
[Out] (-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a
^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*
b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c +
 d*x]))
```

Rubi [A] time = 0.181468, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))}+\frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d}-\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d}+\frac{\left(-6a^2b^2+a^4+5b^4\right)\log(\sin(c+dx))}{a^6d}-$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*b*(a^2 - b^2)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/
(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) + ((a
^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2 + 5*
b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin[c +
 d*x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^5(a+x)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^2x^5} - \frac{2b^4}{a^3x^4} + \frac{-2a^2b^2+3b^4}{a^4x^3} + \frac{4b^2\left(a^2-b^2\right)}{a^5x^2} + \frac{a^4-6a^2b^2+5b^4}{a^6x} - \frac{\left(a^2-b^2\right)^2}{a^5(a+x)^2} + \frac{-a^4+6a^2b^2-5b^4}{a^6(a+x)}\right) dx, x}{d}$$

$$= -\frac{4b\left(a^2-b^2\right)\csc(c+dx)}{a^5d} + \frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+dx)}{4a^2d} + \frac{(a}{}$$

Mathematica [A] time = 6.14389, size = 187, normalized size = 0.99

$$\frac{\left(a^2-b^2\right)^2}{a^5d(a+b\sin(c+dx))} + \frac{\left(2a^2-3b^2\right)\csc^2(c+dx)}{2a^4d} + \frac{\left(-6a^2b^2+a^4+5b^4\right)\log(\sin(c+dx))}{a^6d} - \frac{\left(-6a^2b^2+a^4+5b^4\right)\log(a+}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-4*(a - b)*b*(a + b)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x]
^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d) +
 ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b^2
+ 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b*Sin
[c + d*x]))
```

Maple [A] time = 0.098, size = 282, normalized size = 1.5

$$-\frac{\ln(a+b\sin(dx+c))}{a^2d} + 6\,\frac{\ln(a+b\sin(dx+c))\,b^2}{da^4} - 5\,\frac{\ln(a+b\sin(dx+c))\,b^4}{da^6} + \frac{1}{da\,(a+b\sin(dx+c))} - 2\,\frac{}{da^3\,(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -ln(a+b*sin(d*x+c))/a^2/d+6/d/a^4*ln(a+b*sin(d*x+c))*b^2-5/d/a^6*ln(a+b*sin
(d*x+c))*b^4+1/a/d/(a+b*sin(d*x+c))-2/d/a^3/(a+b*sin(d*x+c))*b^2+1/d/a^5/(a
+b*sin(d*x+c))*b^4-1/4/d/a^2/sin(d*x+c)^4+1/d/a^2/sin(d*x+c)^2-3/2/d/a^4/si
n(d*x+c)^2*b^2+ln(sin(d*x+c))/a^2/d-6/d/a^4*ln(sin(d*x+c))*b^2+5/d/a^6*ln(s
in(d*x+c))*b^4+2/3/d/a^3*b/sin(d*x+c)^3-4/d/a^3*b/sin(d*x+c)+4/d*b^3/a^5/si
n(d*x+c)
```

Maxima [A] time = 1.73361, size = 255, normalized size = 1.36

$$\frac{\frac{5\,a^3b\sin(dx+c)+12\left(a^4-6\,a^2b^2+5\,b^4\right)\sin(dx+c)^4-3\,a^4-6\left(6\,a^3b-5\,ab^3\right)\sin(dx+c)^3+2\left(6\,a^4-5\,a^2b^2\right)\sin(dx+c)^2}{a^5b\sin(dx+c)^5+a^6\sin(dx+c)^4}-\frac{12\left(a^4-6\,a^2b^2+5\,b^4\right)\log(b\sin(dx+c)+a)}{a^6}+}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/12*((5*a^3*b*sin(d*x + c) + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*sin(d*x + c)^4 -
 3*a^4 - 6*(6*a^3*b - 5*a*b^3)*sin(d*x + c)^3 + 2*(6*a^4 - 5*a^2*b^2)*sin(d
*x + c)^2)/(a^5*b*sin(d*x + c)^5 + a^6*sin(d*x + c)^4) - 12*(a^4 - 6*a^2*b^
2 + 5*b^4)*log(b*sin(d*x + c) + a)/a^6 + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(s
in(d*x + c))/a^6)/d
```

Fricas [B] time = 2.29355, size = 1241, normalized size = 6.6

$$\frac{21\,a^5-82\,a^3b^2+60\,ab^4+12\left(a^5-6\,a^3b^2+5\,ab^4\right)\cos(dx+c)^4-2\left(18\,a^5-77\,a^3b^2+60\,ab^4\right)\cos(dx+c)^2-12\left(a^5-6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(21*a^5 - 82*a^3*b^2 + 60*a*b^4 + 12*(a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d
*x + c)^4 - 2*(18*a^5 - 77*a^3*b^2 + 60*a*b^4)*cos(d*x + c)^2 - 12*(a^5 - 6
*a^3*b^2 + 5*a*b^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 -
6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b -
 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x
```

```
+ c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 12*(a^5 - 6*a^3*b^2 + 5*a*b
^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 6*a^3*b^2 + 5*a*
b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b - 6*a^2*b^3 + 5*b
^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-1/2*sin(d*x + c)) - (31*a^4*b - 30*a^2*b^3 - 6*(6*a^4*b - 5*a^2*
b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7*d*cos(d*x
+ c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x + c)^2 + a^6*b
*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.81061, size = 375, normalized size = 1.99

$$\frac{\frac{12\left(a^4-6a^2b^2+5b^4\right)\log(|\sin(dx+c)|)}{a^6}-\frac{12\left(a^4b-6a^2b^3+5b^5\right)\log(|b\sin(dx+c)+a|)}{a^6b}+\frac{12\left(a^4b\sin(dx+c)-6a^2b^3\sin(dx+c)+5b^5\sin(dx+c)+2a^5-8a^3b^2+6ab^4\right)}{(b\sin(dx+c)+a)a^6}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(abs(sin(d*x + c)))/a^6 - 12*(a^4*b -
 6*a^2*b^3 + 5*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6*b) + 12*(a^4*b*sin(d*
x + c) - 6*a^2*b^3*sin(d*x + c) + 5*b^5*sin(d*x + c) + 2*a^5 - 8*a^3*b^2 +
6*a*b^4)/((b*sin(d*x + c) + a)*a^6) - (25*a^4*sin(d*x + c)^4 - 150*a^2*b^2*
sin(d*x + c)^4 + 125*b^4*sin(d*x + c)^4 + 48*a^3*b*sin(d*x + c)^3 - 48*a*b^
3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 18*a^2*b^2*sin(d*x + c)^2 - 8*a^
3*b*sin(d*x + c) + 3*a^4)/(a^6*sin(d*x + c)^4))/d
```

3.187 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=333

$$\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{8a^3b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{a^4b\cos(c+dx)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{(3a+b)\cos(c+dx)}{4d(a+b)^3(1-\sin(c+\cdots}$$

```
[Out] (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)
*d) + (8*a^3*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 -
b^2)^(7/2)*d) + Cos[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])^2) + Cos[c
+ d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])) - ((3*a + b)*Cos[c + d*x])/(4*(a
 + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d
*x])^2) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d*x])) + ((3*a - b)*Cos
[c + d*x])/(4*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4*b*Cos[c + d*x])/((a^2
- b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.626607, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2731, 2650, 2648, 2664, 12, 2660, 618, 204}

$$\frac{2a^5\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{8a^3b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}+\frac{a^4b\cos(c+dx)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{(3a+b)\cos(c+dx)}{4d(a+b)^3(1-\sin(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)
*d) + (8*a^3*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 -
b^2)^(7/2)*d) + Cos[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])^2) + Cos[c
+ d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])) - ((3*a + b)*Cos[c + d*x])/(4*(a
 + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d
*x])^2) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d*x])) + ((3*a - b)*Cos
[c + d*x])/(4*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4*b*Cos[c + d*x])/((a^2
- b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \left(\frac{1}{4(a+b)^2(-1+\sin(c+dx))^2} + \frac{3a+b}{4(a+b)^3(-1+\sin(c+dx))} + \frac{1}{4(a-b)^2(1+\sin(c+dx)}\right.$$

$$= \frac{\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{4(a-b)^2} - \frac{(3a-b)\int \frac{1}{1+\sin(c+dx)}\,dx}{4(a-b)^3} + \frac{\int \frac{1}{(-1+\sin(c+dx))^2}\,dx}{4(a+b)^2} + \frac{(3a+b)\int \frac{1}{-1+\sin(c+dx)}}{4(a+b)^3}$$

$$= \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} - \frac{(3a+b)\cos(c+dx)}{4(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{12(a-b)^2d(1+\sin(c+dx))^2}$$

$$= \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))} - \frac{(3a+b)\cos(c+dx)}{4(a+b)^3d(1-\sin(c+dx))}$$

$$= \frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx}$$

$$= \frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx}$$

$$= \frac{2a^5\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{8a^3b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))}$$

Mathematica [A] time = 1.96747, size = 341, normalized size = 1.02

$$\frac{24a^3\left(a^2+4b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{12a^4b\cos(c+dx)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))} - \frac{4(4a+b)\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((24*a^3*(a^2 + 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a
^2 - b^2)^(7/2) + 1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (
2*Sin[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) - (
4*(4*a + b)*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])) + (2*Sin[(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(-4*a + b
)*Sin[(c + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12
*a^4*b*Cos[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])))/(12*d)
```

Maple [A] time = 0.098, size = 382, normalized size = 1.2

$$-\frac{1}{3\,d\,(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-3}-\frac{1}{2\,d\,(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{a}{d\,(a+b)^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+2\,\frac{}{d\,(a-b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1
)^2+1/d*a/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+2/d*a^3/(a-b)^3/(a+b)^3/(tan(1/2*d
*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)+2/d*a^4/(a-b
)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b+2/d*a^5/(a-
b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b
^2)^(1/2))+8/d*a^3/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*
d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/3/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3
+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^2+1/d*a/(a-b)^3/(tan(1/2*d*x+1/2*c)+1
)
```

Maple [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.50435, size = 1790, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - 2*(7*a^6*b + 2*a^4*b^3 - 1
0*a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 - 2*
b^7)*cos(d*x + c)^2 - 3*((a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c) +
(a^6 + 4*a^4*b^2)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(
d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c
) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)) - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7 - 7*a^5
*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3
 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^
7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3), -1/3*(a^6*b - 3*a
^4*b^3 + 3*a^2*b^5 - b^7 - (7*a^6*b + 2*a^4*b^3 - 10*a^2*b^5 + b^7)*cos(d*x
 + c)^4 - (7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 - 2*b^7)*cos(d*x + c)^2 + 3*((
a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c) + (a^6 + 4*a^4*b^2)*cos(d*x
+ c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d
*x + c))) - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7 - 7*a^5*b^2 + 2*a
^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b
^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 2.7838, size = 548, normalized size = 1.65

$$2\left(\frac{3\left(a^5+4a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{3\left(a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a^4b\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}+\frac{3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*(a^5 + 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*sqrt(a^2 - b^2)) + 3*(a^3*b^2*tan(1/2*d*x + 1/2*c) + a^4*b)/((a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x
+ 1/2*c) + a)) + (3*a^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b^2*tan(1/2*d*x + 1/
2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 6*a*b^3*tan(1/2*d*x + 1/2*c)^4 -
10*a^4*tan(1/2*d*x + 1/2*c)^3 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^4*t
an(1/2*d*x + 1/2*c)^3 + 24*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x
 + 1/2*c) + 9*a^2*b^2*tan(1/2*d*x + 1/2*c) - 10*a^3*b - 2*a*b^3)/((a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

3.188 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=200

$$-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{4ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}$$

```
[Out] (-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*d) - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b
^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*
x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.301028, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2731, 2648, 2664, 12, 2660, 618, 204}

$$-\frac{2a^3\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{4ab^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/2
)*d) - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b
^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c + d*
x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]))
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int\left(-\frac{1}{2(a+b)^2(-1+\sin(c+dx))}+\frac{1}{2(a-b)^2(1+\sin(c+dx))}-\frac{a^2}{\left(a^2-b^2\right)(a+b\sin(c+dx}\right.$$

$$=\frac{\int\frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^2}-\frac{\int\frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^2}-\frac{\left(2ab^2\right)\int\frac{1}{a+b\sin(c+dx)}\,dx}{\left(a^2-b^2\right)^2}-\frac{a^2\int\frac{1}{(a+b\sin(c+dx))^2}\,dx}{a^2-b^2}$$

$$=\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}-\frac{a^2b\cos(c+dx)}{\left(a^2-b^2\right)^2d(a+b\sin(c+dx))}$$

$$=\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}-\frac{a^2b\cos(c+dx)}{\left(a^2-b^2\right)^2d(a+b\sin(c+dx))}$$

$$=-\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}$$

$$=-\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))}$$

$$=-\frac{2a^3\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}-\frac{4ab^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}d}+\frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))}$$

Mathematica [A] time = 1.01231, size = 169, normalized size = 0.84

$$\frac{-\frac{2a\left(a^2+2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{a^2b\cos(c+dx)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a-b)^2\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+\right.\right.}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-2*a*(a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])) + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (a^2*b*
```

```
Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

Maple [A] time = 0.088, size = 282, normalized size = 1.4

$$-\frac{1}{d\,(a+b)^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-2\,\frac{ab^2\tan(1/2\,dx+c/2)}{d\,(a-b)^2\,(a+b)^2\left((\tan(1/2\,dx+c/2))^2\,a+2\,\tan(1/2\,dx+c/2)\,b+a\right)}-2\,\frac{}{d\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-2/d*a/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)-2/d*a^2/(a-b)^2/(a+
b)^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b-2/d*a^3/(a-b)^2/(a
+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2))-4/d*a/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))*b^2-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95834, size = 1261, normalized size = 6.3

$$\left[-\frac{2\,a^4b-4\,a^2b^3+2\,b^5+2\left(2\,a^4b-a^2b^3-b^5\right)\cos(dx+c)^2+\left(\left(a^3b+2\,ab^3\right)\cos(dx+c)\sin(dx+c)+\left(a^4+2\,a^2b^2\right)\cos\right.}{2\left(\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)d\cos(dx+c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x +
c)^2 + ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*cos
(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(
d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqr
t(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(
a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*co
s(d*x + c)), -(a^4*b - 2*a^2*b^3 + b^5 + (2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*
cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^
3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 2.15159, size = 339, normalized size = 1.7

$$-\frac{2\left(\frac{(a^3+2ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\cdots}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\cdots\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*((a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2
 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a
^2*b*tan(1/2*d*x + 1/2*c)^2 + 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*
x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x + 1/2*
c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^4 - 2*
a^2*b^2 + b^4)))/d
```

3.189 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=115

$$-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*Sq
rt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(
a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.452534, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^3d\sqrt{a^2-b^2}}+\frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d}-\frac{2\cot(c+dx)}{a^2d}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*Sq
rt[a^2 - b^2]*d) + (2*b*ArcTanh[Cos[c + d*x]])/(a^3*d) - (2*Cot[c + d*x])/(
a^2*d) + Cot[c + d*x]/(a*d*(a + b*Sin[c + d*x]))
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
```

```
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc(c+dx)\left(-2b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}$$

$$= -\frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{(2b)\int \csc(c+dx)\,dx}{a^3} - \frac{\left(a^2-2b^2\right)\int \frac{1}{a+b\sin(c+dx)}}{a^3}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} - \frac{\left(2\left(a^2-2b^2\right)\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))} + \frac{\left(4\left(a^2-2b^2\right)\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2\left(a^2-2b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}\,d} + \frac{2b\tanh^{-1}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\text{co}}{ad(a+}$$

Mathematica [A] time = 0.745506, size = 139, normalized size = 1.21

$$-\frac{\frac{4\left(a^2-2b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2ab\cos(c+dx)}{a+b\sin(c+dx)} - a\tan\left(\frac{1}{2}(c+dx)\right) + a\cot\left(\frac{1}{2}(c+dx)\right) + 4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 4b\log\left(\right.}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -((4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] + a*Cot[(c + d*x)/2] - 4*b*Log[Cos[(c + d*x)/2]] + 4*b*Log[Sin[(c
 + d*x)/2]] + (2*a*b*Cos[c + d*x])/(a + b*Sin[c + d*x]) - a*Tan[(c + d*x)/2
])/(2*a^3*d)
```

Maple [B] time = 0.086, size = 245, normalized size = 2.1

$$\frac{1}{2\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{b^2\tan\left(1/2\,dx+c/2\right)}{da^3\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a+2\,\tan\left(1/2\,dx+c/2\right)b+a\right)}-2\,\frac{b}{da^2\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a+2\,\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2/d/a^2*tan(1/2*d*x+1/2*c)-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+
1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)-2/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)*b-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c
)+2*b)/(a^2-b^2)^(1/2))+4/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x
+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-1/2/d/a^2/tan(1/2*d*x+1/2*c)-2/d/a^3*b*ln
(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.81611, size = 1681, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*b^3 - (a^2*b
 - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*l
og(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*co
s(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x +
```

```
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^4 - a^2*b^2)*cos(d*x + c) -
2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x
 + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos
(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(
(a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^2)*d*sin(d*x + c) - (a^5*
b - a^3*b^3)*d), (2*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*
b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(a
^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + (a
^4 - a^2*b^2)*cos(d*x + c) - (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^
2 + (a^3*b - a*b^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^2*b^2 -
b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1
/2*cos(d*x + c) + 1/2))/((a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^
2)*d*sin(d*x + c) - (a^5*b - a^3*b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.71518, size = 294, normalized size = 2.56

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\left(a^2-2\,b^2\right)}{\sqrt{a^2-b^2}\,a^3}-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
+ 12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^3) - (4*a*b*tan
```

```
(1/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*b^2*tan(1/2*d*x + 1/
2*c)^2 - 14*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/((a*tan(1/2*d*x + 1/2*c)^3 +
2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3))/d
```

3.190 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=238

$$\frac{2\left(-5a^2b^2+a^4+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d}-\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}-\left(a^2-\right.$$

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^5*Sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^
5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d
*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(
3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a +
 b*Sin[c + d*x]))
```

Rubi [A] time = 0.695298, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2\left(-5a^2b^2+a^4+4b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^5d\sqrt{a^2-b^2}}+\frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d}-\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d}-\left(a^2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```


```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^5*Sqrt[a^2 - b^2]*d) - (b*(3*a^2 - 4*b^2)*ArcTanh[Cos[c + d*x]])/(a^
5*d) + ((7*a^2 - 12*b^2)*Cot[c + d*x])/(3*a^4*d) - ((a^2 - 2*b^2)*Cot[c + d
*x]*Csc[c + d*x])/(a^3*b*d) + ((3*a^2 - 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(
3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a +
 b*Sin[c + d*x]))
```


Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
```

```
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))} + \frac{\int \frac{\csc^3(c+dx)\left(6\left(a^2-2b^2\right)-ab\sin\right.}{a+b\sin}}{3}$$

$$= -\frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\text{cs}}{3ad(a+b\sin}$$

$$= \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)\text{cs}}{3a^2bd(a+b\sin(c+}$$

$$= \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{\left(3a^2-4b^2\right)\cot(c+dx)\text{cs}}{3a^2bd(a+b\sin(c+}$$

$$= -\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)\text{cs}}{a^3bd}$$

$$= -\frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(7a^2-12b^2\right)\cot(c+dx)}{3a^4d} - \frac{\left(a^2-2b^2\right)\cot(c+dx)\text{cs}}{a^3bd}$$

$$= \frac{2\left(a^4-5a^2b^2+4b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}\,d} - \frac{b\left(3a^2-4b^2\right)\tanh^{-1}(\cos(c+dx))}{a^5d} + \frac{\left(7a^2-\right.}{}$$

Mathematica [A] time = 6.29471, size = 403, normalized size = 1.69

$$\frac{\left(3a^2b-4b^3\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{\left(4b^3-3a^2b\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{a^2b\cos(c+dx)-b^3\cos(c+dx)}{a^4d(a+b\sin(c+dx))} + \frac{\csc\left(\frac{1}{2}(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] +
 a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((4*a^2*C
os[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) + (b*
Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a
^2*d) + ((-3*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((3*a^2*b - 4*
b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d) + (S
ec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2]))/(6*a^4*
d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(a^4*d*(a + b*Sin[c + d*x])) +
 (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

Maple [B] time = 0.096, size = 527, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x)
```

```
[Out] 1/24/d/a^2*tan(1/2*d*x+1/2*c)^3-1/4/d/a^3*b*tan(1/2*d*x+1/2*c)^2-5/8/d/a^2*
tan(1/2*d*x+1/2*c)+3/2/d/a^4*b^2*tan(1/2*d*x+1/2*c)+2/d/a^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)-2/d/a^5/(tan(1/2*
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^4+2/d/a^2/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b-2/d/a^4/(tan(1/2*d*x+1/2*c
)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b^3+2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-10/d/a^3/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+8/d/a^5/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-1/24/d/a^2/tan
(1/2*d*x+1/2*c)^3+5/8/d/a^2/tan(1/2*d*x+1/2*c)-3/2/d/a^4/tan(1/2*d*x+1/2*c)
*b^2+1/4/d/a^3*b/tan(1/2*d*x+1/2*c)^2+3/d/a^3*b*ln(tan(1/2*d*x+1/2*c))-4/d/
a^5*b^3*ln(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.10123, size = 2639, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(4*(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 3*((a^2*b - 4*b^3)*cos(d*x +
c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 -
(a^3 - 4*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2
- b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*
sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*
b*sin(d*x + c) - a^2 - b^2)) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) + 3*((3*a^2
*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*co
s(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c
)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 + (3*a^3*b -
 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^3*b - 4*a*
b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*b*d*cos(d*
x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c)), -1/6*(4*
(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 6*((a^2*b - 4*b^3)*cos(d*x + c)^4 + a^
2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2 - (a^3 - 4*
a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c
) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) +
 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 -
 4*b^4)*cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*c
os(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 +
(3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(
-1/2*cos(d*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*a^
3*b - 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a^5*
b*d*cos(d*x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x + c))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.85931, size = 481, normalized size = 2.02

$$\frac{\frac{24\left(3a^2b-4b^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^5}+\frac{48\left(a^4-5a^2b^2+4b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5}+\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}{a^6}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(24*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + 48*(a^4 - 5
*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(
1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan(1/
2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x + 1/
2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 + 48*(a^2*b^2*tan(1/2*d*x + 1/2
*c) - b^4*tan(1/2*d*x + 1/2*c) + a^3*b - a*b^3)/((a*tan(1/2*d*x + 1/2*c)^2
+ 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (132*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
176*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 36*a*b^2*t
an(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*tan(1/2*d*
x + 1/2*c)^3))/d
```

3.191 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=424

$$-\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7 d}-\frac{\left(-135a^2b^2+38a^4+90b^4\right)\cot(c+dx)}{15a^6 d}+\frac{b\left(-40a^2b^2+15a^4+24b^4\right.}{4a^7 d}$$

```
[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]
])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) +
((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((1
5*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) -
(Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b
^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*
Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x
]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.49342, antiderivative size = 424, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^7 d}-\frac{\left(-135a^2b^2+38a^4+90b^4\right)\cot(c+dx)}{15a^6 d}+\frac{b\left(-40a^2b^2+15a^4+24b^4\right.}{4a^7 d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/(a^7*d) + (b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*ArcTanh[Cos[c + d*x]
])/(4*a^7*d) - ((38*a^4 - 135*a^2*b^2 + 90*b^4)*Cot[c + d*x])/(15*a^6*d) +
((4*a^4 - 17*a^2*b^2 + 12*b^4)*Cot[c + d*x]*Csc[c + d*x])/(4*a^5*b*d) - ((1
5*a^4 - 82*a^2*b^2 + 60*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(30*a^4*b^2*d) -
(Cot[c + d*x]*Csc[c + d*x])/(2*b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d*x]*
Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + ((2*a^4 - 12*a^2*b^2 + 9*b
^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^3*b^2*d*(a + b*Sin[c + d*x])) + (3*b*
Cot[c + d*x]*Csc[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x
```

```
]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x]))
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))} - \frac{\cot(}{5ad}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))} + \frac{\left(2a^4-12a^2b^2+9b^4\right)\cot(c+dx)\operatorname{cs}}{6a^3b^2d(a+b\sin(c+dx)}$$

$$= -\frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))} + \frac{a\cot(c+dx}{6b^2d(a+b}$$

$$= \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{\left(15a^4-82a^2b^2+60b^4\right)\cot(c+dx)\csc^2(c}{30a^4b^2d}$$

$$= -\frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{(1}{}$$

$$= -\frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{\left(4a^4-17a^2b^2+12b^4\right)\cot(c+dx)\csc(c+dx)}{4a^5bd} - \frac{(1}{}$$

$$= \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{(4a}{}$$

$$= \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos(c+dx))}{4a^7d} - \frac{\left(38a^4-135a^2b^2+90b^4\right)\cot(c+dx)}{15a^6d} + \frac{(4a}{}$$

$$= -\frac{2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d} + \frac{b\left(15a^4-40a^2b^2+24b^4\right)\tanh^{-1}(\cos(c}{4a^7d}$$

Mathematica [A] time = 1.59178, size = 361, normalized size = 0.85

$$-\frac{1920\left(a^2-6b^2\right)\left(a^2-b^2\right)^{3/2}\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right) + 240b\left(-40a^2b^2+15a^4+24b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 240b\left(-40a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

```
[Out] -(1920*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]] - 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Cos[(c + d*x)/2]] +
```

```
240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Sin[(c + d*x)/2]] + (2*a*Cot[c +
d*x]*Csc[c + d*x]^5*(196*a^5 - 735*a^3*b^2 + 540*a*b^4 - 12*(16*a^5 - 85*a^
3*b^2 + 60*a*b^4)*Cos[2*(c + d*x)] + (92*a^5 - 285*a^3*b^2 + 180*a*b^4)*Cos
[4*(c + d*x)] + 1162*a^4*b*Sin[c + d*x] - 3060*a^2*b^3*Sin[c + d*x] + 1800*
b^5*Sin[c + d*x] - 562*a^4*b*Sin[3*(c + d*x)] + 1470*a^2*b^3*Sin[3*(c + d*x
)] - 900*b^5*Sin[3*(c + d*x)] + 76*a^4*b*Sin[5*(c + d*x)] - 270*a^2*b^3*Sin
[5*(c + d*x)] + 180*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*x]))/(960*a^7*d
)
```

Maple [B] time = 0.108, size = 897, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x)
```

```
[Out] -7/96/d/a^2*tan(1/2*d*x+1/2*c)^3+7/96/d/a^2/tan(1/2*d*x+1/2*c)^3-1/160/d/a^
2/tan(1/2*d*x+1/2*c)^5+1/160/d/a^2*tan(1/2*d*x+1/2*c)^5-2/d/a^2/(tan(1/2*d*
x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)*b-2/d/a/(a^2-b^2)^(1/2)*arctan(1/2*(
2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-15/4/d/a^3*b*ln(tan(1/2*d*x+1/
2*c))-11/16/d/a^2/tan(1/2*d*x+1/2*c)+11/16/d/a^2*tan(1/2*d*x+1/2*c)+16/d/a^
3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*
b^2-1/2/d/a^5*tan(1/2*d*x+1/2*c)^2*b^3+5/2/d/a^6*b^4*tan(1/2*d*x+1/2*c)+1/3
2/d/a^3*b/tan(1/2*d*x+1/2*c)^4+1/2/d*b^3/a^5/tan(1/2*d*x+1/2*c)^2-6/d/a^7*b
^5*ln(tan(1/2*d*x+1/2*c))-2/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)*b^5-1/8/d/a^4/tan(1/2*d*x+1/2*c)^3*b^2-5/2/d/a^6/tan(1/2*d*x+1/2*c
)*b^4-1/32/d/a^3*b*tan(1/2*d*x+1/2*c)^4+1/8/d/a^4*b^2*tan(1/2*d*x+1/2*c)^3-
26/d/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^
(1/2))*b^4+27/8/d/a^4/tan(1/2*d*x+1/2*c)*b^2-1/2/d/a^3*b/tan(1/2*d*x+1/2*c)
^2+10/d/a^5*b^3*ln(tan(1/2*d*x+1/2*c))+1/2/d/a^3*b*tan(1/2*d*x+1/2*c)^2-27/
8/d/a^4*b^2*tan(1/2*d*x+1/2*c)+4/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*
x+1/2*c)*b+a)*b^3+4/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)
*tan(1/2*d*x+1/2*c)*b^4+12/d/a^7/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^6-2/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)*tan(1/2*d*x+1/2*c)*b^6-2/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*
tan(1/2*d*x+1/2*c)*b+a)*b^2*tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.53225, size = 4674, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/120*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 -
 27*a^4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 60*((a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*
cos(d*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^
3*b^2 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a
^3*b^2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^
2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c
)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2)) + 30*(4*a^6 - 17*a^4*b^2 + 12*a^2*b^4)*cos(d
*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 - 15*a^4*b^
2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5*b - 40*a
^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^4 - 2*(
15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos
(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 -
15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*a^5
*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c
)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*lo
g(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 90*a*b^5)*cos(d
*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x + c)^3 + 15*(15*
a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a^7*b*d*cos(d*x
 + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2 - a^7*b*d - (
a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*x + c)), 1/120
*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6 - 27*a^
4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 120*((a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d
*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 - 7*a^3*b^2
```

```
 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 7*a^3*b^
2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d
*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) + 30*(4*a^6 - 17*a^4*b^2 + 12*
a^2*b^4)*cos(d*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)
^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6
)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (1
5*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*
x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos
(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4
 + 24*b^6)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x +
c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^
5)*cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^3*b^3 + 9
0*a*b^5)*cos(d*x + c)^5 - 5*(79*a^5*b - 228*a^3*b^3 + 144*a*b^5)*cos(d*x +
c)^3 + 15*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c))*sin(d*x + c))/(a
^7*b*d*cos(d*x + c)^6 - 3*a^7*b*d*cos(d*x + c)^4 + 3*a^7*b*d*cos(d*x + c)^2
 - a^7*b*d - (a^8*d*cos(d*x + c)^4 - 2*a^8*d*cos(d*x + c)^2 + a^8*d)*sin(d*
x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3698, size = 805, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/480*(120*(15*a^4*b - 40*a^2*b^3 + 24*b^5)*log(abs(tan(1/2*d*x + 1/2*c)))
/a^7 + 960*(a^6 - 8*a^4*b^2 + 13*a^2*b^4 - 6*b^6)*(pi*floor(1/2*(d*x + c)/p
```

```
i + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sq
rt(a^2 - b^2)*a^7) + 960*(a^4*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b^4*tan(1/2*
d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c) + a^5*b - 2*a^3*b^3 + a*b^5)/((a*ta
n(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^7) - (3*a^8*tan(1/2*
d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^8*tan(1/2*d*x + 1/2
*c)^3 + 60*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^7*b*tan(1/2*d*x + 1/2*c)^
2 - 240*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 330*a^8*tan(1/2*d*x + 1/2*c) - 162
0*a^6*b^2*tan(1/2*d*x + 1/2*c) + 1200*a^4*b^4*tan(1/2*d*x + 1/2*c))/a^10 -
(4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 10960*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 +
 6576*b^5*tan(1/2*d*x + 1/2*c)^5 - 330*a^5*tan(1/2*d*x + 1/2*c)^4 + 1620*a^
3*b^2*tan(1/2*d*x + 1/2*c)^4 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 240*a^4*
b*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 35*a^5*tan(
1/2*d*x + 1/2*c)^2 - 60*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a^4*b*tan(1/2*d
*x + 1/2*c) - 3*a^5)/(a^7*tan(1/2*d*x + 1/2*c)^5))/d
```

3.192 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=321

$$-\frac{a^5}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}-\frac{a^4\left(a^2+5b^2\right)}{d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{a^3\left(13a^2b^2+a^4+10b^4\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^5}-$$

```
[Out] -((8*a^2 - 5*a*b - b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^5*d) - ((8*a^2 +
 5*a*b - b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) + (a^3*(a^4 + 13*a^2*
b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) - a^5/(2*(a^2 - b^
2)^3*d*(a + b*Sin[c + d*x])^2) - (a^4*(a^2 + 5*b^2))/((a^2 - b^2)^4*d*(a +
b*Sin[c + d*x])) + (Sec[c + d*x]^4*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin[c
 + d*x]))/(4*(a^2 - b^2)^3*d) - (Sec[c + d*x]^2*(8*a^3*(a^2 + 5*b^2) - b*(2
7*a^4 + 22*a^2*b^2 - b^4)*Sin[c + d*x]))/(8*(a^2 - b^2)^4*d)
```

Rubi [A] time = 0.877422, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 1629}

$$-\frac{a^5}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}-\frac{a^4\left(a^2+5b^2\right)}{d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{a^3\left(13a^2b^2+a^4+10b^4\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^5}-$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((8*a^2 - 5*a*b - b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^5*d) - ((8*a^2 +
 5*a*b - b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) + (a^3*(a^4 + 13*a^2*
b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) - a^5/(2*(a^2 - b^
2)^3*d*(a + b*Sin[c + d*x])^2) - (a^4*(a^2 + 5*b^2))/((a^2 - b^2)^4*d*(a +
b*Sin[c + d*x])) + (Sec[c + d*x]^4*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin[c
 + d*x]))/(4*(a^2 - b^2)^3*d) - (Sec[c + d*x]^2*(8*a^3*(a^2 + 5*b^2) - b*(2
7*a^4 + 22*a^2*b^2 - b^4)*Sin[c + d*x]))/(8*(a^2 - b^2)^4*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
```

```
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^5}{(a+x)^3\left(b^2-x^2\right)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^4(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^3b^6\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3} - \frac{a^2b^4\left(4a^4+3a^2b^2-3b^4\right)}{\left(a^2-b^2\right)^3}\cdots}{(a\cdots}\cdots\right)}{\cdots}$$

$$= \frac{\sec^4(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3 d} - \frac{\sec^2(c+dx)\left(8a^3\left(a^2+5b^2\right)-b\left(27\cdots\right.\right.}{8\left(a^2-b^2\cdots\right.}$$

$$= \frac{\sec^4(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3 d} - \frac{\sec^2(c+dx)\left(8a^3\left(a^2+5b^2\right)-b\left(27\cdots\right.\right.}{8\left(a^2-b^2\cdots\right.}$$

$$= -\frac{\left(8a^2-5ab-b^2\right)\log(1-\sin(c+dx))}{16(a+b)^5 d} - \frac{\left(8a^2+5ab-b^2\right)\log(1+\sin(c+dx))}{16(a-b)^5 d} + \frac{a^3\left(a^4+13\cdots\right.}{\cdots}$$

Mathematica [A] time = 6.34144, size = 304, normalized size = 0.95

$$-\frac{a^5}{2d\left(a^2-b^2\right)^3(a+b\sin(c+dx))^2}-\frac{a^4\left(a^2+5b^2\right)}{d\left(a^2-b^2\right)^4(a+b\sin(c+dx))}+\frac{a^3\left(13a^2b^2+a^4+10b^4\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^5}-$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -((8*a^2 - 5*a*b - b^2)*Log[1 - Sin[c + d*x]])/(16*(a + b)^5*d) - ((8*a^2 +
 5*a*b - b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) + (a^3*(a^4 + 13*a^2*
b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) + 1/(16*(a + b)^3*
d*(1 - Sin[c + d*x])^2) - (7*a + b)/(16*(a + b)^4*d*(1 - Sin[c + d*x])) + 1
/(16*(a - b)^3*d*(1 + Sin[c + d*x])^2) - (7*a - b)/(16*(a - b)^4*d*(1 + Sin
[c + d*x])) - a^5/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) - (a^4*(a^2 +
5*b^2))/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

Maple [A] time = 0.116, size = 465, normalized size = 1.5

$$-\frac{a^5}{2\,d\,(a+b)^3\,(a-b)^3\,(a+b\sin(dx+c))^2}+\frac{a^7\ln(a+b\sin(dx+c))}{d\,(a+b)^5\,(a-b)^5}+13\,\frac{a^5\ln(a+b\sin(dx+c))\,b^2}{d\,(a+b)^5\,(a-b)^5}+10\,\frac{a^3\ln(a+b\,\mathrm{s}}{d\,(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^2+1/d*a^7/(a+b)^5/(a-b)^5*ln(a+
b*sin(d*x+c))+13/d*a^5/(a+b)^5/(a-b)^5*ln(a+b*sin(d*x+c))*b^2+10/d*a^3/(a+b
)^5/(a-b)^5*ln(a+b*sin(d*x+c))*b^4-1/d*a^6/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))
-5/d*a^4/(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))*b^2+1/16/d/(a+b)^3/(sin(d*x+c)-1)
^2+1/16/d/(a+b)^4/(sin(d*x+c)-1)*b+7/16/d/(a+b)^4/(sin(d*x+c)-1)*a-1/2/d/(a
+b)^5*ln(sin(d*x+c)-1)*a^2+5/16/d/(a+b)^5*ln(sin(d*x+c)-1)*a*b+1/16/d/(a+b)
^5*ln(sin(d*x+c)-1)*b^2+1/16/d/(a-b)^3/(1+sin(d*x+c))^2+1/16/d/(a-b)^4/(1+s
in(d*x+c))*b-7/16/d/(a-b)^4/(1+sin(d*x+c))*a-1/2/d/(a-b)^5*ln(1+sin(d*x+c))
*a^2-5/16/d/(a-b)^5*ln(1+sin(d*x+c))*a*b+1/16/d/(a-b)^5*ln(1+sin(d*x+c))*b^
2
```

Maxima [B] time = 1.30265, size = 986, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(16*(a^7 + 13*a^5*b^2 + 10*a^3*b^4)*log(b*sin(d*x + c) + a)/(a^10 - 5*
a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^2 + 5*a*b - b^
2)*log(sin(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5) - (8*a^2 - 5*a*b - b^2)*log(sin(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a
^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(18*a^7 + 72*a^5*b^2 + 6*a^3*b^4 +
 (8*a^6*b + 67*a^4*b^3 + 22*a^2*b^5 - b^7)*sin(d*x + c)^5 + 2*(6*a^7 + 41*a
^5*b^2 + 2*a^3*b^4 - a*b^6)*sin(d*x + c)^4 - (5*a^6*b + 159*a^4*b^3 + 27*a^
2*b^5 + b^7)*sin(d*x + c)^3 - 4*(8*a^7 + 37*a^5*b^2 + 4*a^3*b^4 - a*b^6)*si
n(d*x + c)^2 - (a^6*b - 86*a^4*b^3 - 11*a^2*b^5)*sin(d*x + c))/(a^10 - 4*a^
8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8 + (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6
- 4*a^2*b^8 + b^10)*sin(d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a
^3*b^7 + a*b^9)*sin(d*x + c)^5 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4 - 16*a^4*b^
6 + 9*a^2*b^8 - 2*b^10)*sin(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 -
 4*a^3*b^7 + a*b^9)*sin(d*x + c)^3 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*
a^4*b^6 + 6*a^2*b^8 - b^10)*sin(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b
^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)))/d
```

Fricas [B] time = 6.42709, size = 2206, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a^9 - 16*a^7*b^2 + 24*a^5*b^4 - 16*a^3*b^6 + 4*a*b^8 - 4*(6*a^9 +
35*a^7*b^2 - 39*a^5*b^4 - 3*a^3*b^6 + a*b^8)*cos(d*x + c)^4 - 16*(a^9 - 3*a
^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 - 16*((a^7*b^2 + 13*a^5*b^4 +
10*a^3*b^6)*cos(d*x + c)^6 - 2*(a^8*b + 13*a^6*b^3 + 10*a^4*b^5)*cos(d*x +
c)^4*sin(d*x + c) - (a^9 + 14*a^7*b^2 + 23*a^5*b^4 + 10*a^3*b^6)*cos(d*x +
c)^4)*log(b*sin(d*x + c) + a) + ((8*a^7*b^2 + 45*a^6*b^3 + 104*a^5*b^4 + 12
5*a^4*b^5 + 80*a^3*b^6 + 23*a^2*b^7 - b^9)*cos(d*x + c)^6 - 2*(8*a^8*b + 45
*a^7*b^2 + 104*a^6*b^3 + 125*a^5*b^4 + 80*a^4*b^5 + 23*a^3*b^6 - a*b^8)*cos
(d*x + c)^4*sin(d*x + c) - (8*a^9 + 45*a^8*b + 112*a^7*b^2 + 170*a^6*b^3 +
184*a^5*b^4 + 148*a^4*b^5 + 80*a^3*b^6 + 22*a^2*b^7 - b^9)*cos(d*x + c)^4)*
log(sin(d*x + c) + 1) + ((8*a^7*b^2 - 45*a^6*b^3 + 104*a^5*b^4 - 125*a^4*b^
5 + 80*a^3*b^6 - 23*a^2*b^7 + b^9)*cos(d*x + c)^6 - 2*(8*a^8*b - 45*a^7*b^2
 + 104*a^6*b^3 - 125*a^5*b^4 + 80*a^4*b^5 - 23*a^3*b^6 + a*b^8)*cos(d*x + c
```

```
)^4*sin(d*x + c) - (8*a^9 - 45*a^8*b + 112*a^7*b^2 - 170*a^6*b^3 + 184*a^5*
b^4 - 148*a^4*b^5 + 80*a^3*b^6 - 22*a^2*b^7 + b^9)*cos(d*x + c)^4)*log(-sin
(d*x + c) + 1) - 2*(2*a^8*b - 8*a^6*b^3 + 12*a^4*b^5 - 8*a^2*b^7 + 2*b^9 +
(8*a^8*b + 59*a^6*b^3 - 45*a^4*b^5 - 23*a^2*b^7 + b^9)*cos(d*x + c)^4 - (11
*a^8*b - 36*a^6*b^3 + 42*a^4*b^5 - 20*a^2*b^7 + 3*b^9)*cos(d*x + c)^2)*sin(
d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b
^12)*d*cos(d*x + c)^6 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5
*a^3*b^9 - a*b^11)*d*cos(d*x + c)^4*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a
^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 3.70755, size = 790, normalized size = 2.46

$$\frac{\frac{16\left(a^7b+13a^5b^3+10a^3b^5\right)\log(|b\sin(dx+c)+a|)}{a^{10}b-5a^8b^3+10a^6b^5-10a^4b^7+5a^2b^9-b^{11}} - \frac{\left(8a^2+5ab-b^2\right)\log(|\sin(dx+c)+1|)}{a^5-5a^4b+10a^3b^2-10a^2b^3+5ab^4-b^5} - \frac{\left(8a^2-5ab-b^2\right)\log(|\sin(dx+c)-1|)}{a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5} - \frac{2\left(8a^6b\sin(dx+c)^5+67a^4\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/16*(16*(a^7*b + 13*a^5*b^3 + 10*a^3*b^5)*log(abs(b*sin(d*x + c) + a))/(a^
10*b - 5*a^8*b^3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) - (8*a^2 + 5
*a*b - b^2)*log(abs(sin(d*x + c) + 1))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2
*b^3 + 5*a*b^4 - b^5) - (8*a^2 - 5*a*b - b^2)*log(abs(sin(d*x + c) - 1))/(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(8*a^6*b*sin(d*
x + c)^5 + 67*a^4*b^3*sin(d*x + c)^5 + 22*a^2*b^5*sin(d*x + c)^5 - b^7*sin(
d*x + c)^5 + 12*a^7*sin(d*x + c)^4 + 82*a^5*b^2*sin(d*x + c)^4 + 4*a^3*b^4*
sin(d*x + c)^4 - 2*a*b^6*sin(d*x + c)^4 - 5*a^6*b*sin(d*x + c)^3 - 159*a^4*
b^3*sin(d*x + c)^3 - 27*a^2*b^5*sin(d*x + c)^3 - b^7*sin(d*x + c)^3 - 32*a^
```

```
7*sin(d*x + c)^2 - 148*a^5*b^2*sin(d*x + c)^2 - 16*a^3*b^4*sin(d*x + c)^2 +
 4*a*b^6*sin(d*x + c)^2 - a^6*b*sin(d*x + c) + 86*a^4*b^3*sin(d*x + c) + 11
*a^2*b^5*sin(d*x + c) + 18*a^7 + 72*a^5*b^2 + 6*a^3*b^4)/((a^8 - 4*a^6*b^2
+ 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c)^3 + a*sin(d*x + c)^2 - b*sin
(d*x + c) - a)^2))/d
```

3.193 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$\frac{a^3}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{a^2\left(a^2+3b^2\right)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{a\left(8a^2b^2+a^4+3b^4\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}+\sec$$

```
[Out] ((2*a - b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^4*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c +
 d*x]])/((a^2 - b^2)^4*d) + a^3/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2)
+ (a^2*(a^2 + 3*b^2))/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x
]^2*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin[c + d*x]))/(2*(a^2 - b^2)^3*d)
```

Rubi [A] time = 0.482381, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2721, 1647, 1629}

$$\frac{a^3}{2d\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{a^2\left(a^2+3b^2\right)}{d\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{a\left(8a^2b^2+a^4+3b^4\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^4}+\sec$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a - b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^4*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c +
 d*x]])/((a^2 - b^2)^4*d) + a^3/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2)
+ (a^2*(a^2 + 3*b^2))/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) + (Sec[c + d*x
]^2*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin[c + d*x]))/(2*(a^2 - b^2)^3*d)
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)^3\left(b^2-x^2\right)^2}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\sec^2(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^3b^4\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3}-\frac{a^2b^2\left(2a^4-3a^2b^2-3b^4\right)}{\left(a^2-b^2\right)^3}}{(a+x)^3(b}\right.}{}$$

$$= \frac{\sec^2(c+dx)\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int\left(\frac{b^2(-2a+b)}{2(a+b)^4(b-x)}-\frac{2a^3b^2}{\left(a^2-b^2\right)^2(a+x)^3}\right.\right.}{}$$

$$= \frac{(2a-b)\log(1-\sin(c+dx))}{4(a+b)^4 d} + \frac{(2a+b)\log(1+\sin(c+dx))}{4(a-b)^4 d} - \frac{a\left(a^4+8a^2b^2+3b^4\right)\log(a+}{\left(a^2-b^2\right)^4 d}$$

Mathematica [A] time = 2.189, size = 196, normalized size = 0.84

$$\frac{\frac{2a^3}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))^2}+\frac{4a^2\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{4a\left(8a^2b^2+a^4+3b^4\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^4}-\frac{1}{(a+b)^3(\sin(c+dx)-1)}+\frac{1}{(a-b)^3(\sin(c+dx)+1)}+\frac{(2a-b)\,\mathrm{l}}{}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (((2*a - b)*Log[1 - Sin[c + d*x]])/(a + b)^4 + ((2*a + b)*Log[1 + Sin[c + d
*x]])/(a - b)^4 - (4*a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])/(
a^2 - b^2)^4 - 1/((a + b)^3*(-1 + Sin[c + d*x])) + 1/((a - b)^3*(1 + Sin[c
+ d*x])) + (2*a^3)/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) + (4*a^2*(a^2 + 3
*b^2))/((a^2 - b^2)^3*(a + b*Sin[c + d*x])))/(4*d)
```

Maple [A] time = 0.109, size = 323, normalized size = 1.4

$$\frac{a^3}{2\,d\,(a+b)^2\,(a-b)^2\,(a+b\sin(dx+c))^2}-\frac{a^5\ln(a+b\sin(dx+c))}{d\,(a+b)^4\,(a-b)^4}-8\,\frac{a^3\ln(a+b\sin(dx+c))\,b^2}{d\,(a+b)^4\,(a-b)^4}-3\,\frac{a\ln(a+b\sin(d\cdots}{d\,(a+b)^4\,(a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d*a^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^2-1/d*a^5/(a+b)^4/(a-b)^4*ln(a+b
*sin(d*x+c))-8/d*a^3/(a+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))*b^2-3/d*a/(a+b)^4/(
a-b)^4*ln(a+b*sin(d*x+c))*b^4+1/d*a^4/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))+3/d*
a^2/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))*b^2-1/4/d/(a+b)^3/(sin(d*x+c)-1)+1/2/d
/(a+b)^4*ln(sin(d*x+c)-1)*a-1/4/d/(a+b)^4*ln(sin(d*x+c)-1)*b+1/4/d/(a-b)^3/
(1+sin(d*x+c))+1/2/d/(a-b)^4*ln(1+sin(d*x+c))*a+1/4/d/(a-b)^4*ln(1+sin(d*x+
c))*b
```

Maxima [A] time = 1.87626, size = 595, normalized size = 2.56

$$-\frac{\frac{4\left(a^5+8\,a^3b^2+3\,ab^4\right)\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{(2\,a+b)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{(2\,a-b)\log(\sin(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{2\left(4\,a^5+8\,a\cdots\right.}{a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6-\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)\cdots}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(4*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^
2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (2*a + b)*log(sin(d*x + c) + 1)/(a^4 - 4
*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*log(sin(d*x + c) - 1)/(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(4*a^5 + 8*a^3*b^2 - (2*a^4*b +
```

```
9*a^2*b^3 + b^5)*sin(d*x + c)^3 - (3*a^5 + 10*a^3*b^2 - a*b^4)*sin(d*x + c)
^2 + (a^4*b + 11*a^2*b^3)*sin(d*x + c))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*
b^6 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*sin(d*x + c)^4 - 2*(a^7*b - 3
*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*sin(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b
^7)*sin(d*x + c)))/d
```

Fricas [B] time = 3.96383, size = 1727, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + 2*(3*a^7 + 7*a^5*b^2 - 11*a
^3*b^4 + a*b^6)*cos(d*x + c)^2 + 4*((a^5*b^2 + 8*a^3*b^4 + 3*a*b^6)*cos(d*x
 + c)^4 - 2*(a^6*b + 8*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^2*sin(d*x + c) - (
a^7 + 9*a^5*b^2 + 11*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2)*log(b*sin(d*x + c)
+ a) - ((2*a^5*b^2 + 9*a^4*b^3 + 16*a^3*b^4 + 14*a^2*b^5 + 6*a*b^6 + b^7)*c
os(d*x + c)^4 - 2*(2*a^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2*b^
5 + a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (2*a^7 + 9*a^6*b + 18*a^5*b^2 + 23
*a^4*b^3 + 22*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*cos(d*x + c)^2)*log(sin
(d*x + c) + 1) - ((2*a^5*b^2 - 9*a^4*b^3 + 16*a^3*b^4 - 14*a^2*b^5 + 6*a*b^
6 - b^7)*cos(d*x + c)^4 - 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4
+ 6*a^2*b^5 - a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (2*a^7 - 9*a^6*b + 18*a^
5*b^2 - 23*a^4*b^3 + 22*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6 - b^7)*cos(d*x + c)^
2)*log(-sin(d*x + c) + 1) - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (2*a^6
*b + 7*a^4*b^3 - 8*a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 -
 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^4 - 2*(a^9*b - 4*
a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)^2*sin(d*x + c) - (a
^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^
2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^3(c + dx)}{(a + b\sin(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

Giac [B] time = 2.10026, size = 626, normalized size = 2.7

$$-\frac{\frac{4\left(a^{5}b+8\,a^{3}b^{3}+3\,ab^{5}\right)\log(|b\sin(dx+c)+a|)}{a^{8}b-4\,a^{6}b^{3}+6\,a^{4}b^{5}-4\,a^{2}b^{7}+b^{9}}-\frac{(2\,a+b)\log(|\sin(dx+c)+1|)}{a^{4}-4\,a^{3}b+6\,a^{2}b^{2}-4\,ab^{3}+b^{4}}-\frac{(2\,a-b)\log(|\sin(dx+c)-1|)}{a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}}+\frac{2\left(a^{5}\sin(dx+c)^{2}+8\,a^{3}b^{2}\sin(dx+c)^{2}+3\,ab^{4}\sin(d\right.}{\left(a^{8}-4\,a^{6}b^{2}+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*(a^5*b + 8*a^3*b^3 + 3*a*b^5)*log(abs(b*sin(d*x + c) + a))/(a^8*b -
 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (2*a + b)*log(abs(sin(d*x + c)
+ 1))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*log(abs(sin(d
*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(a^5*sin(d*x
+ c)^2 + 8*a^3*b^2*sin(d*x + c)^2 + 3*a*b^4*sin(d*x + c)^2 - 3*a^4*b*sin(d*
x + c) + 2*a^2*b^3*sin(d*x + c) + b^5*sin(d*x + c) - 6*a^3*b^2 - 6*a*b^4)/(
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)) - 2*(
3*a^5*b^2*sin(d*x + c)^2 + 24*a^3*b^4*sin(d*x + c)^2 + 9*a*b^6*sin(d*x + c)
^2 + 8*a^6*b*sin(d*x + c) + 52*a^4*b^3*sin(d*x + c) + 12*a^2*b^5*sin(d*x +
c) + 6*a^7 + 26*a^5*b^2 + 4*a^3*b^4)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*
b^6 + b^8)*(b*sin(d*x + c) + a)^2))/d
```

3.194 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=149

$$-\frac{a}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{a^2+b^2}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{a\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\log(1-\sin(c}{2d(a+b}$$

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^3*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)^3
*d) + (a*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) - a/(2*(a
^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (a^2 + b^2)/((a^2 - b^2)^2*d*(a + b*S
in[c + d*x]))
```

Rubi [A] time = 0.132163, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 801}

$$-\frac{a}{2d\left(a^2-b^2\right)(a+b\sin(c+dx))^2}-\frac{a^2+b^2}{d\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{a\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{d\left(a^2-b^2\right)^3}-\frac{\log(1-\sin(c}{2d(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -Log[1 - Sin[c + d*x]]/(2*(a + b)^3*d) - Log[1 + Sin[c + d*x]]/(2*(a - b)^3
*d) + (a*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^3*d) - a/(2*(a
^2 - b^2)*d*(a + b*Sin[c + d*x])^2) - (a^2 + b^2)/((a^2 - b^2)^2*d*(a + b*S
in[c + d*x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+x)^3\left(b^2-x^2\right)}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)^3(b-x)} + \frac{a}{(a-b)(a+b)(a+x)^3} + \frac{a^2+b^2}{(a-b)^2(a+b)^2(a+x)^2} + \frac{a^3+3ab^2}{(a-b)^3(a+b)^3(a+x)} - \frac{1}{2(a-b)^3(b+x)}\right) dx\right.}{d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^3d} - \frac{\log(1+\sin(c+dx))}{2(a-b)^3d} + \frac{a\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^3 d} - \frac{}{2\left(a^2\right.}$$

Mathematica [A] time = 2.02021, size = 213, normalized size = 1.43

$$\frac{\frac{2b}{\left(a^2-b^2\right)(a+b\sin(c+dx))} - \frac{4ab\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2} + a\left(\frac{b\left(\frac{\left(a^2-b^2\right)\left(-5a^2-4ab\sin(c+dx)+b^2\right)}{(a+b\sin(c+dx))^2}+2\left(3a^2+b^2\right)\log(a+b\sin(c+dx))\right)}{\left(a^2-b^2\right)^3} + \frac{\log(1-\sin(c+dx))}{(a+b)^3} - \frac{\log(\sin(}{(a-}\right.}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-(Log[1 - Sin[c + d*x]]/(a + b)^2) + Log[1 + Sin[c + d*x]]/(a - b)^2 - (4*
a*b*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^2 + (2*b)/((a^2 - b^2)*(a + b*Sin[
c + d*x])) + a*(Log[1 - Sin[c + d*x]]/(a + b)^3 - Log[1 + Sin[c + d*x]]/(a
- b)^3 + (b*(2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]] + ((a^2 - b^2)*(-5*a^2
 + b^2 - 4*a*b*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2))/(a^2 - b^2)^3))/(2*b
*d)
```

Maple [A] time = 0.1, size = 198, normalized size = 1.3

$$-\frac{a}{2d(a+b)(a-b)(a+b\sin(dx+c))^2} - \frac{a^2}{d(a+b)^2(a-b)^2(a+b\sin(dx+c))} - \frac{b^2}{d(a+b)^2(a-b)^2(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/d*a/(a+b)/(a-b)/(a+b*sin(d*x+c))^2-1/d/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c)
)*a^2-1/d/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))*b^2+1/d*a^3/(a+b)^3/(a-b)^3*ln(a
+b*sin(d*x+c))+3/d*a/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))*b^2-1/2/d/(a+b)^3*l
n(sin(d*x+c)-1)-1/2*ln(1+sin(d*x+c))/(a-b)^3/d
```

Maxima [A] time = 1.91543, size = 308, normalized size = 2.07

$$\frac{\frac{2\left(a^3+3\,ab^2\right)\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{3\,a^3+ab^2+2\left(a^2b+b^3\right)\sin(dx+c)}{a^6-2\,a^4b^2+a^2b^4+\left(a^4b^2-2\,a^2b^4+b^6\right)\sin(dx+c)^2+2\left(a^5b-2\,a^3b^3+ab^5\right)\sin(dx+c)}-\frac{\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a^3 + 3*a*b^2)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) - (3*a^3 + a*b^2 + 2*(a^2*b + b^3)*sin(d*x + c))/(a^6 - 2*a^4*b^2 +
 a^2*b^4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(d*x + c)^2 + 2*(a^5*b - 2*a^3*b^
3 + a*b^5)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) - log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

Fricas [B] time = 2.6488, size = 995, normalized size = 6.68

$$3\,a^5-2\,a^3b^2-ab^4-2\left(a^5+4\,a^3b^2+3\,ab^4-\left(a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^2+2\left(a^4b+3\,a^2b^3\right)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^5 - 2*a^3*b^2 - a*b^4 - 2*(a^5 + 4*a^3*b^2 + 3*a*b^4 - (a^3*b^2 +
3*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^2*b^3)*sin(d*x + c))*log(b*sin(d*x
 + c) + a) + (a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a^3*
b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2 + 3*
a^2*b^3 + a*b^4)*sin(d*x + c))*log(sin(d*x + c) + 1) + (a^5 - 3*a^4*b + 4*a
^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*
cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x + c))*lo
```

```
g(-sin(d*x + c) + 1) + 2*(a^4*b - b^5)*sin(d*x + c))/((a^6*b^2 - 3*a^4*b^4
+ 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*
b^7)*d*sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.58231, size = 347, normalized size = 2.33

$$\frac{\frac{2\left(a^3b+3ab^3\right)\log(|b\sin(dx+c)+a|)}{a^6b-3a^4b^3+3a^2b^5-b^7}-\frac{\log(|\sin(dx+c)+1|)}{a^3-3a^2b+3ab^2-b^3}-\frac{\log(|\sin(dx+c)-1|)}{a^3+3a^2b+3ab^2+b^3}-\frac{3a^3b^2\sin(dx+c)^2+9ab^4\sin(dx+c)^2+8a^4b\sin(dx+c)+18a^2b^3\sin(dx+c)-}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)(b\sin(dx+c)+a)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3*b + 3*a*b^3)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 +
3*a^2*b^5 - b^7) - log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) - log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3*b^
2*sin(d*x + c)^2 + 9*a*b^4*sin(d*x + c)^2 + 8*a^4*b*sin(d*x + c) + 18*a^2*b
^3*sin(d*x + c) - 2*b^5*sin(d*x + c) + 6*a^5 + 7*a^3*b^2 - a*b^4)/((a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*(b*sin(d*x + c) + a)^2))/d
```

3.195 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=75

$$\frac{1}{a^2d(a+b\sin(c+dx))}-\frac{\log(a+b\sin(c+dx))}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}+\frac{1}{2ad(a+b\sin(c+dx))^2}$$

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[a + b*Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a +
 b*Sin[c + d*x])^2) + 1/(a^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.060479, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 44}

$$\frac{1}{a^2d(a+b\sin(c+dx))}-\frac{\log(a+b\sin(c+dx))}{a^3d}+\frac{\log(\sin(c+dx))}{a^3d}+\frac{1}{2ad(a+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] Log[Sin[c + d*x]]/(a^3*d) - Log[a + b*Sin[c + d*x]]/(a^3*d) + 1/(2*a*d*(a +
 b*Sin[c + d*x])^2) + 1/(a^2*d*(a + b*Sin[c + d*x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a^3x} - \frac{1}{a(a+x)^3} - \frac{1}{a^2(a+x)^2} - \frac{1}{a^3(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(a+b\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+b\sin(c+dx))^2} + \frac{1}{a^2d(a+b\sin(c+dx))}$$

Mathematica [A] time = 0.261819, size = 60, normalized size = 0.8

$$\frac{\frac{a(3a+2b\sin(c+dx))}{(a+b\sin(c+dx))^2} - 2\log(a+b\sin(c+dx)) + 2\log(\sin(c+dx))}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*Log[Sin[c + d*x]] - 2*Log[a + b*Sin[c + d*x]] + (a*(3*a + 2*b*Sin[c + d*
x]))/(a + b*Sin[c + d*x])^2)/(2*a^3*d)
```

Maple [A] time = 0.046, size = 74, normalized size = 1.

$$\frac{\ln(\sin(dx+c))}{a^3d} - \frac{\ln(a+b\sin(dx+c))}{a^3d} + \frac{1}{2\,da\,(a+b\sin(dx+c))^2} + \frac{1}{a^2d\,(a+b\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c))^3,x)
```

```
[Out] ln(sin(d*x+c))/a^3/d-ln(a+b*sin(d*x+c))/a^3/d+1/2/a/d/(a+b*sin(d*x+c))^2+1/
a^2/d/(a+b*sin(d*x+c))
```

Maxima [A] time = 1.54714, size = 109, normalized size = 1.45

$$\frac{\frac{2\,b\sin(dx+c)+3\,a}{a^2b^2\sin(dx+c)^2+2\,a^3b\sin(dx+c)+a^4} - \frac{2\,\log(b\sin(dx+c)+a)}{a^3} + \frac{2\,\log(\sin(dx+c))}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((2*b*sin(d*x + c) + 3*a)/(a^2*b^2*sin(d*x + c)^2 + 2*a^3*b*sin(d*x + c
) + a^4) - 2*log(b*sin(d*x + c) + a)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

Fricas [B] time = 2.00358, size = 365, normalized size = 4.87

$$-\frac{2\,ab\sin\left(dx+c\right)+3\,a^{2}+2\left(b^{2}\cos\left(dx+c\right)^{2}-2\,ab\sin\left(dx+c\right)-a^{2}-b^{2}\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\left(b^{2}\cos\left(dx+c\right)^{2}-\right.}{2\left(a^{3}b^{2}d\cos\left(dx+c\right)^{2}-2\,a^{4}bd\sin\left(dx+c\right)-\left(a^{5}+a^{3}b^{2}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*sin(d*x + c) + 3*a^2 + 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
c) - a^2 - b^2)*log(b*sin(d*x + c) + a) - 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2)*log(-1/2*sin(d*x + c)))/(a^3*b^2*d*cos(d*x + c)^2 -
2*a^4*b*d*sin(d*x + c) - (a^5 + a^3*b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.74614, size = 93, normalized size = 1.24

$$-\frac{\frac{2\,\log\left(\left|b\sin\left(dx+c\right)+a\right|\right)}{a^{3}}-\frac{2\,\log\left(\left|\sin\left(dx+c\right)\right|\right)}{a^{3}}-\frac{2\,ab\sin\left(dx+c\right)+3\,a^{2}}{\left(b\sin\left(dx+c\right)+a\right)^{2}a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*log(abs(b*sin(d*x + c) + a))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (
2*a*b*sin(d*x + c) + 3*a^2)/((b*sin(d*x + c) + a)^2*a^3))/d
```

3.196 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$-\frac{a^2-3b^2}{a^4d(a+b\sin(c+dx))}-\frac{a^2-b^2}{2a^3d(a+b\sin(c+dx))^2}-\frac{\left(a^2-6b^2\right)\log(\sin(c+dx))}{a^5d}+\frac{\left(a^2-6b^2\right)\log(a+b\sin(c+dx))}{a^5d}+3$$

```
[Out] (3*b*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^3*d) - ((a^2 - 6*b^2)*Log[
Sin[c + d*x]])/(a^5*d) + ((a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) -
(a^2 - b^2)/(2*a^3*d*(a + b*Sin[c + d*x])^2) - (a^2 - 3*b^2)/(a^4*d*(a + b*
Sin[c + d*x]))
```

Rubi [A] time = 0.133294, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$-\frac{a^2-3b^2}{a^4d(a+b\sin(c+dx))}-\frac{a^2-b^2}{2a^3d(a+b\sin(c+dx))^2}-\frac{\left(a^2-6b^2\right)\log(\sin(c+dx))}{a^5d}+\frac{\left(a^2-6b^2\right)\log(a+b\sin(c+dx))}{a^5d}+3$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*b*Csc[c + d*x])/(a^4*d) - Csc[c + d*x]^2/(2*a^3*d) - ((a^2 - 6*b^2)*Log[
Sin[c + d*x]])/(a^5*d) + ((a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]])/(a^5*d) -
(a^2 - b^2)/(2*a^3*d*(a + b*Sin[c + d*x])^2) - (a^2 - 3*b^2)/(a^4*d*(a + b*
Sin[c + d*x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{b^2-x^2}{x^3(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{b^2}{a^3x^3}-\frac{3b^2}{a^4x^2}+\frac{-a^2+6b^2}{a^5x}+\frac{a^2-b^2}{a^3(a+x)^3}+\frac{a^2-3b^2}{a^4(a+x)^2}+\frac{a^2-6b^2}{a^5(a+x)}\right) dx, x, b\sin(c+dx)\right)}{d}$$
$$= \frac{3b\csc(c+dx)}{a^4d}-\frac{\csc^2(c+dx)}{2a^3d}-\frac{\left(a^2-6b^2\right)\log(\sin(c+dx))}{a^5d}+\frac{\left(a^2-6b^2\right)\log(a+b\sin(c+$$

Mathematica [A] time = 0.931238, size = 121, normalized size = 0.83

$$-\frac{\frac{2a\left(a^2-3b^2\right)}{a+b\sin(c+dx)}+2\left(a^2-6b^2\right)\log(\sin(c+dx))-2\left(a^2-6b^2\right)\log(a+b\sin(c+dx))+\frac{a^2(a-b)(a+b)}{(a+b\sin(c+dx))^2}+a^2\csc^2(c+dx)-6a}{2a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(-6*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - 6*b^2)*Log[Sin[c + d*
x]] - 2*(a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]] + (a^2*(a - b)*(a + b))/(a +
b*Sin[c + d*x])^2 + (2*a*(a^2 - 3*b^2))/(a + b*Sin[c + d*x]))/(2*a^5*d)
```

Maple [A] time = 0.111, size = 194, normalized size = 1.3

$$\frac{\ln(a+b\sin(dx+c))}{a^3d}-6\,\frac{\ln(a+b\sin(dx+c))\,b^2}{da^5}-\frac{1}{a^2d\,(a+b\sin(dx+c))}+3\,\frac{b^2}{da^4\,(a+b\sin(dx+c))}-\frac{1}{2\,da\,(a+b\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x)
```

```
[Out] ln(a+b*sin(d*x+c))/a^3/d-6/d/a^5*ln(a+b*sin(d*x+c))*b^2-1/a^2/d/(a+b*sin(d*
x+c))+3/d/a^4/(a+b*sin(d*x+c))*b^2-1/2/a/d/(a+b*sin(d*x+c))^2+1/2/d/a^3/(a+
b*sin(d*x+c))^2*b^2-1/2/d/a^3/sin(d*x+c)^2-ln(sin(d*x+c))/a^3/d+6/d/a^5*ln(
```

```
sin(d*x+c))*b^2+3/d/a^4*b/sin(d*x+c)
```

Maxima [A] time = 1.92069, size = 211, normalized size = 1.46

$$\frac{\frac{4\,a^2b\sin(dx+c)-2\left(a^2b-6\,b^3\right)\sin(dx+c)^3-a^3-3\left(a^3-6\,ab^2\right)\sin(dx+c)^2}{a^4b^2\sin(dx+c)^4+2\,a^5b\sin(dx+c)^3+a^6\sin(dx+c)^2}+\frac{2\left(a^2-6\,b^2\right)\log(b\sin(dx+c)+a)}{a^5}-\frac{2\left(a^2-6\,b^2\right)\log(\sin(dx+c))}{a^5}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a^2*b*sin(d*x + c) - 2*(a^2*b - 6*b^3)*sin(d*x + c)^3 - a^3 - 3*(a^
3 - 6*a*b^2)*sin(d*x + c)^2)/(a^4*b^2*sin(d*x + c)^4 + 2*a^5*b*sin(d*x + c)
^3 + a^6*sin(d*x + c)^2) + 2*(a^2 - 6*b^2)*log(b*sin(d*x + c) + a)/a^5 - 2*
(a^2 - 6*b^2)*log(sin(d*x + c))/a^5)/d
```

Fricas [B] time = 2.3421, size = 911, normalized size = 6.28

$$-\frac{4\,a^4-18\,a^2b^2-3\left(a^4-6\,a^2b^2\right)\cos(dx+c)^2-2\left(\left(a^2b^2-6\,b^4\right)\cos(dx+c)^4+a^4-5\,a^2b^2-6\,b^4-\left(a^4-4\,a^2b^2-12\,b^4\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4 - 18*a^2*b^2 - 3*(a^4 - 6*a^2*b^2)*cos(d*x + c)^2 - 2*((a^2*b^2
 - 6*b^4)*cos(d*x + c)^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 12*
b^4)*cos(d*x + c)^2 + 2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c)^2
)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 2*((a^2*b^2 - 6*b^4)*cos(d*x + c)
^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 12*b^4)*cos(d*x + c)^2 +
2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1
/2*sin(d*x + c)) - 2*(a^3*b + 6*a*b^3 + (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*s
in(d*x + c))/(a^5*b^2*d*cos(d*x + c)^4 - (a^7 + 2*a^5*b^2)*d*cos(d*x + c)^2
 + (a^7 + a^5*b^2)*d - 2*(a^6*b*d*cos(d*x + c)^2 - a^6*b*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.80037, size = 208, normalized size = 1.43

$$-\frac{\frac{2\left(a^2-6b^2\right)\log(|\sin(dx+c)|)}{a^5}-\frac{2\left(a^2b-6b^3\right)\log(|b\sin(dx+c)+a|)}{a^5b}+\frac{2a^2b\sin(dx+c)^3-12b^3\sin(dx+c)^3+3a^3\sin(dx+c)^2-18ab^2\sin(dx+c)^2-4a^2b\sin(dx+c)}{\left(b\sin(dx+c)^2+a\sin(dx+c)\right)^2a^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a^2 - 6*b^2)*log(abs(sin(d*x + c)))/a^5 - 2*(a^2*b - 6*b^3)*log(ab
s(b*sin(d*x + c) + a))/(a^5*b) + (2*a^2*b*sin(d*x + c)^3 - 12*b^3*sin(d*x +
 c)^3 + 3*a^3*sin(d*x + c)^2 - 18*a*b^2*sin(d*x + c)^2 - 4*a^2*b*sin(d*x +
c) + a^3)/((b*sin(d*x + c)^2 + a*sin(d*x + c))^2*a^4))/d
```

3.197 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{-6a^2b^2+a^4+5b^4}{a^6d(a+b\sin(c+dx))}+\frac{\left(a^2-b^2\right)^2}{2a^5d(a+b\sin(c+dx))^2}+\frac{\left(a^2-3b^2\right)\csc^2(c+dx)}{a^5d}-\frac{2b\left(3a^2-5b^2\right)\csc(c+dx)}{a^6d}+\frac{\left(-12a^2b^2+a\right.}{}$$

```
[Out] (-2*b*(3*a^2 - 5*b^2)*Csc[c + d*x])/(a^6*d) + ((a^2 - 3*b^2)*Csc[c + d*x]^2
)/(a^5*d) + (b*Csc[c + d*x]^3)/(a^4*d) - Csc[c + d*x]^4/(4*a^3*d) + ((a^4 -
 12*a^2*b^2 + 15*b^4)*Log[Sin[c + d*x]])/(a^7*d) - ((a^4 - 12*a^2*b^2 + 15*
b^4)*Log[a + b*Sin[c + d*x]])/(a^7*d) + (a^2 - b^2)^2/(2*a^5*d*(a + b*Sin[c
 + d*x])^2) + (a^4 - 6*a^2*b^2 + 5*b^4)/(a^6*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.211009, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2721, 894}

$$\frac{-6a^2b^2+a^4+5b^4}{a^6d(a+b\sin(c+dx))}+\frac{\left(a^2-b^2\right)^2}{2a^5d(a+b\sin(c+dx))^2}+\frac{\left(a^2-3b^2\right)\csc^2(c+dx)}{a^5d}-\frac{2b\left(3a^2-5b^2\right)\csc(c+dx)}{a^6d}+\frac{\left(-12a^2b^2+a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-2*b*(3*a^2 - 5*b^2)*Csc[c + d*x])/(a^6*d) + ((a^2 - 3*b^2)*Csc[c + d*x]^2
)/(a^5*d) + (b*Csc[c + d*x]^3)/(a^4*d) - Csc[c + d*x]^4/(4*a^3*d) + ((a^4 -
 12*a^2*b^2 + 15*b^4)*Log[Sin[c + d*x]])/(a^7*d) - ((a^4 - 12*a^2*b^2 + 15*
b^4)*Log[a + b*Sin[c + d*x]])/(a^7*d) + (a^2 - b^2)^2/(2*a^5*d*(a + b*Sin[c
 + d*x])^2) + (a^4 - 6*a^2*b^2 + 5*b^4)/(a^6*d*(a + b*Sin[c + d*x]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(b^2-x^2\right)^2}{x^5(a+x)^3}\,dx, x, b\sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^4}{a^3x^5} - \frac{3b^4}{a^4x^4} + \frac{2b^2\left(-a^2+3b^2\right)}{a^5x^3} + \frac{2\left(3a^2b^2-5b^4\right)}{a^6x^2} + \frac{a^4-12a^2b^2+15b^4}{a^7x} - \frac{\left(a^2-b^2\right)^2}{a^5(a+x)^3} + \frac{-a^4+6a^2b^2-5b^4}{a^6(a+x)^2}\right.\right.}{d}$$

$$= -\frac{2b\left(3a^2-5b^2\right)\csc(c+dx)}{a^6d} + \frac{\left(a^2-3b^2\right)\csc^2(c+dx)}{a^5d} + \frac{b\csc^3(c+dx)}{a^4d} - \frac{\csc^4(c+dx)}{4a^3d} + \cdots$$

Mathematica [A] time = 5.32652, size = 195, normalized size = 0.88

$$\frac{\frac{4a\left(-6a^2b^2+a^4+5b^4\right)}{a+b\sin(c+dx)} + \frac{2\left(a^3-ab^2\right)^2}{(a+b\sin(c+dx))^2} + 4a^2\left(a^2-3b^2\right)\csc^2(c+dx) - 8ab\left(3a^2-5b^2\right)\csc(c+dx) + 4\left(-12a^2b^2+a^4+15b^4\right)\cdots}{4a^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-8*a*b*(3*a^2 - 5*b^2)*Csc[c + d*x] + 4*a^2*(a^2 - 3*b^2)*Csc[c + d*x]^2 +
 4*a^3*b*Csc[c + d*x]^3 - a^4*Csc[c + d*x]^4 + 4*(a^4 - 12*a^2*b^2 + 15*b^4
)*Log[Sin[c + d*x]] - 4*(a^4 - 12*a^2*b^2 + 15*b^4)*Log[a + b*Sin[c + d*x]]
 + (2*(a^3 - a*b^2)^2)/(a + b*Sin[c + d*x])^2 + (4*a*(a^4 - 6*a^2*b^2 + 5*b
^4))/(a + b*Sin[c + d*x]))/(4*a^7*d)
```

Maple [A] time = 0.114, size = 348, normalized size = 1.6

$$-\frac{\ln(a+b\sin(dx+c))}{a^3d} + 12\frac{\ln(a+b\sin(dx+c))b^2}{da^5} - 15\frac{\ln(a+b\sin(dx+c))b^4}{da^7} + \frac{1}{a^2d(a+b\sin(dx+c))} - 6\frac{\cdots}{da^4(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -ln(a+b*sin(d*x+c))/a^3/d+12/d/a^5*ln(a+b*sin(d*x+c))*b^2-15/d/a^7*ln(a+b*s
in(d*x+c))*b^4+1/a^2/d/(a+b*sin(d*x+c))-6/d/a^4/(a+b*sin(d*x+c))*b^2+5/d/a^
6/(a+b*sin(d*x+c))*b^4+1/2/a/d/(a+b*sin(d*x+c))^2-1/d/a^3/(a+b*sin(d*x+c))^
2*b^2+1/2/d/a^5/(a+b*sin(d*x+c))^2*b^4-1/4/d/a^3/sin(d*x+c)^4+1/d/a^3/sin(d
*x+c)^2-3/d/a^5/sin(d*x+c)^2*b^2+ln(sin(d*x+c))/a^3/d-12/d/a^5*ln(sin(d*x+c
))*b^2+15/d/a^7*ln(sin(d*x+c))*b^4+1/d/a^4*b/sin(d*x+c)^3-6/d/a^4*b/sin(d*x
+c)+10/d*b^3/a^6/sin(d*x+c)
```

Maxima [A] time = 1.61205, size = 319, normalized size = 1.44

$$\frac{\frac{2\,a^4 b\sin(dx+c)+4\left(a^4b-12\,a^2b^3+15\,b^5\right)\sin(dx+c)^5-a^5+6\left(a^5-12\,a^3b^2+15\,ab^4\right)\sin(dx+c)^4-4\left(4\,a^4b-5\,a^2b^3\right)\sin(dx+c)^3+\left(4\,a^5-5\,a^3b^2\right)\sin(dx+c)^2}{a^6b^2\sin(dx+c)^6+2\,a^7b\sin(dx+c)^5+a^8\sin(dx+c)^4}-\frac{4\left(a^4-12\,a^2\right.}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*((2*a^4*b*sin(d*x + c) + 4*(a^4*b - 12*a^2*b^3 + 15*b^5)*sin(d*x + c)^5
 - a^5 + 6*(a^5 - 12*a^3*b^2 + 15*a*b^4)*sin(d*x + c)^4 - 4*(4*a^4*b - 5*a^
2*b^3)*sin(d*x + c)^3 + (4*a^5 - 5*a^3*b^2)*sin(d*x + c)^2)/(a^6*b^2*sin(d*
x + c)^6 + 2*a^7*b*sin(d*x + c)^5 + a^8*sin(d*x + c)^4) - 4*(a^4 - 12*a^2*b
^2 + 15*b^4)*log(b*sin(d*x + c) + a)/a^7 + 4*(a^4 - 12*a^2*b^2 + 15*b^4)*lo
g(sin(d*x + c))/a^7)/d
```

Fricas [B] time = 2.42375, size = 1708, normalized size = 7.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(9*a^6 - 77*a^4*b^2 + 90*a^2*b^4 + 6*(a^6 - 12*a^4*b^2 + 15*a^2*b^4)*c
os(d*x + c)^4 - (16*a^6 - 149*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^2 + 4*((a
^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*b^2 - 3*a^2*b^4
 - 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x + c)^4 + (2*a^6
 - 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 12*a^3*b^3
```

```
+ 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(a^5*b - 12
*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a)
- 4*((a^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*b^2 - 3*
a^2*b^4 - 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x + c)^4 +
 (2*a^6 - 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 12*a
^3*b^3 + 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(a^5
*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x
+ c)) - 2*(5*a^5*b + 14*a^3*b^3 - 30*a*b^5 - 2*(a^5*b - 12*a^3*b^3 + 15*a*b
^5)*cos(d*x + c)^4 - 2*(2*a^5*b + 19*a^3*b^3 - 30*a*b^5)*cos(d*x + c)^2)*si
n(d*x + c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9 + 3*a^7*b^2)*d*cos(d*x + c)^4
+ (2*a^9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9 + a^7*b^2)*d - 2*(a^8*b*d*cos
(d*x + c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*b*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 2.10396, size = 441, normalized size = 2.

$$\frac{\frac{12\left(a^4-12a^2b^2+15b^4\right)\log(|\sin(dx+c)|)}{a^7}-\frac{12\left(a^4b-12a^2b^3+15b^5\right)\log(|b\sin(dx+c)+a|)}{a^7b}+\frac{6\left(3a^4b^2\sin(dx+c)^2-36a^2b^4\sin(dx+c)^2+45b^6\sin(dx+c)^2+8a^5b\,\mathrm{s}\right.}{(b\sin(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/12*(12*(a^4 - 12*a^2*b^2 + 15*b^4)*log(abs(sin(d*x + c)))/a^7 - 12*(a^4*b
 - 12*a^2*b^3 + 15*b^5)*log(abs(b*sin(d*x + c) + a))/(a^7*b) + 6*(3*a^4*b^2
*sin(d*x + c)^2 - 36*a^2*b^4*sin(d*x + c)^2 + 45*b^6*sin(d*x + c)^2 + 8*a^5
*b*sin(d*x + c) - 84*a^3*b^3*sin(d*x + c) + 100*a*b^5*sin(d*x + c) + 6*a^6
- 50*a^4*b^2 + 56*a^2*b^4)/((b*sin(d*x + c) + a)^2*a^7) - (25*a^4*sin(d*x +
 c)^4 - 300*a^2*b^2*sin(d*x + c)^4 + 375*b^4*sin(d*x + c)^4 + 72*a^3*b*sin(
```

```
d*x + c)^3 - 120*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 36*a^2*b^2*
sin(d*x + c)^2 - 12*a^3*b*sin(d*x + c) + 3*a^4)/(a^7*sin(d*x + c)^4))/d
```

3.198 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=474

$$\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{8a^4b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{}{2d\,(\ldots}$$

```
[Out] (8*a^4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(
9/2)*d) + (12*a^2*b^2*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/((a^2 - b^2)^(9/2)*d) + (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + Cos[c + d*x]/(12*(a + b)
^3*d*(1 - Sin[c + d*x])^2) - (3*a*Cos[c + d*x])/(4*(a + b)^4*d*(1 - Sin[c +
 d*x])) + Cos[c + d*x]/(12*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(
12*(a - b)^3*d*(1 + Sin[c + d*x])^2) + (3*a*Cos[c + d*x])/(4*(a - b)^4*d*(1
 + Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4
*b*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (3*a^5*b*Cos[
c + d*x])/(2*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (4*a^3*b^3*Cos[c + d*x
])/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.871767, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2731, 2650, 2648, 2664, 2754, 12, 2660, 618, 204}

$$\frac{a^4\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{8a^4b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{9/2}}+\frac{}{2d\,(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (8*a^4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(
9/2)*d) + (12*a^2*b^2*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/((a^2 - b^2)^(9/2)*d) + (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + Cos[c + d*x]/(12*(a + b)
^3*d*(1 - Sin[c + d*x])^2) - (3*a*Cos[c + d*x])/(4*(a + b)^4*d*(1 - Sin[c +
 d*x])) + Cos[c + d*x]/(12*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(
12*(a - b)^3*d*(1 + Sin[c + d*x])^2) + (3*a*Cos[c + d*x])/(4*(a - b)^4*d*(1
```

```
 + Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4
*b*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (3*a^5*b*Cos[
c + d*x])/(2*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (4*a^3*b^3*Cos[c + d*x
])/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \left(\frac{1}{4(a+b)^3(-1+\sin(c+dx))^2} + \frac{3a}{4(a+b)^4(-1+\sin(c+dx))} + \frac{1}{4(a-b)^3(1+\sin(c+dx))^2}\right.$$

$$= -\frac{(3a)\int \frac{1}{1+\sin(c+dx)}\,dx}{4(a-b)^4} + \frac{\int \frac{1}{(1+\sin(c+dx))^2}\,dx}{4(a-b)^3} + \frac{(3a)\int \frac{1}{-1+\sin(c+dx)}\,dx}{4(a+b)^4} + \frac{\int \frac{1}{(-1+\sin(c+dx))^2}\,dx}{4(a+b)^3} +$$

$$= \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2} - \frac{3a\cos(c+dx)}{4(a+b)^4d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{12(a-b)^3d(1+\sin(c+dx))^2}$$

$$= \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2} - \frac{3a\cos(c+dx)}{4(a+b)^4d(1-\sin(c+dx))} + \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))}$$

$$= \frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d} + \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2} - \frac{3a\cos(c+}{4(a+b)^4d(1-\sin}$$

$$= \frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d} + \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2} - \frac{3a\cos(c+}{4(a+b)^4d(1-\sin}$$

$$= \frac{8a^4b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d} + \frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d} + \frac{\cos(c+}{12(a+b)^3d(1-\mathrm{s}}$$

$$= \frac{8a^4b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d} + \frac{12a^2b^2\left(a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}d} + \frac{a^4\left(2a^2+b^2\right)\tan}{(a^2-}$$

Mathematica [A] time = 1.043, size = 351, normalized size = 0.74

$$\frac{\frac{96a^2\left(21a^2b^2+2a^4+12b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{9/2}} - \frac{\sec^3(c+dx)\left(22a^5b^2\sin(c+dx)-91a^5b^2\sin(3(c+dx))-17a^5b^2\sin(5(c+dx))-264a^3b^4\sin(c+dx)-244a^3b^4\sin(\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((96*a^2*(2*a^4 + 21*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt
[a^2 - b^2]])/(a^2 - b^2)^(9/2) - (Sec[c + d*x]^3*(-264*a^6*b - 358*a^4*b^3
 + 8*a^2*b^5 - 16*b^7 - 8*(44*a^6*b + 55*a^4*b^3 + 8*a^2*b^5 - 2*b^7)*Cos[2
*(c + d*x)] - 2*(28*a^6*b + 89*a^4*b^3 - 12*a^2*b^5)*Cos[4*(c + d*x)] + 22*
a^5*b^2*Sin[c + d*x] - 264*a^3*b^4*Sin[c + d*x] + 32*a*b^6*Sin[c + d*x] + 3
2*a^7*Sin[3*(c + d*x)] - 91*a^5*b^2*Sin[3*(c + d*x)] - 244*a^3*b^4*Sin[3*(c
 + d*x)] - 12*a*b^6*Sin[3*(c + d*x)] - 17*a^5*b^2*Sin[5*(c + d*x)] - 76*a^3
*b^4*Sin[5*(c + d*x)] - 12*a*b^6*Sin[5*(c + d*x)]))/((a^2 - b^2)^4*(a + b*S
in[c + d*x])^2))/(96*d)
```

Maple [B] time = 0.131, size = 922, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/3/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1
)^2+1/d/(a+b)^4/(tan(1/2*d*x+1/2*c)-1)*a-1/2/d/(a+b)^4/(tan(1/2*d*x+1/2*c)-
1)*b+5/d*a^5/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b
+a)^2*tan(1/2*d*x+1/2*c)^3*b^2+6/d*a^3/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4+4/d*a^6/(a-b)^4/(a
+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c
)^2*b+15/d*a^4/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3+14/d*b^5/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan(1/2*d*x+1/2*c)^2+11/d*a^5/(a-b)^
4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1
/2*c)*b^2+22/d*a^3/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4+4/d*a^6/(a-b)^4/(a+b)^4/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b+7/d*a^4/(a-b)^4/(a+b)^4/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3+2/d*a^6/(a-b)^4/(a+b)^4/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))+21/d*a^4/(
a-b)^4/(a+b)^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2
-b^2)^(1/2))*b^2+12/d*b^4/(a-b)^4/(a+b)^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*t
an(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2-1/3/d/(a-b)^3/(tan(1/2*d*x+1/2*
c)+1)^3+1/2/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)^2+1/d/(a-b)^4/(tan(1/2*d*x+1/2
*c)+1)*a+1/2/d/(a-b)^4/(tan(1/2*d*x+1/2*c)+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.85301, size = 2765, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 - 2*(28*a^8*b
 + 61*a^6*b^3 - 101*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 4*(8*a^8*b - 25*
a^6*b^3 + 27*a^4*b^5 - 11*a^2*b^7 + b^9)*cos(d*x + c)^2 - 3*((2*a^6*b^2 + 2
1*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^5 - 2*(2*a^7*b + 21*a^5*b^3 + 12*a^3*b
^5)*cos(d*x + c)^3*sin(d*x + c) - (2*a^8 + 23*a^6*b^2 + 33*a^4*b^4 + 12*a^2
*b^6)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 -
2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
 + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 + (17*a^7*b^
2 + 59*a^5*b^4 - 64*a^3*b^6 - 12*a*b^8)*cos(d*x + c)^4 - 2*(4*a^9 - 9*a^7*b
^2 + 3*a^5*b^4 + 5*a^3*b^6 - 3*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*
b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x +
c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11
)*d*cos(d*x + c)^3*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^
8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^3), 1/6*(2*a^8*b - 8*a^6*b^3 + 12*a^4
*b^5 - 8*a^2*b^7 + 2*b^9 - (28*a^8*b + 61*a^6*b^3 - 101*a^4*b^5 + 12*a^2*b^
7)*cos(d*x + c)^4 - 2*(8*a^8*b - 25*a^6*b^3 + 27*a^4*b^5 - 11*a^2*b^7 + b^9
)*cos(d*x + c)^2 - 3*((2*a^6*b^2 + 21*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^5
- 2*(2*a^7*b + 21*a^5*b^3 + 12*a^3*b^5)*cos(d*x + c)^3*sin(d*x + c) - (2*a^
8 + 23*a^6*b^2 + 33*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^3)*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^9 - 8*a^
7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 + (17*a^7*b^2 + 59*a^5*b^4 - 64*a^
3*b^6 - 12*a*b^8)*cos(d*x + c)^4 - 2*(4*a^9 - 9*a^7*b^2 + 3*a^5*b^4 + 5*a^3
*b^6 - 3*a*b^8)*cos(d*x + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a
```

```
^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d*cos(d*x + c)^5 - 2*(a^11*b - 5*a
^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3*sin
(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)
*d*cos(d*x + c)^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 3.39856, size = 853, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^6 + 21*a^4*b^2 + 12*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*
sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 4*a^
6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*(5*a^5*b^2*tan(1/
2*d*x + 1/2*c)^3 + 6*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1/2*d*x +
 1/2*c)^2 + 15*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 + 14*a^2*b^5*tan(1/2*d*x + 1/
2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/2*c) + 22*a^3*b^4*tan(1/2*d*x + 1/2*c)
+ 4*a^6*b + 7*a^4*b^3)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a*
tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 2*(3*a^5*tan(1/
2*d*x + 1/2*c)^5 + 24*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 9*a*b^4*tan(1/2*d*x
+ 1/2*c)^5 - 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b^3*tan(1/2*d*x + 1/2*
c)^4 - 3*b^5*tan(1/2*d*x + 1/2*c)^4 - 10*a^5*tan(1/2*d*x + 1/2*c)^3 - 56*a^
3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 36*a^4*b*ta
n(1/2*d*x + 1/2*c)^2 + 36*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*d*
x + 1/2*c) + 24*a^3*b^2*tan(1/2*d*x + 1/2*c) + 9*a*b^4*tan(1/2*d*x + 1/2*c)
 - 15*a^4*b - 20*a^2*b^3 - b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
 b^8)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

3.199 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=350

$$-\frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\cdots}{2d\left(a^2\cdots\right.}$$

```
[Out] (-4*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^
(7/2)*d) - (a^2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/((a^2 - b^2)^(7/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(2*(a + b)^3*
d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) - (
a^2*b*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) - (3*a^3*b*C
os[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (2*a*b^3*Cos[c + d*
x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.538886, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2731, 2648, 2664, 2754, 12, 2660, 618, 204}

$$-\frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{4a^2b^2\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{7/2}}-\frac{\cdots}{2d\left(a^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^
(7/2)*d) - (a^2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/((a^2 - b^2)^(7/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(2*(a + b)^3*
d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])) - (
a^2*b*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) - (3*a^3*b*C
os[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (2*a*b^3*Cos[c + d*
x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int\left(-\frac{1}{2(a+b)^3(-1+\sin(c+dx))}+\frac{1}{2(a-b)^3(1+\sin(c+dx))}-\frac{a^2}{\left(a^2-b^2\right)(a+b\sin(c+dx)\right.$$

$$=\frac{\int\frac{1}{1+\sin(c+dx)}\,dx}{2(a-b)^3}-\frac{\int\frac{1}{-1+\sin(c+dx)}\,dx}{2(a+b)^3}-\frac{\left(2ab^2\right)\int\frac{1}{(a+b\sin(c+dx))^2}\,dx}{\left(a^2-b^2\right)^2}-\frac{a^2\int\frac{1}{(a+b\sin(c+dx))^3}\,dx}{a^2-b^2}-$$

$$=\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))}-\frac{a^2b\cos(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\sin(c+dx))^2}$$

$$=\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))}-\frac{a^2b\cos(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\sin(c+dx))^2}$$

$$=-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}+\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c}$$

$$=-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}+\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c}$$

$$=-\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}-\frac{2b^2\left(3a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}+\frac{\cos(c+d}{2(a+b)^3d(1-\sin}$$

$$=-\frac{4a^2b^2\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}-\frac{a^2\left(2a^2+b^2\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}d}-\frac{2b^2\left(3a^2+b^2\right)\tan}{\left(a^2-b\right.}$$

Mathematica [A] time = 3.29141, size = 212, normalized size = 0.61

$$\frac{-\frac{2\left(11a^2b^2+2a^4+2b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}-\frac{ab\cos(c+dx)\left(b\left(3a^2+4b^2\right)\sin(c+dx)+4a^3+3ab^2\right)}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a-b)^3\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+\cdots\right.\right.}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 + 11*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
))) - (a*b*Cos[c + d*x]*(4*a^3 + 3*a*b^2 + b*(3*a^2 + 4*b^2)*Sin[c + d*x]))
/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.115, size = 766, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)-5/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)
^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*a^3*b^2-2/d*b^4/(a-b)
^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*
x+1/2*c)^3-4/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)
*b+a)^2*tan(1/2*d*x+1/2*c)^2*a^4*b-11/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/
2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2*tan(1/2*d*x+1/2*c)^2-6/d*b^5/(a-b)
^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+
1/2*c)^2-11/d/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*
b+a)^2*tan(1/2*d*x+1/2*c)*a^3*b^2-10/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2
*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a*tan(1/2*d*x+1/2*c)-4/d/(a-b)^3/(a+b)^
3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^4*b-3/d*b^3/(a-b)^3
/(a+b)^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*a^2-2/d/(a-b)^
3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))*a^4-11/d*b^2/(a-b)^3/(a+b)^3/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*a^2-2/d*b^4/(a-b)^3/(a+b)^3/(a^2-b^2)^(1
/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/d/(a-b)^3/(t
```

```
an(1/2*d*x+1/2*c)+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.32169, size = 2067, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(8*a^6*b + a^4*b^3 - 11
*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos(d*
x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c) - (
2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*log
(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)
^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 -
2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a
^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^
9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c
) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x
 + c)), 1/2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (8*a^6*b + a^4*b^3 -
 11*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos
(d*x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c)
- (2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*a
rctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - (2*a^7 - 6*a^
5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^
2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*co
s(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(
d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b
```

```
^8 + b^10)*d*cos(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.9558, size = 518, normalized size = 1.48

$$-\frac{\frac{\left(2a^4+11a^2b^2+2b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{a^2-b^2}}+\frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b-b^3\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)}+\frac{5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\cdots}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((2*a^4 + 11*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + a
rctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan
(1/2*d*x + 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(ta
n(1/2*d*x + 1/2*c)^2 - 1)) + (5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^4*ta
n(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 11*a^2*b^3*tan(1/2*
d*x + 1/2*c)^2 + 6*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/
2*c) + 10*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b + 3*a^2*b^3)/((a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c)
+ a)^2))/d
```

3.200 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$-\frac{(-9a^2b^2+2a^4+6b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\,(a^2-b^2)^{3/2}}-\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3d\,(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{2a^2d\,(a^2-b^2)\,(a+b\sin(c+dx))}+\frac{3b\tanh}{}$$

```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((
5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a
 + b*Sin[c + d*x])^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 0.788453, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2723, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{(-9a^2b^2+2a^4+6b^4)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^4d\,(a^2-b^2)^{3/2}}-\frac{(5a^2-6b^2)\cot(c+dx)}{2a^3d\,(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{2a^2d\,(a^2-b^2)\,(a+b\sin(c+dx))}+\frac{3b\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^4*(a^2 - b^2)^(3/2)*d)) + (3*b*ArcTanh[Cos[c + d*x]])/(a^4*d) - ((
5*a^2 - 6*b^2)*Cot[c + d*x])/(2*a^3*(a^2 - b^2)*d) + Cot[c + d*x]/(2*a*d*(a
 + b*Sin[c + d*x])^2) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(2*a^2*(a^2 - b^2)*d
*(a + b*Sin[c + d*x]))
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[((a + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2))/Sin[e + f*x]
^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3056

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((A*b^2 + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]
)))
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
```

```
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\csc^2(c+dx)\left(1-\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^2(c+dx)\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} + \frac{\int \frac{\csc^2(c+dx)\left(5a^4-11a^2b^2+6b^4-ab\right.}{}}{2}$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} +$$

$$= -\frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-3b^2\right)\cot(c+dx)}{2a^2\left(a^2-b^2\right)d(a+b\sin(c+dx))} -$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-\right.}{2a^2\left(a^2-b\right.}$$

$$= \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{\left(2a^2-\right.}{2a^2\left(a^2-b\right.}$$

$$= -\frac{\left(2a^4-9a^2b^2+6b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4\left(a^2-b^2\right)^{3/2}d} + \frac{3b\tanh^{-1}(\cos(c+dx))}{a^4d} - \frac{\left(5a^2-6b^2\right)\cot(\right.}{2a^3\left(a^2-b^2\right.}$$

Mathematica [A] time = 5.78381, size = 195, normalized size = 0.97

$$\frac{-\frac{2\left(-9a^2b^2+2a^4+6b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + \frac{ab\left(4b^2-3a^2\right)\cos(c+dx)}{(a-b)(a+b)(a+b\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{(a+b\sin(c+dx))^2} + a\tan\left(\frac{1}{2}(c+dx)\right) - a\cot\left(\frac{1}{2}(c+dx)\right) - 6}{2a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(a^2 - b^2)^(3/2) - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]]
```

```
- 6*b*Log[Sin[(c + d*x)/2]] - (a^2*b*Cos[c + d*x])/(a + b*Sin[c + d*x])^2 +
 (a*b*(-3*a^2 + 4*b^2)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*x]))
 + a*Tan[(c + d*x)/2])/(2*a^4*d)
```

Maple [B] time = 0.116, size = 729, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2/d/a^3*tan(1/2*d*x+1/2*c)-5/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*
c)*b+a)^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3*b^2+6/d/a^3/(tan(1/2*d*x+1/2*c)^
2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3-4/d/(tan
(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b/(a^2-b^2)*tan(1/2*d*x+1/2
*c)^2-3/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)
/a^2*tan(1/2*d*x+1/2*c)^2+10/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/
2*c)*b+a)^2*b^5/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-11/d/(tan(1/2*d*x+1/2*c)^2*a
+2*tan(1/2*d*x+1/2*c)*b+a)^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)*b^2+14/d/a^3/(t
an(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^4/(a^2-b^2)*tan(1/2*d*x
+1/2*c)-4/d/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/(a^2-b^2)*b
+5/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3/(a^2-b^2)-
2/d/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)
)+9/d/a^2/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)
^(1/2))*b^2-6/d/a^4/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)
/(a^2-b^2)^(1/2))*b^4-1/2/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^4*b*ln(tan(1/2*d*x
+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.54849, size = 3044, normalized size = 15.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - 2*(8*a^6*b - 1
7*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 +
12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^
4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)
*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*s
in(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*
sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) -
2*(2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 6*(2*a^5*b^2 - 4*a
^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b
- a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin
(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^5*b^2 - 4*a^3*b^4 + 2*a*b^6
 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3 - a^2*
b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c)^2
- 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d*co
s(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c)), -1/2*
((5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - (8*a^6*b - 17*a^4*b^3
+ 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3 + 12*a*b^5 -
 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7*a^4*b^2 - 3*
a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c)^2)*sin(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
+ c))) - (2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 3*(2*a^5*b^
2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 +
(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)
^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*(2*a^5*b^2 - 4*a^3*b^4 +
2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^4*b^3
 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c)
)*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*cos(d*x
+ c)^2 - 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4 + a^4*b^
6)*d*cos(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(d*x + c))
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 1.55189, size = 458, normalized size = 2.27

$$-\frac{\frac{2\left(2a^4-9a^2b^2+6b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^6-a^4b^2\right)\sqrt{a^2-b^2}}+\frac{2\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b^3\tan\left(\right.\right.}{\left(a^6-a^4b^2\right)\left(a\tan\left(\frac{1}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a
) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2)*
sqrt(a^2 - b^2)) + 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*
x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^3*tan(1/2*d*x + 1/2
*c)^2 - 10*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1
4*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b - 5*a^2*b^3)/((a^6 - a^4*b^2)*(a*tan
(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs(tan(1/
2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 - (6*b*tan(1/2*d*x + 1/2*c)
 - a)/(a^4*tan(1/2*d*x + 1/2*c)))/d
```

3.201 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=289

$$\frac{\left(-19a^2b^2+2a^4+20b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}+\frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d}-\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d}-\frac{(a}{}$$

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*
a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(6*a^3*b*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 1.07153, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2724, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{\left(-19a^2b^2+2a^4+20b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^6d\sqrt{a^2-b^2}}+\frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d}-\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d}-\frac{(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b
^2]])/(a^6*Sqrt[a^2 - b^2]*d) - (b*(9*a^2 - 20*b^2)*ArcTanh[Cos[c + d*x]])/
(2*a^6*d) + ((17*a^2 - 60*b^2)*Cot[c + d*x])/(6*a^5*d) - ((a^2 - 5*b^2)*Cot
[c + d*x]*Csc[c + d*x])/(a^4*b*d) + ((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d
*x])/(6*a^2*b*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*
a*d*(a + b*Sin[c + d*x])^2) + ((3*a^2 - 20*b^2)*Cot[c + d*x]*Csc[c + d*x])/
(6*a^3*b*d*(a + b*Sin[c + d*x]))
```


Rule 2724

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a*f*Sin[e
```

```
 + f*x]^3), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m +
 1)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 -
b^2*m*(m - 2))*Sin[e + f*x]^2, x])/Sin[e + f*x]^3, x], x] - Simp[((3*a^2 +
b^2*(m - 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(3*a^2*b*f*(m + 1)*
Sin[e + f*x]^2), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[
m, -1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\int \frac{\csc^3(c+dx)\left(2\left(3a^2-10b^2\right)-2ab\right.}{(a+b\,\mathrm{s}}}{}$$

$$= \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} + \frac{\left(3a^2-20b^2\right)\cot(c+dx)}{6a^3bd(a+b\sin(c}$$

$$= -\frac{\left(a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} - \frac{\cot(c+dx)\,\mathrm{cs}}{3ad(a+b\sin}$$

$$= \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{\left(3a^2-5b^2\right)\cot(c+dx)\,\mathrm{c}}{6a^2bd(a+b\sin(c+}$$

$$= \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{a^4bd} + \frac{\left(3a^2-5b^2\right)\cot(c+dx)\,\mathrm{c}}{6a^2bd(a+b\sin(c+}$$

$$= -\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+dx}{a^4bd}$$

$$= -\frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{\left(17a^2-60b^2\right)\cot(c+dx)}{6a^5d} - \frac{\left(a^2-5b^2\right)\cot(c+dx}{a^4bd}$$

$$= \frac{\left(2a^4-19a^2b^2+20b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}\,d} - \frac{b\left(9a^2-20b^2\right)\tanh^{-1}(\cos(c+dx))}{2a^6d} + \frac{(17}{}$$

Mathematica [A] time = 6.21126, size = 459, normalized size = 1.59

$$\frac{(9a^2b-20b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d}+\frac{(20b^3-9a^2b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d}+\frac{3a^2b\cos(c+dx)-8b^3\cos(c+dx)}{2a^5d(a+b\sin(c+dx))}+\frac{a^2b\,\mathrm{c}}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2]
 + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((2*a^2
*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d) + (
3*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(
24*a^3*d) + ((-9*a^2*b + 20*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + ((9*a^2
*b - 20*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]^2)/(8
*a^4*d) + (Sec[(c + d*x)/2]*(-2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/
2]))/(3*a^5*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(2*a^4*d*(a + b*Si
n[c + d*x])^2) + (3*a^2*b*Cos[c + d*x] - 8*b^3*Cos[c + d*x])/(2*a^5*d*(a +
b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*d)
```

Maple [B] time = 0.134, size = 780, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d/a^3*tan(1/2*d*x+1/2*c)^3-3/8/d/a^4*b*tan(1/2*d*x+1/2*c)^2-5/8/d/a^3*
tan(1/2*d*x+1/2*c)+3/d/a^5*b^2*tan(1/2*d*x+1/2*c)+5/d/a^3/(tan(1/2*d*x+1/2*
c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^2-10/d/a^5/(tan(1
/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4+4/d*
b/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a^2*tan(1/2*d*x+1/2*c
)^2-1/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x
+1/2*c)^2*b^3-18/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*
tan(1/2*d*x+1/2*c)^2*b^5+11/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2
*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^2-26/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/
2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)*b^4+4/d/a^2/(tan(1/2*d*x+1/2*c)^2*a+
2*tan(1/2*d*x+1/2*c)*b+a)^2*b-9/d/a^4/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2*b^3+2/d/a^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*
```

```
c)+2*b)/(a^2-b^2)^(1/2))-19/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d
*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2+20/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*
a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4-1/24/d/a^3/tan(1/2*d*x+1/2*c
)^3+5/8/d/a^3/tan(1/2*d*x+1/2*c)-3/d/a^5/tan(1/2*d*x+1/2*c)*b^2+3/8/d/a^4*b
/tan(1/2*d*x+1/2*c)^2+9/2/d/a^4*b*ln(tan(1/2*d*x+1/2*c))-10/d/a^6*b^3*ln(ta
n(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.54187, size = 4578, normalized size = 15.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3
*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 38*a^3*b^3
+ 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*
b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 +
 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*cos(d*x + c)^4 - (2*a^6 - 15*a^
4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*
cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x
+ c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b
^4 + 20*a*b^6)*cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*
a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4
 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 +
(9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 -
38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1
/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 2
```

```
0*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c
)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5
+ 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos
(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b -
59*a^4*b^3 + 45*a^2*b^5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4
*(a^9*b - a^7*b^3)*d*cos(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a
^6*b^4)*d*cos(d*x + c)^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 +
(a^10 - a^6*b^4)*d)*sin(d*x + c)), 1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*
b^6)*cos(d*x + c)^5 - 4*(4*a^7 + 3*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x
 + c)^3 - 6*(4*a^5*b - 38*a^3*b^3 + 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20
*a*b^5)*cos(d*x + c)^4 - 4*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2
 + (2*a^6 - 17*a^4*b^2 + a^2*b^4 + 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^
6)*cos(d*x + c)^4 - (2*a^6 - 15*a^4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)
^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 -
b^2)*cos(d*x + c))) + 6*(2*a^7 - 3*a^5*b^2 - 19*a^3*b^4 + 20*a*b^6)*cos(d*x
 + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 +
 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x +
 c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^
5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*c
os(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(18*a^5*b^2 -
58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^
4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^
4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)
^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2) - 2*(2*(14*a^6*b - 59*a^4*b^3 + 45*a^2*b^
5)*cos(d*x + c)^3 - 3*(11*a^6*b - 41*a^4*b^3 + 30*a^2*b^5)*cos(d*x + c))*si
n(d*x + c))/(2*(a^9*b - a^7*b^3)*d*cos(d*x + c)^4 - 4*(a^9*b - a^7*b^3)*d*c
os(d*x + c)^2 + 2*(a^9*b - a^7*b^3)*d + ((a^8*b^2 - a^6*b^4)*d*cos(d*x + c)
^4 - (a^10 + a^8*b^2 - 2*a^6*b^4)*d*cos(d*x + c)^2 + (a^10 - a^6*b^4)*d)*si
n(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

Giac [A] time = 2.05061, size = 609, normalized size = 2.11

$$\frac{12\left(9\,a^2b-20\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^6}+\frac{24\left(2\,a^4-19\,a^2b^2+20\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^6}+\frac{24\left(5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,ab^4\tan\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(12*(9*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 24*(2*a^4
- 19*a^2*b^2 + 20*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a
*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) + 24*(5*
a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*
tan(1/2*d*x + 1/2*c)^2 - a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*b^5*tan(1/2*d*
x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 26*a*b^4*tan(1/2*d*x + 1/2
*c) + 4*a^4*b - 9*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1
/2*c) + a)^2*a^6) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d*x + 1/2
*c)^2 - 15*a^6*tan(1/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2*c))/a^9
- (198*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a
^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*a^2*b*tan(1
/2*d*x + 1/2*c) + a^3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

3.202 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=492

$$-\frac{\sqrt{a^2-b^2}\left(-29a^2b^2+2a^4+42b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{\left(-645a^2b^2+91a^4+630b^4\right)\cot(c+dx)}{30a^7d}+\frac{b\left(-200a^2b^2+45\right.}{}$$

```
[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcT
anh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*
x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/
(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^
2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((
5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a
+ b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*S
in[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])
^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b
^2*d*(a + b*Sin[c + d*x]))
```

Rubi [A] time = 2.12688, antiderivative size = 492, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2726, 3055, 3001, 3770, 2660, 618, 204}

$$-\frac{\sqrt{a^2-b^2}\left(-29a^2b^2+2a^4+42b^4\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{a^8d}-\frac{\left(-645a^2b^2+91a^4+630b^4\right)\cot(c+dx)}{30a^7d}+\frac{b\left(-200a^2b^2+45\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```


```
[Out] -((Sqrt[a^2 - b^2]*(2*a^4 - 29*a^2*b^2 + 42*b^4)*ArcTan[(b + a*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(a^8*d)) + (b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcT
anh[Cos[c + d*x]])/(8*a^8*d) - ((91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*
x])/(30*a^7*d) + ((8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]*Csc[c + d*x])/
(8*a^6*b*d) - ((15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2
)/(30*a^5*b^2*d) - (Cot[c + d*x]*Csc[c + d*x])/(3*b*d*(a + b*Sin[c + d*x])^
2) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + ((
```

```
5*a^4 - 60*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(60*a^3*b^2*d*(a
+ b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*S
in[c + d*x])^2) - (Cot[c + d*x]*Csc[c + d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])
^2) + ((4*a^4 - 54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(12*a^4*b
^2*d*(a + b*Sin[c + d*x]))
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> -Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(5*a*f*Sin[e
 + f*x]^5), x] + (Dist[1/(20*a^2*b^2*m*(m - 1)), Int[((a + b*Sin[e + f*x])^
m*Simp[60*a^4 - 44*a^2*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*
m*(20*a^2 - b^2*m*(m - 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(
m - 4) - 20*a^2*b^2*(m - 1)*(2*m + 1))*Sin[e + f*x]^2, x])/Sin[e + f*x]^4,
x], x] + Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*m*Sin[e + f*
x]^2), x] + Simp[(a*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*m*(m
- 1)*Sin[e + f*x]^3), x] - Simp[(b*(m - 4)*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^(m + 1))/(20*a^2*f*Sin[e + f*x]^4), x]) /; FreeQ[{a, b, e, f, m}, x] && N
eQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c
 - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a
 + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*
(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b
*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^
2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c
, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ
[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]
) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || E
qQ[a, 0])))
```

Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2} - \frac{\cot}{5a}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{\left(5a^4-60a^2b^2+63b^4\right)\cot(c+dx}{60a^3b^2d(a+b\sin(c+d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2} + \frac{\left(5a^4-60a^2b^2+63b^4\right)\cot(c+dx}{60a^3b^2d(a+b\sin(c+d}$$

$$= -\frac{\left(15a^4-187a^2b^2+210b^4\right)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d} - \frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2} + \frac{a\cot(c+}{12b^2d(a}$$

$$= \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c+dx)}{8a^6bd} - \frac{\left(15a^4-187a^2b^2+210b^4\right)\cot(c+dx)\csc}{30a^5b^2d}$$

$$= -\frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d} + \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c+dx)}{8a^6bd} -$$

$$= -\frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d} + \frac{\left(8a^4-79a^2b^2+84b^4\right)\cot(c+dx)\csc(c+dx)}{8a^6bd} -$$

$$= \frac{b\left(45a^4-200a^2b^2+168b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d}$$

$$= \frac{b\left(45a^4-200a^2b^2+168b^4\right)\tanh^{-1}(\cos(c+dx))}{8a^8d} - \frac{\left(91a^4-645a^2b^2+630b^4\right)\cot(c+dx)}{30a^7d}$$

$$= -\frac{\sqrt{a^2-b^2}\left(2a^4-29a^2b^2+42b^4\right)\tan^{-1}\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d} + \frac{b\left(45a^4-200a^2b^2+168b^4\right)\tan}{8a^8d}$$

Mathematica [A] time = 1.70473, size = 448, normalized size = 0.91

$$\frac{-\frac{3840\left(-31a^4b^2+71a^2b^4+2a^6-42b^6\right)\tan^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 480b\left(-200a^2b^2+45a^4+168b^4\right)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + 480b\left(-200a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-3840*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*ArcTan[(b + a*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 480*b*(45*a^4 - 200*a^2*b^2 + 1
68*b^4)*Log[Cos[(c + d*x)/2]] - 480*b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*Log[
Sin[(c + d*x)/2]] + (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(-784*a^6 + 3256*a^4*b
^2 + 7860*a^2*b^4 - 12600*b^6 + 2*(384*a^6 - 2131*a^4*b^2 - 6315*a^2*b^4 +
9450*b^6)*Cos[2*(c + d*x)] + (-368*a^6 + 824*a^4*b^2 + 6060*a^2*b^4 - 7560*
b^6)*Cos[4*(c + d*x)] + 182*a^4*b^2*Cos[6*(c + d*x)] - 1290*a^2*b^4*Cos[6*(
c + d*x)] + 1260*b^6*Cos[6*(c + d*x)] - 8156*a^5*b*Sin[c + d*x] + 42270*a^3
*b^3*Sin[c + d*x] - 37800*a*b^5*Sin[c + d*x] + 3956*a^5*b*Sin[3*(c + d*x)]
- 20715*a^3*b^3*Sin[3*(c + d*x)] + 18900*a*b^5*Sin[3*(c + d*x)] - 608*a^5*b
*Sin[5*(c + d*x)] + 3975*a^3*b^3*Sin[5*(c + d*x)] - 3780*a*b^5*Sin[5*(c + d
*x)]))/(b + a*Csc[c + d*x])^2)/(3840*a^8*d)
```

Maple [B] time = 0.138, size = 1252, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x)
```

```
[Out] -13/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^5-21/d/a^8*
b^5*ln(tan(1/2*d*x+1/2*c))+1/4/d/a^5*b^2*tan(1/2*d*x+1/2*c)^3-5/4/d/a^6*tan
(1/2*d*x+1/2*c)^2*b^3+15/2/d/a^7*b^4*tan(1/2*d*x+1/2*c)+3/64/d/a^4*b/tan(1/
2*d*x+1/2*c)^4-3/64/d/a^4*b*tan(1/2*d*x+1/2*c)^4-15/2/d/a^7/tan(1/2*d*x+1/2
*c)*b^4-5/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2
*d*x+1/2*c)^3*b^2-1/160/d/a^3/tan(1/2*d*x+1/2*c)^5+1/160/d/a^3*tan(1/2*d*x+
1/2*c)^5-45/8/d/a^4*b*ln(tan(1/2*d*x+1/2*c))+19/d/a^5/(tan(1/2*d*x+1/2*c)^2
*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^4-4/d*b/(tan(1/2*d*x+
1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2/a^2*tan(1/2*d*x+1/2*c)^2+9/d/a^4/(ta
n(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^2*b^3+2
1/d/a^6/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/2
*c)^2*b^5-11/d/a^3/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(
1/2*d*x+1/2*c)*b^2+49/d/a^5/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+
a)^2*tan(1/2*d*x+1/2*c)*b^4+31/d/a^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^2-71/d/a^6/(a^2-b^2)^(1/2)*arctan(1/2*
(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))*b^4+27/4/d/a^5/tan(1/2*d*x+1/
2*c)*b^2-3/4/d/a^4*b/tan(1/2*d*x+1/2*c)^2+25/d/a^6*b^3*ln(tan(1/2*d*x+1/2*c
))+3/4/d/a^4*b*tan(1/2*d*x+1/2*c)^2-27/4/d/a^5*b^2*tan(1/2*d*x+1/2*c)-4/d/a
^2/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b+17/d/a^4/(tan(1/2*
```

```
d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*b^3-2/d/a^2/(a^2-b^2)^(1/2)*arct
an(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-7/96/d/a^3*tan(1/2*d*x
+1/2*c)^3+7/96/d/a^3/tan(1/2*d*x+1/2*c)^3+11/16/d/a^3*tan(1/2*d*x+1/2*c)-11
/16/d/a^3/tan(1/2*d*x+1/2*c)-14/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x
+1/2*c)*b+a)^2*tan(1/2*d*x+1/2*c)^3*b^6+5/4/d/a^6*b^3/tan(1/2*d*x+1/2*c)^2-
26/d/a^8/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan(1/2*d*x+1/
2*c)^2*b^7-38/d/a^7/(tan(1/2*d*x+1/2*c)^2*a+2*tan(1/2*d*x+1/2*c)*b+a)^2*tan
(1/2*d*x+1/2*c)*b^6+42/d/a^8/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2))*b^6-1/4/d/a^5/tan(1/2*d*x+1/2*c)^3*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 6.51664, size = 6091, normalized size = 12.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^
7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 3
7*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 60*(2*(2*a^5*b - 29*a
^3*b^3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*
a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 4
2*a*b^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6
 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*
b^4 + 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x +
 c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2)) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*cos(d*x + c)
```

+ 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)*cos(d*x + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^3 + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(2*a^9*b*d*cos(d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c)^2 - 2*a^9*b*d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x + c)^4 + (2*a^10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*x + c)), -1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a^7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 - 37*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 120*(2*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x + c)^6 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 - 45*a^2*b^4 + 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 126*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*cos(d*x + c) + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 200*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) - 2*((608*a^6*b - 3975*a^4*b^3 + 3780*a^2*b^5)*cos(d*x + c)^5 - 5*(289*a^6*b - 1632*a^4*b^3 + 1512*a^2*b^5)*cos(d*x + c)^3 + 15*(53*a^6*b - 279*a^4*b^3 + 252*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(

```
2*a^9*b*d*cos(d*x + c)^6 - 6*a^9*b*d*cos(d*x + c)^4 + 6*a^9*b*d*cos(d*x + c
)^2 - 2*a^9*b*d + (a^8*b^2*d*cos(d*x + c)^6 - (a^10 + 3*a^8*b^2)*d*cos(d*x
+ c)^4 + (2*a^10 + 3*a^8*b^2)*d*cos(d*x + c)^2 - (a^10 + a^8*b^2)*d)*sin(d*
x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.85019, size = 987, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/960*(120*(45*a^4*b - 200*a^2*b^3 + 168*b^5)*log(abs(tan(1/2*d*x + 1/2*c)
))/a^8 + 960*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*(pi*floor(1/2*(d*x
+ c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)
))/(sqrt(a^2 - b^2)*a^8) + 960*(5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 19*a^3*b
^4*tan(1/2*d*x + 1/2*c)^3 + 14*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1
/2*d*x + 1/2*c)^2 - 9*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^5*tan(1/2*d
*x + 1/2*c)^2 + 26*b^7*tan(1/2*d*x + 1/2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/
2*c) - 49*a^3*b^4*tan(1/2*d*x + 1/2*c) + 38*a*b^6*tan(1/2*d*x + 1/2*c) + 4*
a^6*b - 17*a^4*b^3 + 13*a^2*b^5)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d
*x + 1/2*c) + a)^2*a^8) - (12330*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 54800*a^2*b
^3*tan(1/2*d*x + 1/2*c)^5 + 46032*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*tan(
1/2*d*x + 1/2*c)^4 + 6480*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 7200*a*b^4*tan(1
/2*d*x + 1/2*c)^4 - 720*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1200*a^2*b^3*tan(1/2
*d*x + 1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 240*a^3*b^2*tan(1/2*d*x +
 1/2*c)^2 + 45*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^8*tan(1/2*d*x + 1/2*c
)^5) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^11*b*tan(1/2*d*x + 1/2*c)^4 -
```

```
70*a^12*tan(1/2*d*x + 1/2*c)^3 + 240*a^10*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*
a^11*b*tan(1/2*d*x + 1/2*c)^2 - 1200*a^9*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a
^12*tan(1/2*d*x + 1/2*c) - 6480*a^10*b^2*tan(1/2*d*x + 1/2*c) + 7200*a^8*b^
4*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.203 $\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=271

$$\frac{3a^2b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{fg(p+2)}+\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\right.}{fg(p+1)}$$

```
[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^2*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyper
geometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*
(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^3*(Cos[e + f*x]^2)^((1 + p)/2)
*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e +
 f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a*b^2*Hypergeometric2F
1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^
3*(3 + p))
```

Rubi [A] time = 0.379927, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2722, 3476, 364, 2602, 2577, 2591}

$$\frac{3a^2b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2};\sin^2(e+fx)\right)}{fg(p+2)}+\frac{a^3(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\right.}{fg(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```


```
[Out] (a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^2*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyper
geometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*
(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^3*(Cos[e + f*x]^2)^((1 + p)/2)
*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[e +
 f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a*b^2*Hypergeometric2F
1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^
3*(3 + p))
```


Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
```

```
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx = \int \left(a^3(g\tan(e+fx))^p + 3a^2b\sin(e+fx)(g\tan(e+fx))^p + 3ab^2\sin^2(e+f\right.$$

$$= a^3\int (g\tan(e+fx))^p\,dx + \left(3a^2b\right)\int \sin(e+fx)(g\tan(e+fx))^p\,dx + \left(3ab^2\right.$$

$$= \frac{\left(a^3g\right)\operatorname{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{\left(3ab^2g\right)\operatorname{Subst}\left(\int \frac{x^{2+p}}{\left(g^2+x^2\right)^2}\,dx,\right.}{f}$$

$$= \frac{a^3\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{3a^2b\cos^2(e+fx)}{}$$

Mathematica [C] time = 18.4028, size = 4791, normalized size = 17.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^3*(2 + p)*Appell
F1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2
*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 + p/2,
p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1
 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[
1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e +
 f*x)/2]))*(g*Tan[e + f*x])^p*(-(b^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8 - a
^3*Sin[e + f*x]^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p + ((3*I)/8)*b^3*Sin[2*(e
+ f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*b^3*Sin[2*(e + f*x)]^2*Sin[3*(
e + f*x)]*Tan[e + f*x]^p)/8 - (I/8)*b^3*Sin[2*(e + f*x)]^3*Sin[3*(e + f*x)]
*Tan[e + f*x]^p + Cos[e + f*x]^3*(a^3*Cos[3*(e + f*x)]*Tan[e + f*x]^p - I*a
^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^3*((I/8)*b^3*Cos[3*(
e + f*x)]*Tan[e + f*x]^p + (b^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8) + Sin[e
 + f*x]^2*((-3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + ((3*I)/2)*a^2*b*S
in[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*((-3*a*b^2*
Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Sin[3
*(e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*T
an[e + f*x]^p)/4) + Cos[2*(e + f*x)]^2*((-3*b^3*Sin[3*(e + f*x)]*Tan[e + f*
x]^p)/8 - (3*a*b^2*Sin[e + f*x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*I)
/8)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + Cos[3*(e + f*x)]
```

*(((-3*I)/8)*b^3*Tan[e + f*x]^p - ((3*I)/4)*a*b^2*Sin[e + f*x]*Tan[e + f*x]
^p - (3*b^3*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/8)) + Cos[3*(e + f*x)]*((-I/8)
*b^3*Tan[e + f*x]^p - I*a^3*Sin[e + f*x]^3*Tan[e + f*x]^p - (3*b^3*Sin[2*(e
 + f*x)]*Tan[e + f*x]^p)/8 + ((3*I)/8)*b^3*Sin[2*(e + f*x)]^2*Tan[e + f*x]^
p + (b^3*Sin[2*(e + f*x)]^3*Tan[e + f*x]^p)/8 + Sin[e + f*x]^2*(((-3*I)/2)*
a^2*b*Tan[e + f*x]^p - (3*a^2*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p)/2) + Sin[e
 + f*x]*(((-3*I)/4)*a*b^2*Tan[e + f*x]^p - (3*a*b^2*Sin[2*(e + f*x)]*Tan[e
+ f*x]^p)/2 + ((3*I)/4)*a*b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)) + Cos[e +
 f*x]^2*((3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 + 3*a^3*Sin[e + f*x]*S
in[3*(e + f*x)]*Tan[e + f*x]^p - ((3*I)/2)*a^2*b*Sin[2*(e + f*x)]*Sin[3*(e
+ f*x)]*Tan[e + f*x]^p + Cos[3*(e + f*x)]*(((3*I)/2)*a^2*b*Tan[e + f*x]^p +
 (3*I)*a^3*Sin[e + f*x]*Tan[e + f*x]^p + (3*a^2*b*Sin[2*(e + f*x)]*Tan[e +
f*x]^p)/2) + Cos[2*(e + f*x)]*(((-3*I)/2)*a^2*b*Cos[3*(e + f*x)]*Tan[e + f*
x]^p - (3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2)) + Cos[e + f*x]*(((3*I)
/4)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*I)*a^3*Sin[e + f*x]^2*Sin[3*
(e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[
e + f*x]^p)/2 - ((3*I)/4)*a*b^2*Sin[2*(e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e +
 f*x]^p + Cos[2*(e + f*x)]^2*((-3*a*b^2*Cos[3*(e + f*x)]*Tan[e + f*x]^p)/4
+ ((3*I)/4)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*((3*I)*a^
2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p + 3*a^2*b*Sin[2*(e + f*x)]*Sin[3*(e + f
*x)]*Tan[e + f*x]^p) + Cos[3*(e + f*x)]*((-3*a*b^2*Tan[e + f*x]^p)/4 - 3*a^
3*Sin[e + f*x]^2*Tan[e + f*x]^p + ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Tan[e +
f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/4 + Sin[e + f*x]*(-3*a
^2*b*Tan[e + f*x]^p + (3*I)*a^2*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p)) + Cos[2
*(e + f*x)]*(((-3*I)/2)*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*I)*a^2*b
*Sin[e + f*x]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*a*b^2*Sin[2*(e + f*x)]*S
in[3*(e + f*x)]*Tan[e + f*x]^p)/2 + Cos[3*(e + f*x)]*((3*a*b^2*Tan[e + f*x]
^p)/2 + 3*a^2*b*Sin[e + f*x]*Tan[e + f*x]^p - ((3*I)/2)*a*b^2*Sin[2*(e + f*
x)]*Tan[e + f*x]^p))) + Cos[2*(e + f*x)]*((3*b^3*Sin[3*(e + f*x)]*Tan[e + f
*x]^p)/8 + (3*a^2*b*Sin[e + f*x]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 - ((3
*I)/4)*b^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p - (3*b^3*Sin[2*
(e + f*x)]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8 + Sin[e + f*x]*((3*a*b^2*Si
n[3*(e + f*x)]*Tan[e + f*x]^p)/2 - ((3*I)/2)*a*b^2*Sin[2*(e + f*x)]*Sin[3*(
e + f*x)]*Tan[e + f*x]^p) + Cos[3*(e + f*x)]*(((3*I)/8)*b^3*Tan[e + f*x]^p
+ ((3*I)/2)*a^2*b*Sin[e + f*x]^2*Tan[e + f*x]^p + (3*b^3*Sin[2*(e + f*x)]*T
an[e + f*x]^p)/4 - ((3*I)/8)*b^3*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p + Sin[e
+ f*x]*(((3*I)/2)*a*b^2*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]*Tan[e +
f*x]^p)/2)))))/(f*(1 + p)*(2 + p)*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p
*Sec[e + f*x]^2*Tan[(e + f*x)/2]*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3
+ p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*Appel
lF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] -
6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e +
 f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2,

```
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]))*Tan[e + f*x]
^(-1 + p))/((1 + p)*(2 + p)) + (Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f
*x)/2]^2)^p*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)
/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2,
(3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF
1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1
 + p)*(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2]))*Tan[(e + f*x)/2]))*Tan[e + f*x]^p)/((1 + p)*(2 + p))
 + (2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + p)*Tan[(e + f*x)/2]*(-(Sec[
(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(e + f*x)/2]^2*Tan[(e + f*x
)/2])*(a^3*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2,
 -Tan[(e + f*x)/2]^2] + 2*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p
)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF1[(1 +
 p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + p)*
(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]))*Tan[(e + f*x)/2]))*Tan[e + f*x]^p)/((1 + p)*(2 + p)) + (2*
(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^3*(2 + p)*(-(((1 +
p)*AppellF1[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 + p)*Ap
pellF1[1 + (1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 2*b*(((1 + p)*
(3*a^2*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2] + 4*b^2*(AppellF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2] - AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]))*Sec[(e + f*x)/2]^2)/2 + 6*a*b*(2 + p)*((-2*(1 + p)*AppellF
1[1 + (1 + p)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 +
(1 + p)/2, 1 + p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - 6*a*b*(2 + p)*((-3*(1 + p
)*AppellF1[1 + (1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*Appe
llF1[1 + (1 + p)/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (1 + p)*Tan[(e +
 f*x)/2]*(3*a^2*((-2*(1 + p/2)*AppellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2
) + ((1 + p/2)*p*AppellF1[2 + p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2)) + 4*b^2
*((-3*(1 + p/2)*AppellF1[2 + p/2, p, 4, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + (4*(1 + p/2
)*AppellF1[2 + p/2, p, 5, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*AppellF1[2 +
```

```
 p/2, 1 + p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) - ((1 + p/2)*p*AppellF1[2 + p/2, 1 +
p, 4, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*
Tan[(e + f*x)/2])/(2 + p/2)))))*Tan[e + f*x]^p)/((1 + p)*(2 + p))))
```

Maple [F] time = 1.514, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^{3}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(3\,ab^{2}\cos\left(fx+e\right)^{2}-a^{3}-3\,ab^{2}+\left(b^{3}\cos\left(fx+e\right)^{2}-3\,a^{2}b-b^{3}\right)\sin\left(fx+e\right)\right)\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 - 3
*a^2*b - b^3)*sin(f*x + e))*(g*tan(f*x + e))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{3}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

3.204 $\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=186

$$\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\ldots\right.}{fg(p+2)}$$

```
[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (2*a*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^2*Hypergeometric2F1[2, (3 + p)/2
, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))
```

Rubi [A] time = 0.242273, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {2722, 3476, 364, 2602, 2577, 2591}

$$\frac{a^2(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\ldots\right.}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
 f*x])^(1 + p))/(f*g*(1 + p)) + (2*a*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyperge
ometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g
*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^2*Hypergeometric2F1[2, (3 + p)/2
, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p))
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

IntegerQ[n]

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \int \Big(a^2(g\tan(e+fx))^p + 2ab\sin(e+fx)(g\tan(e+fx))^p + b^2\sin^2(e+fx)(g$$

$$= a^2\int (g\tan(e+fx))^p\,dx + (2ab)\int \sin(e+fx)(g\tan(e+fx))^p\,dx + b^2\int \text{si}$$

$$= \frac{(a^2g)\,\text{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{(b^2g)\,\text{Subst}\left(\int \frac{x^{2+p}}{(g^2+x^2)^2}\,dx, x, g\right.}{f}$$

$$= \frac{a^2\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{2ab\cos^2(e+fx)^{\frac{1+p}{2}}}{}$$

Mathematica [C] time = 14.2085, size = 2464, normalized size = 13.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^2*(2 + p)*Appell
F1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4
*b*(b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] - b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*(g*Tan[e + f*
x])^p*(-(b^2*Cos[2*(e + f*x)]^3*Tan[e + f*x]^p)/4 + (I/4)*b^2*Sin[2*(e + f*
x)]*Tan[e + f*x]^p + I*a^2*Sin[e + f*x]^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p +
 (b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/2 - (I/4)*b^2*Sin[2*(e + f*x)]^3*T
an[e + f*x]^p + Cos[e + f*x]^2*(a^2*Cos[2*(e + f*x)]*Tan[e + f*x]^p - I*a^2
*Sin[2*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^2*((b^2*Tan[e + f*x]^p
)/2 + a*b*Sin[e + f*x]*Tan[e + f*x]^p - (I/4)*b^2*Sin[2*(e + f*x)]*Tan[e +
f*x]^p) + Sin[e + f*x]*(I*a*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p + a*b*Sin[2*(
e + f*x)]^2*Tan[e + f*x]^p) + Cos[2*(e + f*x)]*(-(b^2*Tan[e + f*x]^p)/4 - a
*b*Sin[e + f*x]*Tan[e + f*x]^p - a^2*Sin[e + f*x]^2*Tan[e + f*x]^p - (b^2*S
in[2*(e + f*x)]^2*Tan[e + f*x]^p)/4) + Cos[e + f*x]*((-I)*a*b*Cos[2*(e + f*
x)]^2*Tan[e + f*x]^p + a*b*Sin[2*(e + f*x)]*Tan[e + f*x]^p + 2*a^2*Sin[e +
f*x]*Sin[2*(e + f*x)]*Tan[e + f*x]^p - I*a*b*Sin[2*(e + f*x)]^2*Tan[e + f*x
]^p + Cos[2*(e + f*x)]*(I*a*b*Tan[e + f*x]^p + (2*I)*a^2*Sin[e + f*x]*Tan[e
 + f*x]^p))))/(f*(1 + p)*(2 + p)*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*
Sec[e + f*x]^2*Tan[(e + f*x)/2]*(a^2*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 +
 p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b*(b*(2 + p)*AppellF1[(
```

```
1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*(2
+ p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e + f*x]^(-1 + p))/((1 + p)*(2 +
p)) + (Sec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(a^2*(2 + p)*
AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + 4*b*(b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2
, -Tan[(e + f*x)/2]^2] - b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan
[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2
 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e +
 f*x]^p)/((1 + p)*(2 + p)) + (2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^(-1 + p
)*Tan[(e + f*x)/2]*(-(Sec[(e + f*x)/2]^2*Sin[e + f*x]) + Cos[e + f*x]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])*(a^2*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 +
 p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b*(b*(2 + p)*AppellF1[(
1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - b*(2
+ p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -T
an[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*Tan[e + f*x]^p)/((1 + p)*(2 + p)) + (
2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^2*(2 + p)*(-(((1
+ p)*AppellF1[1 + (1 + p)/2, p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + (p*(1 + p)*
AppellF1[1 + (1 + p)/2, 1 + p, 1, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) + 4*b*((a*(1 +
 p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2]*Sec[(e + f*x)/2]^2)/2 + a*(1 + p)*Tan[(e + f*x)/2]*((-2*(1 + p/2)*Appell
F1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e
+ f*x)/2]^2*Tan[(e + f*x)/2])/(2 + p/2) + ((1 + p/2)*p*AppellF1[2 + p/2, 1
+ p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^
2*Tan[(e + f*x)/2])/(2 + p/2)) + b*(2 + p)*((-2*(1 + p)*AppellF1[1 + (1 + p
)/2, p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e +
 f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)/2, 1
 + p, 2, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f
*x)/2]^2*Tan[(e + f*x)/2])/(3 + p)) - b*(2 + p)*((-3*(1 + p)*AppellF1[1 + (
1 + p)/2, p, 4, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec
[(e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p) + (p*(1 + p)*AppellF1[1 + (1 + p)
/2, 1 + p, 3, 1 + (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(
e + f*x)/2]^2*Tan[(e + f*x)/2])/(3 + p))))*Tan[e + f*x]^p)/((1 + p)*(2 + p)
)))
```

Maple [F] time = 1.446, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^{2}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}\right)\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(g*tan(f*x
+ e))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(g\tan\left(e+fx\right)\right)^{p}\left(a+b\sin\left(e+fx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)
```

```
[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{2}\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

3.205 $\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx$

Optimal. Leaf size=129

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2}\right.}{fg(p+2)}$$

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f
*x])^(1 + p))/(f*g*(1 + p)) + (b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometri
c2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e
 + f*x])^(1 + p))/(f*g*(2 + p))
```

Rubi [A] time = 0.147865, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2722, 3476, 364, 2602, 2577}

$$\frac{a(g\tan(e+fx))^{p+1}\,{}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,{}_2F_1\left(\frac{p+1}{2},\frac{p+2}{2};\frac{p+4}{2}\right.}{fg(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f
*x])^(1 + p))/(f*g*(1 + p)) + (b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeometri
c2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Tan[e
 + f*x])^(1 + p))/(f*g*(2 + p))
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx = \int \left(a(g\tan(e+fx))^p + b\sin(e+fx)(g\tan(e+fx))^p\right)dx$$

$$= a\int (g\tan(e+fx))^p\,dx + b\int \sin(e+fx)(g\tan(e+fx))^p\,dx$$

$$= \frac{(ag)\operatorname{Subst}\left(\int \frac{x^p}{g^2+x^2}\,dx, x, g\tan(e+fx)\right)}{f} + \frac{\left(b\cos^{1+p}(e+fx)\sin^{-1-p}(e+fx\right.}{}$$

$$= \frac{a\,{}_2F_1\left(1,\frac{1+p}{2};\frac{3+p}{2};-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{b\cos^2(e+fx)^{\frac{1+p}{2}}\,{}_2F}{}$$

Mathematica [C] time = 8.26249, size = 849, normalized size = 6.58

$$\frac{}{f\left(-16p\cos\left(\frac{1}{2}(e+fx)\right)\csc^3(e+fx)\sec(e+fx)\left(a(p+2)F_1\left(\frac{p+1}{2};p,1;\frac{p+3}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2b\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

```
[Out] (2*(a + b*Sin[e + f*x])*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 + p)/2, p,
1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*Appell
F1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e
+ f*x)/2])*(g*Tan[e + f*x])^p)/(f*(Sec[(e + f*x)/2]^2*(a*(2 + p)*AppellF1[(
1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(
1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/
2]^2]*Tan[(e + f*x)/2]) - 16*p*Cos[(e + f*x)/2]*Csc[e + f*x]^3*Sec[e + f*x]
*Sin[(e + f*x)/2]^5*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1[1 + p/2, p, 2, 2 +
 p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]) + 2*p*Csc[
e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1,
 (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1
[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e +
f*x)/2]) + 2*(1 + p)*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(b*AppellF1[1 + p/
2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (a*(2 + p)*(-A
ppellF1[(3 + p)/2, p, 2, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + p*AppellF1[(3 + p)/2, 1 + p, 1, (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
+ f*x)/2]^2])*Tan[(e + f*x)/2])/(3 + p) + (2*b*(2 + p)*(-2*AppellF1[2 + p/2
, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + p*AppellF1[2 +
p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e +
f*x)/2]^2)/(4 + p))))
```

Maple [F] time = 0.904, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(g\tan\left(e+fx\right)\right)^{p}\left(a+b\sin\left(e+fx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))**p,x)
```

```
[Out] Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sin\left(fx+e\right)+a\right)\left(g\tan\left(fx+e\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

3.206 $\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$

Optimal. Leaf size=284

$$\frac{b\cos(e+fx)\sin^2(e+fx)^{-p/2}(g\tan(e+fx))^pF_1\left(\frac{1-p}{2};-\frac{p}{2},1;\frac{3-p}{2};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{f(p-1)\left(b^2-a^2\right)}+\frac{ag\sin^2(e+fx)^{\frac{1-p}{2}}(g\tan(e}{}$$

```
[Out] (a*g*(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))^((-1 + p)/2)*Hypergeometric2F1
[(1 - p)/2, (1 - p)/2, (3 - p)/2, (Cos[e + f*x]^2 - (b^2*Cos[e + f*x]^2)/(-
a^2 + b^2))/(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))]*(Sin[e + f*x]^2)^((1 -
 p)/2)*(g*Tan[e + f*x])^(-1 + p))/((a^2 - b^2)*f*(-1 + p)) + (b*AppellF1[(1
 - p)/2, -p/2, 1, (3 - p)/2, Cos[e + f*x]^2, (b^2*Cos[e + f*x]^2)/(-a^2 + b
^2)]*Cos[e + f*x]*(g*Tan[e + f*x])^p)/((-a^2 + b^2)*f*(-1 + p)*(Sin[e + f*x
]^2)^(p/2))
```

Rubi [F] time = 0.0480098, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]
```

```
[Out] Defer[Int][(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]), x]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx=\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$$

Mathematica [B] time = 13.4983, size = 864, normalized size = 3.04

$$\frac{}{a^2bf(p+1)(p+2)(a+b\sin(e+fx))\left(\frac{\sec^2(e+fx)\left(\left(a^2-b^2\right)(p+1)F_1\left(\frac{p+2}{2};-\frac{1}{2},1;\frac{p+4}{2};-\tan^2(e+fx),\frac{\left(b^2-a^2\right)\tan^2(e+fx)}{a^2}\right)\tan(e+fx)+a\left(b(p+2)\,{}_2F_1\left(1,\right.\right.\right)}{a^2b(p+2)}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]
```

```
[Out] (Tan[e + f*x]^(1 + p)*(g*Tan[e + f*x])^p*((a^2 - b^2)*(1 + p)*AppellF1[(2 +
 p)/2, -1/2, 1, (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a
^2]*Tan[e + f*x] + a*(b*(2 + p)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2,
(-1 + b^2/a^2)*Tan[e + f*x]^2] - a*(1 + p)*Hypergeometric2F1[1/2, 1 + p/2,
2 + p/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*f*(1 + p)*(2 + p)*(a + b*S
in[e + f*x])*((Sec[e + f*x]^2*Tan[e + f*x]^p*((a^2 - b^2)*(1 + p)*AppellF1[
(2 + p)/2, -1/2, 1, (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^
2)/a^2]*Tan[e + f*x] + a*(b*(2 + p)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)
/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - a*(1 + p)*Hypergeometric2F1[1/2, 1 + p
/2, 2 + p/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*(2 + p)) + (Tan[e + f*
x]^(1 + p)*((a^2 - b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 1, (4 + p)/2, -Ta
n[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2 + (a^2 - b^
2)*(1 + p)*Tan[e + f*x]*((2*(-a^2 + b^2)*(2 + p)*AppellF1[1 + (2 + p)/2, -1
/2, 2, 1 + (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*S
ec[e + f*x]^2*Tan[e + f*x])/(a^2*(4 + p)) + ((2 + p)*AppellF1[1 + (2 + p)/2
, 1/2, 1, 1 + (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2
]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + p)) + a*(-(a*(1 + p)*Hypergeometric2F1[
1/2, 1 + p/2, 2 + p/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2) - 2*a*(1 + p/2)*(1
+ p)*Sec[e + f*x]^2*(-Hypergeometric2F1[1/2, 1 + p/2, 2 + p/2, -Tan[e + f*x
]^2] + 1/Sqrt[1 + Tan[e + f*x]^2]) + b*(1 + p)*(2 + p)*Csc[e + f*x]*Sec[e +
 f*x]*(-Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f
*x]^2] + (1 - (-1 + b^2/a^2)*Tan[e + f*x]^2)^(-1)))))/(a^2*b*(1 + p)*(2 + p
))))
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{a+b\sin\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)
```

```
[Out] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(g\tan\left(fx+e\right)\right)^{p}}{b\sin\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(e+fx\right)\right)^{p}}{a+b\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e)),x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{b\sin\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

3.207 $\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=737

$$-\frac{2ab\cos(e+fx)\sin^2(e+fx)^{-q/2}(g\tan(e+fx))^q F_1\left(\frac{1-q}{2};-\frac{q}{2},2;\frac{3-q}{2};\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{b^2-a^2}\right)}{f(q-1)\left(a^2-b^2\right)^2}+\frac{a^2\sin(e+fx)\cos(e+f}{}$$

```
[Out] (a^2*Cos[e + f*x]*(1 - Cos[e + f*x]^2)^((-1 + q)/2)*(1 - (b^2*Cos[e + f*x]^
2)/(-a^2 + b^2))^(-2 + (3 - q)/2 + (-1 + q)/2)*((2*(a^2 - b^2) + b^2*(1 + q
)*Cos[e + f*x]^2)*HurwitzLerchPhi[-((a^2*Cot[e + f*x]^2)/(a^2 - b^2)), 1, (
1 - q)/2] - b^2*(-1 + q)*Cos[e + f*x]^2*HurwitzLerchPhi[-((a^2*Cot[e + f*x]
^2)/(a^2 - b^2)), 1, (3 - q)/2])*Sin[e + f*x]*(Sin[e + f*x]^2)^((-1 - q)/2)
*(g*Tan[e + f*x])^q)/(2*(a^2 - b^2)^2*(-a^2 + b^2)*f) - (a^2*Cos[e + f*x]*(
1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))^((-1 + q)/2)*Hypergeometric2F1[(1 -
q)/2, (1 - q)/2, (3 - q)/2, (Cos[e + f*x]^2 - (b^2*Cos[e + f*x]^2)/(-a^2 +
b^2))/(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))]*Sin[e + f*x]*(Sin[e + f*x]^2
)^((-1 - q)/2)*(g*Tan[e + f*x])^q)/((a^2 - b^2)^2*f*(-1 + q)) + (b^2*Cos[e
+ f*x]*(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))^((-1 + q)/2)*Hypergeometric2
F1[(1 - q)/2, (1 - q)/2, (3 - q)/2, (Cos[e + f*x]^2 - (b^2*Cos[e + f*x]^2)/
(-a^2 + b^2))/(1 - (b^2*Cos[e + f*x]^2)/(-a^2 + b^2))]*Sin[e + f*x]*(Sin[e
+ f*x]^2)^((-1 - q)/2)*(g*Tan[e + f*x])^q)/((a^2 - b^2)^2*f*(-1 + q)) - (2*
a*b*AppellF1[(1 - q)/2, -q/2, 2, (3 - q)/2, Cos[e + f*x]^2, (b^2*Cos[e + f*
x]^2)/(-a^2 + b^2)]*Cos[e + f*x]*(g*Tan[e + f*x])^q)/((a^2 - b^2)^2*f*(-1 +
 q)*(Sin[e + f*x]^2)^(q/2))
```

Rubi [F] time = 0.045093, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] Defer[Int][(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$$

Mathematica [A] time = 14.2268, size = 908, normalized size = 1.23

$$a^3\left(a^2-b^2\right)f(p+1)(p+2)(a+b\sin(e+fx))^2\left(\frac{\sec^2(e+fx)\left(a(p+2)\left(\left(a^2+b^2\right){}_2F_1\left(1,\frac{p+1}{2};\frac{p+3}{2};\frac{\left(b^2-a^2\right)\tan^2(e+fx)}{a^2}\right)-2b^2\,{}_2F_1\left(2,\frac{p+1}{2};\frac{p+3}{2};\frac{\left(b^2-a^2\right)\text{ta}}{a^2}\right.\right.\right.}{a^3(}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] (Tan[e + f*x]^(1 + p)*(g*Tan[e + f*x])^p*(a*(2 + p)*((a^2 + b^2)*Hypergeome
tric2F1[1, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - 2*b^2
*Hypergeometric2F1[2, (1 + p)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a
^2]) + 2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Ta
n[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Tan[e + f*x]))/(a^3*(a^2 -
 b^2)*f*(1 + p)*(2 + p)*(a + b*Sin[e + f*x])^2*((Sec[e + f*x]^2*Tan[e + f*x
]^p*(a*(2 + p)*((a^2 + b^2)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, ((-a
^2 + b^2)*Tan[e + f*x]^2)/a^2] - 2*b^2*Hypergeometric2F1[2, (1 + p)/2, (3 +
 p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]) + 2*b*(-a^2 + b^2)*(1 + p)*Appel
lF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f
*x]^2)/a^2]*Tan[e + f*x]))/(a^3*(a^2 - b^2)*(2 + p)) + (Tan[e + f*x]^(1 + p
)*(2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e
+ f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*Sec[e + f*x]^2 + 2*b*(-a^2 + b
^2)*(1 + p)*Tan[e + f*x]*((4*(-a^2 + b^2)*(2 + p)*AppellF1[1 + (2 + p)/2, -
1/2, 3, 1 + (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2]*
Sec[e + f*x]^2*Tan[e + f*x])/(a^2*(4 + p)) + ((2 + p)*AppellF1[1 + (2 + p)/
2, 1/2, 2, 1 + (4 + p)/2, -Tan[e + f*x]^2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^
2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + p)) + a*(2 + p)*(-2*b^2*(1 + p)*Csc[e
+ f*x]*Sec[e + f*x]*(-Hypergeometric2F1[2, (1 + p)/2, (3 + p)/2, ((-a^2 + b
^2)*Tan[e + f*x]^2)/a^2] + (1 - ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2)^(-2)) +
(a^2 + b^2)*(1 + p)*Csc[e + f*x]*Sec[e + f*x]*(-Hypergeometric2F1[1, (1 + p
)/2, (3 + p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] + (1 - ((-a^2 + b^2)*Tan
```

```
[e + f*x]^2)/a^2)^(-1))))))/(a^3*(a^2 - b^2)*(1 + p)*(2 + p))))
```

Maple [F] time = 0.677, size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(a+b\sin\left(fx+e\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)

[Out] int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(g\tan\left(fx+e\right)\right)^{p}}{b^{2}\cos\left(fx+e\right)^{2}-2\,ab\sin\left(fx+e\right)-a^{2}-b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="fricas")

[Out] integral(-(g*tan(f*x + e))^p/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2
 - b^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(e+fx\right)\right)^{p}}{\left(a+b\sin\left(e+fx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e))**2,x)
```

```
[Out] Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(g\tan\left(fx+e\right)\right)^{p}}{\left(b\sin\left(fx+e\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)
```

3.208 $\int (a+b\sin(e+fx))^m (g\tan(e+fx))^p\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left((g\tan(e+fx))^p(a+b\sin(e+fx))^m,x\right)$$

```
[Out] Unintegrable[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

Rubi [A] time = 0.0403311, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

Rubi steps

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

Mathematica [A] time = 2.69509, size = 0, normalized size = 0.

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

```
[Out] Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

Maple [A] time = 0.862, size = 0, normalized size = 0.

$$\int \left(a+b\sin\left(fx+e\right)\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

```
[Out] int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sin\left(fx+e\right)+a\right)^{m}\left(g\tan\left(fx+e\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))**m*(g*tan(f*x+e))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sin(fx+e)+a)^m (g\tan(fx+e))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```